力学的最初问题

金晓峰
复旦大学物理系
xfjin@fudan.edu.cn

1. 缘起
2. 亚里士多德
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4. 对当下教学的启示

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1. 缘起

George Gamow

It is very difficult to trace the origin of the science of physics, just as it is difficult to trace the origin of many great rivers…

The springs which gave birth to the great river of physical science were scattered all over the surface of the earth inhabited by *Homo sapiens*, i.e., thinking man. It seems, however, that most of them were concentrated on the southern tip of the Balkan peninsula inhabited by the people now known as "ancient Greeks".

W. Windelband

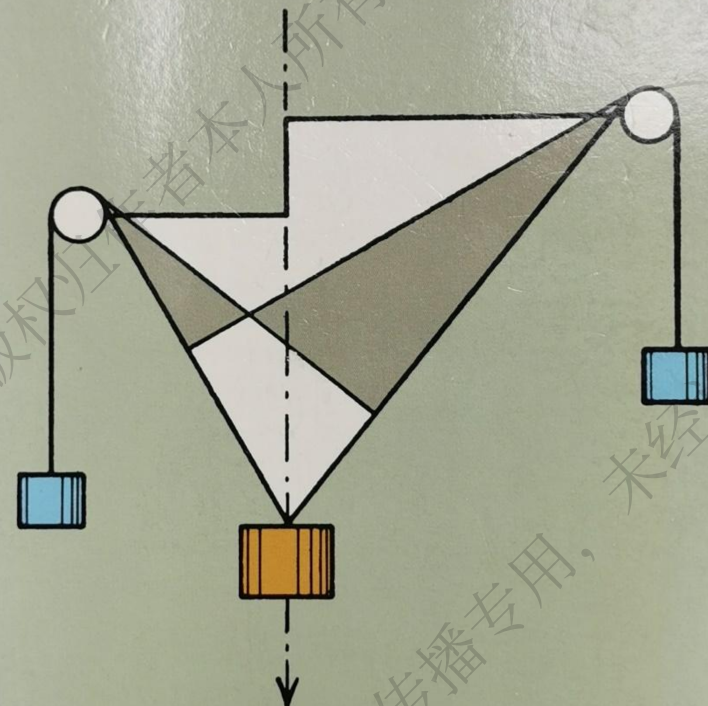
希腊人把简单的认知提升到了系统知识或“科学”的层次。希腊人不满足于实践经验的积累，也不满足于因宗教需要而产生的玄想，他们开始为了科学本身的缘故而寻求科学。像技术一样，科学作为一种独立事业从其他文化活动中分离出来。

物理：不仅仅是技巧，更是文化！

不仅仅教技巧，还要了解语境！

“仁”、“义”

A HISTORY OF MECHANICS



René Dugas

2. 亚里士多德

For lack of more ancient records, history of mechanics starts with Aristotle (384-322 B. C.) or, more accurately, with the author of the probably apocryphal treatise called *Problems of Mechanics (Μηχανικὰ προβλήματα)*. This is, in fact, a text-book of practical mechanics devoted to the study of simple machines.

In this treatise the *power* of the agency that sets a body in motion is defined as the product of the weight or the mass of the body—the Ancients always confused these concepts—and the velocity of the motion which the body acquires. This law makes it possible to formulate the condition of equilibrium of a straight lever with two unequal arms which carry unequal weights at their ends. Indeed, when the lever rotates the velocities of the weights will be proportional to the lengths of their supporting arms, for in these circumstances the powers of the two opposing powers cancel each other out.

To Aristotle himself, just as much in his *Treatise on the Heavens*, (*Περὶ οὐρανοῦ*) as in his *Physics*, concepts belonging to mechanics were not differentiated from concepts having a more general significance. Thus the notion of movement included both changes of position and changes of kind, of physical or chemical state. Aristotle's law of powers, which he called δύναμις or ἰσχὺς, is formulated in Chapter V of Book VII of his Physics in the following way.

"Let the motive agency be α, the moving body β, the distance travelled γ and the time taken by the displacement be δ. Then an equal power, namely the power α, will move half of β along a path twice γ in the same time, or it will move it through the distance γ in half the time δ. For in this way the proportions will be maintained."

Aristotle imposed a simple restriction on the application of this rule—a small power should not be able to move too heavy a body, "for then one man alone would be sufficient to set a ship in motion."

F, m, d, t

This same law of powers reappears in Book III of the *Treatise on the Heavens*. Its application to statics may be regarded as the origin of the *principle of virtual velocities* which will be encountered much later.

In another place Aristotle made a distinction between *natural motions* and *violent motions*.

The fall of heavy bodies, for example, is a natural motion, while the motion of a projectile is a violent one.

To each thing corresponds a natural place. In this place its substantial form achieves perfection—it is disposed in such a way that it is subject as completely as possible to influences which are favourable, and so that it avoids those which are inimical. If something is moved from its natural place it tends to return there, for everything tends to perfection. If it already occupies its natural place it remains there at rest and can only be torn away by violence.

In a precise way, for Aristotle, the position of a body is the internal surface of the bodies which surround it. To his most faithful commentators, the natural place of the earth is the concave surface which defines the bottom of the sea, joined in part to the lower surface of the atmosphere, the natural place of the air.[1]

Concerning the natural motion of falling bodies, Aristotle maintained in Book I of the *Treatise on the Heavens* that the " relation which weights have to each other is reproduced inversely in their durations of fall. If a weight falls from a certain height in so much time, a weight which is twice as great will fall from the same height in half the time. "

In his *Physics* (Part V), Aristotle remarked on the acceleration of falling heavy bodies. A body is attracted towards its natural place by means of its heaviness. The closer the body comes to the ground, the more that property increases.

If the natural place of heavy bodies is the centre of the World, the natural place of light bodies is the region contiguous with the Sphere of the Moon. Heavenly bodies are not subject to the laws applicable to terrestrial ones—every star is a body as it were divine, moved by its own divinity.

We return to terrestrial mechanics. All *violent* motion is essentially impermanent. This is one of the axioms which the Schoolmen were to repeat—*Nullum violentum potest esse perpetuum*. Once a projectile is thrown, the motive agency which assures the continuity of the motion resides in the air which has been set in motion. Aristotle then assumes that, in contrast to solid bodies, air spontaneously preserves the impulsion which it receives when the projectile is thrown, and that it can in consequence act as the motive agency during the projectile's flight.

This opinion may seem all the more paradoxical in view of the fact that Aristotle remarked, elsewhere, on the resistance of the medium. This resistance increases in direct proportion to the density of the medium. "If air is twice as tenuous as water, the same moving body will spend twice as much time in travelling a certain path in water as in travelling the same path in air."

Aristotle also concerned himself with the *composition of motions*. "Let a moving body be simultaneously actuated by two motions that are such that the distances travelled in the same time are in a constant proportion. Then it will move along the diagonal of a parallelogram which has as sides two lines whose lengths are in this constant relation to each other." On the other hand, if the ratio between the two component distances travelled by the moving body in the same time varies from one instant to another, the body cannot have a rectilinear motion. "In such a way a curved path is generated when the moving body is animated by two motions whose proportion does not remain constant from one instant to another."

These propositions relate to what we now call kinematics. But Aristotle immediately inferred from them dynamical results concerning the composition of forces. The connection between the two disciplines is not given, but as Duhem has indicated, it is easily supplied by making use of the law of powers—a fundamental principle of aristotelian dynamics. In particular, let us consider a heavy moving body describing some curve in a vertical plane. It is clear that the body is actuated by two motions simultaneously. Of these, one produces a vertical descent while the other, according to the position of the body on its trajectory, results in an increase or a decrease of the distance from the centre. In Aristotle's sense, the body will have a natural falling motion due to gravity, and will be carried horizontally in a violent motion. Consider different moving bodies unequally distant from the centre of a circle and on the same radius. Let this radius, in falling, rotate about the centre. Then it may be inferred that for each body the relation of the natural to the violent motion remains the same. "The contemplation of this equality held Aristotle's attention for a long time. He appears to have seen in it a somewhat mysterious correlation with the law of the equilibrium of levers." [1]

Aristotle believed in the *impossibility of a vacuum* (*Physics*, Book IV, Chapter XI) on the grounds that, in a vacuum, no natural motion, that is to say no tendency towards a natural place, would be possible. Incidentally this idea led him to formulate a principle analogous to that of inertia, and to justify this in the same way as that used by the great physicists of the XVIIIth Century.

" It is impossible to say why a body that has been set in motion in a vacuum should ever come to rest ; why, indeed, it should come to rest at one place rather than at another. As a consequence, it will either necessarily stay at rest or, if in motion, will move indefinitely unless some obstacle comes into collision with it. "

Aristotle's ideas on gravitation and the figure of the Earth merit our attention, if only because of the influence which they have had on the development of the principles of mechanics. First we shall quote from the *Treatise on the Heavens* (Book II, Chapter XIV). " Since the centres of the Universe and of the Earth coincide, one should ask oneself towards which of these heavy bodies and even the parts of the Earth are attracted. Are they attracted towards this point because it is the centre of the Universe or because it is the centre of the Earth ? It is the centre of the Universe towards which they must be attracted. . . . Consequently heavy bodies are attracted towards the centre of the Earth, but only fortuitously, because this centre is at the centre of the Universe. "

3. 阿基米德

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surface and the spigot. Why? Because air pressure can not support a column of water more than 10 m high.

13–7 Buoyancy and Archimedes' Principle

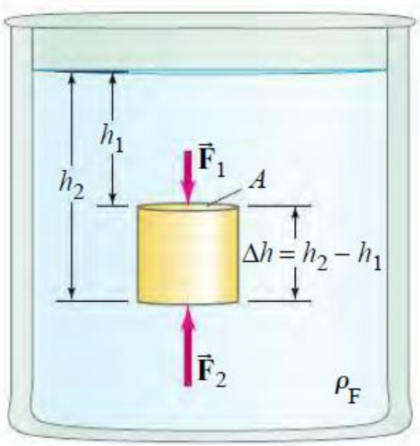
Objects submerged in a fluid appear to weigh less than they do when outside the fluid. For example, a large rock that you would have difficulty lifting off the ground can often be easily lifted from the bottom of a stream. When the rock breaks through the surface of the water, it suddenly seems to be much heavier. Many objects, such as wood, float on the surface of water. These are two examples of *buoyancy*. In each example, the force of gravity is acting downward. But in addition, an upward *buoyant force* is exerted by the liquid. The buoyant force on fish and underwater divers (as in the Chapter-Opening photo) almost exactly balances the force of gravity downward, and allows them to "hover" in equilibrium.

The buoyant force occurs because the pressure in a fluid increases with depth. Thus the upward pressure on the bottom surface of a submerged object is greater than the downward pressure on its top surface. To see this effect, consider a cylinder of height Δh whose top and bottom ends have an area A and which is completely submerged in a fluid of density ρ_F, as shown in Fig. 13–13. The fluid exerts a pressure $P_1 = \rho_F g h_1$ at the top surface of the cylinder (Eq. 13–3). The force due to this pressure on top of the cylinder is $F_1 = P_1 A = \rho_F g h_1 A$, and it is directed downward. Similarly, the fluid exerts an upward force on the bottom of the cylinder equal to $F_2 = P_2 A = \rho_F g h_2 A$. The net force on the cylinder exerted by the fluid pressure, which is the **buoyant force**, $\vec{\mathbf{F}}_B$, acts upward and has the magnitude

$$\begin{aligned} F_B = F_2 - F_1 &= \rho_F g A (h_2 - h_1) \\ &= \rho_F g A \, \Delta h \\ &= \rho_F V g \\ &= m_F g, \end{aligned}$$

where $V = A \, \Delta h$ is the volume of the cylinder, the product $\rho_F V$ is the mass of the fluid displaced, and $\rho_F V g = m_F g$ is the weight of fluid which takes up a volume equal to the volume of the cylinder. Thus the buoyant force on the cylinder is equal to the weight of fluid displaced by the cylinder.

FIGURE 13–13 Determination of the buoyant force.



This result is valid no matter what the shape of the object. Its discovery is credited to Archimedes (287?–212 B.C.), and it is called **Archimedes' principle**: *the buoyant force on an object immersed in a fluid is equal to the weight of the fluid displaced by that object.*

By "fluid displaced," we mean a volume of fluid equal to the submerged volume of the object (or that part of the object that is submerged). If the object is placed in a glass or tub initially filled to the brim with water, the water that flows over the top represents the water displaced by the object.

We can derive Archimedes' principle in general by the following simple but elegant argument. The irregularly shaped object D shown in Fig. 13–14a is acted on by the force of gravity (its weight, $m\vec{\mathbf{g}}$, downward) and the buoyant force, $\vec{\mathbf{F}}_B$, upward. We wish to determine F_B. To do so, we next consider a body (D′ in Fig. 13–14b), this time made of the fluid itself, with the same shape and size as the original object, and located at the same depth. You might think of this body of fluid as being separated from the rest of the fluid by an imaginary membrane. The buoyant force F_B on this body of fluid will be exactly the same as that on the original object since the surrounding fluid, which exerts F_B, is in exactly the same configuration. This body of fluid D′ is in equilibrium (the fluid as a whole is at rest). Therefore, $F_B = m'g$, where $m'g$ is the weight of the body of fluid. Hence the buoyant force F_B is equal to the weight of the body of fluid whose volume equals the volume of the original submerged object, which is Archimedes' principle.

Archimedes' discovery was made by experiment. What we have done in the last two paragraphs is to show that Archimedes' principle can be derived from Newton's laws.

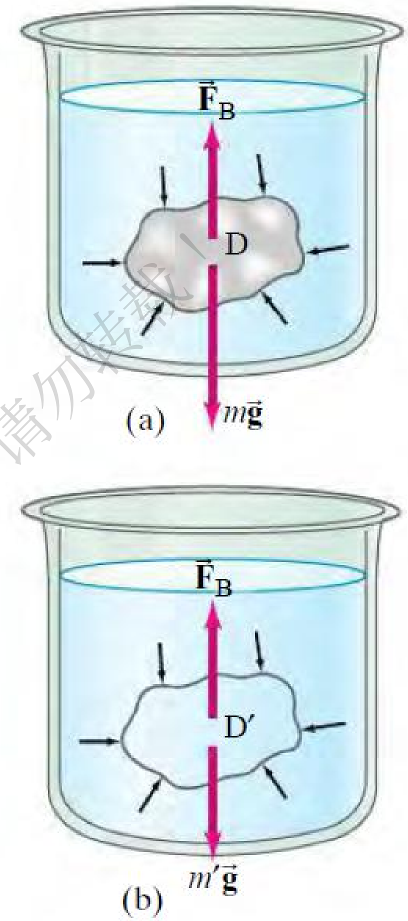


FIGURE 13–14
Archimedes' principle.

We shall now concern ourselves with Archimedes' treatise on *Floating Bodies*. The author starts from the following hypothesis—

"The nature of a fluid is such that if its parts are equivalently placed and continuous with each other, that which is the least compressed is driven along by that which is the more compressed. Each part of the fluid is compressed by the fluid which is above it in a vertical direction, whether the fluid is falling somewhere or whether it is driven from one place to another."

From this starting point, the following propositions derive in a logical sequence.

Proposition I. — If a surface is intersected by a plane which always passes through the same point and if the section is a circumference (of a circle) having this fixed point as its centre, the surface is that of a sphere.

Proposition II. — The surface of any fluid at rest is spherical and the centre of this surface is the same as the centre of the Earth.

This result had already, as we have seen, been enunciated by Aristotle.

Proposition III. — If a body whose weight is equal to that of the same volume of a fluid (α) is immersed in that fluid, it will sink until no part of it remains above the surface, but will not descend further.

We shall reproduce the proof of this proposition, which is the origin of Archimedes' Principle.

" Let a body have the same heaviness as a liquid. If this is possible, suppose that the body is placed in the fluid with part of it above the surface. Let the fluid be at rest. Suppose that a plane which passes through the centre of the Earth intersects the fluid and the

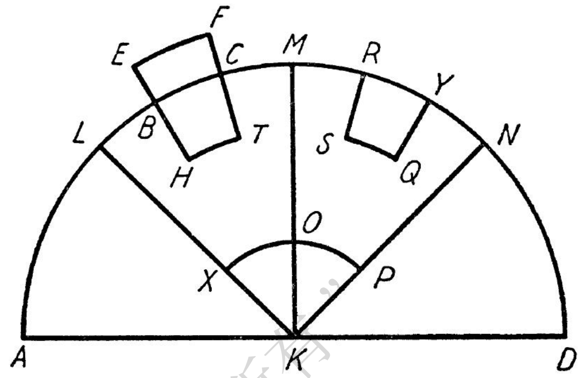


Fig. 3

body immersed in it in such a way that the section of the fluid is *ABCD* and the section of the body is *EHTF*. Let *K* be the centre of the Earth, *BHTC* be the part of the body which is immersed in the fluid and *BEFC* the part which projects out of it. Construct a pyramid whose base is a parallelogram in the surface of the fluid (α) and whose apex is the centre of the Earth. Let the intersection of the faces of the pyramid by the plane containing the arc *ABCD* be *KL* and *KM*. In the fluid, and below *EFTH* draw another spherical surface *XOP* having the point *K* as its centre, in such a way that *XOP* is the section of the surface by the plane containing the arc *ABCD*. Take another pyramid equal to the first, with which it is contiguous and continuous, and such that the sections of its faces are *KM* and *KN*. Suppose that there is, in the fluid, another solid *RSQY* which is made of the fluid and is equal and similar to *BHTC*, that part of the body

EHTF which is immersed in the fluid. The portions of the fluid which are contained by the surface *XO* in the first pyramid and the surface *OP* in the second pyramid are equally placed and continuous with each other. But they are not equally compressed. For the portions of the fluid contained in *XO* are compressed by the body *EHTF* and also by the fluid contained by the surfaces *LM*, *XO* and those of the pyramid. The parts contained in *PO* are compressed by the solid *RSQY* and by the fluid contained by the surfaces *OP*, *MN* and those of the pyramid. But the weight of the fluid contained between *MN* and *OP* is less than the combined heaviness of the fluid between *LM* and *XO* and the solid. For the solid *RSQY* is smaller than the solid *EHTF*—*RSQY* is equal to *BHTC*—and it has been assumed that the body immersed has, volume for volume, the same heaviness as the fluid. If therefore one takes away the parts which are equal to each other, the remainder will be unequal. Consequently it is clear that the part of the fluid contained in the surface *OP* will be driven along by the part of the fluid contained in the surface *XO*, and that the fluid will not remain at rest. Therefore, no part of the body which has been immersed will remain above the surface. However, the body will not fall further. For the body has the same heaviness as the fluid and the equivalently placed parts of the fluid compress it similarily."

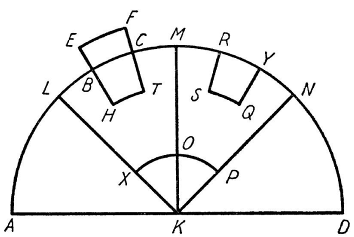


Fig. 3

Proposition IV. — If a body which is lighter than a fluid is placed in this fluid, a part of the body will remain above the surface.

(Proof analogous to that of Proposition III.)

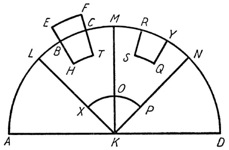


Fig. 3

Proposition V. — If a body which is lighter than a fluid is placed in the fluid, it will be immersed to such an extent that a volume of fluid which is equal to the volume of the part of the body immersed has the same weight as the whole body.

The diagram is the same as the preceding one (Proposition III).

" Let the liquid be at rest and the body *EHTF* be lighter than the fluid. If the fluid is at rest, parts which are equivalently placed will be similarly compressed. Then the fluid contained by each of the surfaces *XO* and *OP* is compressed by an equal weight. But, if the body *BHTC* is excluded, the weight of fluid in the first pyramid is equal, with the exclusion of the fluid *RSQY*, to the weight of fluid in the second pyramid. Therefore it is clear that the weight of the body *EHTF* is equal to the weight of the fluid *RSQY*. From which it follows that a volume of fluid equal to that of the body which is immersed has the same weight as the whole body."

Proposition VI. — If a body which is lighter than a fluid is totally and forcibly immersed in it, the body will be thrust upwards with a force equal to the difference between its weight and that of an equal volume of fluid.

Proposition VII. — If a body is placed in a fluid which is lighter than itself, it will fall to the bottom. In the fluid the body will be lighter by an amount which is the weight of the fluid which has the same volume as the body itself.

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“名师出高徒”

4. 对当下教学的启示

A Einstein

“We now know that science cannot grow out of empiricism alone, that in the constructions of science we need to use free invention which only *a posteriori* can be confronted with experience as to its usefulness. This fact could elude earlier generations, to whom theoretical creation seemed to grow inductively out of empiricism without the creative influence of a free construction of concepts. The more primitive the status of science is the more readily can the scientist live under the illusion that he is a pure empiricist.

Thank you