$$C_{AB}(n) = \int_0^\infty d\tau \, e^{i\pi n\tau/\beta} C_{AB}(\tau) \quad \begin{cases} n \text{ even}; & \text{boson} \\ n \text{ odd} & \text{fermion} \end{cases}$$

Lesson 9

$$\underset{\Sigma}{C_{AB}(i\omega_n)} = \int_0^\infty d\tau \, e^{i\omega_n\tau} C_{AB}(\tau)$$

$$\omega_n \equiv \frac{n\pi}{\beta} = \begin{cases} \frac{2k\pi}{\beta} & \text{boson} \\ \frac{(2k+1)\pi}{\beta} & \text{fermion} \end{cases}$$

$$\boxed{\beta = \frac{1}{k_B T}}$$

4.85

Matsubara Green's function $\longleftrightarrow$ retarded Green's function

Lehmann representation:

$$C^R_{AB}(\omega) = \frac{1}{Z}\sum_{nn'} \frac{\langle n|A|n'\rangle\langle n'|B|n\rangle}{\omega + E_n - E_{n'} + i\eta} \cdot \left[e^{-\beta E_n} - (\pm) e^{-\beta E_{n'}}\right]$$

$+$ Boson

$-$ Fermion

Matasubara Green's function: $\tau \in [0, \beta]$

$$C_{AB}(\tau) = \frac{-1}{Z} \mathrm{Tr}\left[e^{-\beta H} e^{\tau H} A e^{-\tau H} B\right]$$

$$= \frac{-1}{Z}\sum_{nn'} e^{-\beta E_n} \langle n|A|n'\rangle\langle n'|B|n\rangle e^{\tau(E_n - E_{n'})}$$

$$C_{AB}(i\omega_n) = \int_0^\beta d\tau \, e^{i\omega_n\tau} \frac{-1}{Z}\sum_{nn'} e^{-\beta E_n}\langle n|A|n'\rangle\langle n'|B|n\rangle e^{\tau(E_n - E_{n'})}$$

$$= \frac{-1}{Z} \sum_{nn'} e^{-\beta E_n} \frac{\langle n|A|n'\rangle\langle n'|B|n\rangle}{i\omega_n + E_n - E_{n'}} \left[e^{i\omega_n\beta} e^{\beta(E_n - E_{n'})} - 1 \right]$$

Boson $\omega_n \beta = 2k\pi$

Fermion $\omega_n \beta = (2k+1)\pi$

$$= \frac{-1}{Z} \sum_{nn'} e^{-\beta E_n} \frac{\langle n|A|n'\rangle\langle n'|B|n\rangle}{i\omega_n + E_n - E_{n'}} \left[\pm e^{\beta(E_n - E_{n'})} - 1 \right]$$

Boson

$$= \frac{1}{Z} \sum_{nn'} \frac{\langle n|A|n'\rangle\langle n'|B|n\rangle}{i\omega_n + E_n - E_{n'}} \left[e^{-\beta E_n} - (\pm) e^{-\beta E_{n'}} \right]$$

Fermion

(4.86)

$$C_{AB}(\tilde{z}) = \frac{1}{Z} \sum_{nn'} \frac{\langle n|A|n'\rangle\langle n'|B|n\rangle}{\tilde{z} + E_n - E_{n'}} \left[e^{-\beta E_n} - (\pm) e^{-\beta E_{n'}} \right]$$

$$C^R_{AB}(\omega) = C_{AB}(i\omega_n \to \omega + i\eta) \quad (4.87)$$

$$C^A_{AB}(\omega) = C_{AB}(i\omega_n \to \omega - i\eta) \quad (4.88)$$

Single particle Matsubara Green's function.

$$\mathcal{G}(\underbrace{r\sigma}_{\nu}\tau, \underbrace{r'\sigma'}_{\nu'}\tau') = -\langle T_\tau (\underbrace{\Psi_\sigma(r\tau)}_{c_\nu(\tau)} \underbrace{\Psi^+_{\sigma'}(r'\tau')}_{c^+_{\nu'}(\tau')}) \rangle$$

$$H_0 = \sum_\nu \xi_\nu c^+_\nu c_\nu$$

$$c_\nu(\tau) = e^{\tau H_0} c_\nu e^{-\tau H_0} = e^{-\xi_\nu \tau} c_\nu$$

$$C_\nu^+(\tau) = e^{\tau H_0} C_\nu e^{-\tau H_0} = e^{\xi_\nu \tau} C_\nu^+ \qquad 4.89$$

$$\mathcal{G}_0(\nu, \tau-\tau') = -\langle T_\tau \left(C_\nu(\tau)\, C_\nu^+(\tau') \right) \rangle$$

$$= -\left[\theta(\tau-\tau') \underline{\langle C_\nu C_\nu^+ \rangle} \pm \theta(\tau'-\tau) \langle C_\nu^+ C_\nu \rangle \right] e^{-\xi_\nu(\tau-\tau')}$$

$$\mathcal{G}_{0,F}(\nu, \underline{\tau-\tau'}) = -\left[\theta(\tau-\tau') \left[1 - n_F(\xi_\nu) \right] - \theta(\tau'-\tau)\, n_F(\xi_\nu) \right] \cdot e^{-\xi_\nu(\tau-\tau')} \qquad 4.90$$

$$C_\nu C_\nu^+ - C_\nu^+ C_\nu = 1$$

$$\nearrow \langle C_\nu C_\nu^+ \rangle = 1 + \langle C_\nu^+ C_\nu \rangle = 1 + n_B(\xi_\nu)$$

$$\mathcal{G}_{0,F}(\nu, i\omega_n) = \underline{\int_0^\beta d\tau\, e^{i\omega_n \tau}}\, \mathcal{G}_{0,F}(\nu, \tau) \qquad \underline{\omega_n = \frac{(2n+1)\pi}{\beta}}$$

$$= -\left[1 - n_F(\xi_\nu) \right] \int_0^\beta d\tau\, e^{i\omega_n \tau} e^{-\xi_\nu \tau}$$

$$= -\left[\underline{1 - n_F(\xi_\nu)} \right] \cdot \frac{1}{i\omega_n - \xi_\nu} \underline{\left[\overset{-1}{\underline{e^{i\omega_n \beta}}} e^{-\xi_\nu \beta} - 1 \right]}$$

$$= \frac{1}{i\omega_n - \xi_\nu} \qquad 4.91$$

$$\mathcal{G}_{0B}(\nu, \tau-\tau') = -\left[\theta(\tau-\tau') \left(1 + n_B(\xi_\nu) \right) + \theta(\tau'-\tau)\, n_B(\xi_\nu) \right] \cdot e^{-\xi_\nu(\tau-\tau')}$$

$$\mathcal{G}_{0,B}(\nu, i\omega_n) = \int_0^\beta d\tau\, e^{i\omega_n \tau}\, \mathcal{G}_{0B}(\nu, \tau) \qquad \omega_n \beta = 2n\pi$$

$$= \underline{-\left[1 + \underline{n_B(\xi_\nu)} \right]} \frac{1}{i\omega_n - \xi_\nu} \underline{\left[\underset{1}{\underline{e^{i\omega_n \beta}}} e^{-\xi_\nu \beta} - 1 \right]}$$

$$= \frac{1}{i\omega_n - \xi_\nu} \qquad 4.92$$

fermion / Boson.

$$G_0^R(\nu,\omega) = \frac{1}{\omega - \xi_\nu + i\eta}$$

Evaluation of Matsubara Sum

$$S_1(\nu,\tau) = \frac{1}{\beta}\sum_{i\omega_n} \underline{g(\nu, i\omega_n)} e^{i\omega_n\tau}, \quad \underline{4.93}$$

$\tau \in (0,\beta)$

$$S_2(\nu_1, \nu_2; i\omega_n, \tau) = \frac{1}{\beta}\sum_{ik_n} \underline{g_0(\nu_1, ik_n)\, g_0(\nu_2, ik_n + i\omega_n)} e^{ik_n\tau}, \quad \underline{4.94}$$

$\tau \in (0,\beta)$

general forms:

$$S^F(\tau) = \frac{1}{\beta}\sum_{ik_n} g(ik_n)\, e^{ik_n\tau}; \quad 0<\tau<\beta$$

$$S^B(\tau) = \frac{1}{\beta}\sum_{i\omega_n} g(i\omega_n)\, e^{i\omega_n\tau}; \quad \underline{4.95}$$

Fermi distribution:

$$n_F(z) = \frac{1}{e^{\beta z}+1}; \quad \text{Poles } z = i\frac{(2n+1)\pi}{\beta}$$

$\underline{4.96}$

$$n_B(z) = \frac{1}{e^{\beta z}-1}; \quad \text{poles } z = \frac{i2n\pi}{\beta}$$

if $\underline{g(z) \text{ has no singularity}}$ in the contour; $n_F(z)$ singularity ik_n

$g(z)$ no singularity, $n_B(z)$ singularity $i\omega_n$;

$$\oint dz\, n_F(z)\, g(z) = 2\pi i \operatorname*{Res}_{z=ik_n}[n_F(z)\, g(z)] = -\frac{2\pi i}{\beta} g(ik_n) \quad 4.97$$

(4.98)

$$\oint dz\, n_B(z)\, g(z) = 2\pi i \operatorname*{Res}_{z=i\omega_n}[n_B(z)\, g(z)] = \frac{2\pi i}{\beta} g(i\omega_n)$$

$$S^F(\tau) = \frac{1}{\beta}\sum_{ik_n} g(ik_n)\, e^{ik_n\tau}; \qquad g(z)\, e^{\tau z}$$

$$S^B(\tau) = \frac{1}{\beta}\sum_{i\omega_n} g(i\omega_n)\, e^{i\omega_n\tau};$$

Contour $g(z)$ no singularity.

We can obtain:

$$S^F(\tau) = \frac{1}{\beta}\sum_{ik_n} g(ik_n)\, e^{ik_n\tau} = -\oint \frac{dz}{2\pi i}\, n_F(z)\, g(z)\, e^{\tau z}$$

$$S^B(\tau) = \frac{1}{\beta}\sum_{i\omega_n} g(i\omega_n)\, e^{i\omega_n\tau} = \oint \frac{dz}{2\pi i}\, n_B(z)\, g(z)\, e^{\tau z}$$

(4.99)

Summation with poles of $g(z)$ in the contour.

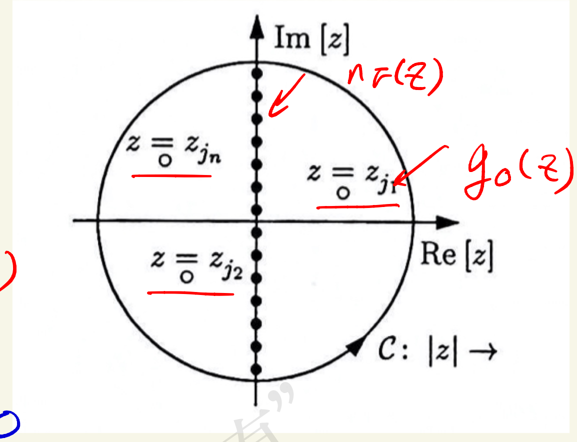
$$S_0^F(\tau) = \frac{1}{\beta}\sum_{ik_n} g_0(ik_n)\, e^{ik_n\tau}, \quad \tau \in (0,\beta)$$

$$g_0(z) = \prod_j \frac{1}{z - z_j} \quad 4.100$$

Poles $z = z_j$

$z = ik_n$

$$\int_{C_\infty} \frac{dz}{2\pi i}\, n_F(z) \cdot g_0(z) \cdot e^{\tau z} = 0$$



$$n_F(z)\, e^{\tau z} = \frac{e^{\tau z}}{e^{\beta z}+1} \qquad \tau \in (0, \beta)$$

$$|\mathrm{Re}\, z| \to \infty \quad \begin{cases} \mathrm{Re}\, z > 0 \,; & e^{-(\beta-\tau)\mathrm{Re}\, z} \to 0 \\ \mathrm{Re}\, z < 0 \,; & e^{\tau \mathrm{Re}\, z} \to 0 \end{cases}$$

Fermion

$$0 = \int_{C_\infty} \frac{dz}{2\pi i}\, n_F(z)\, g_0(z)\, e^{\tau z}$$

$$= -\frac{1}{\beta} \sum_{ik_n} g_0(ik_n)\, e^{ik_n \tau} + \sum_j \operatorname*{Res}_{z=z_j} [g_0(z)]\, n_F(z_j)\, e^{z_j \tau}$$

$$\boxed{\begin{aligned} S_0^F(\tau) &= \frac{1}{\beta} \sum_{ik_n} g_0(ik_n)\, e^{ik_n \tau} \\ &= \sum_j \mathrm{Res}[g_0(z)]\, n_F(z_j)\, e^{z_j \tau} \end{aligned}}$$

☆ 4.101

Fermion.

$$\boxed{\begin{aligned} S_0^B(\tau) &= \frac{1}{\beta} \sum_{i\omega_n} g_0(i\omega_n) \cdot e^{i\omega_n \tau} \\ &= -\sum_j \operatorname*{Res}_{z=z_j} [g_0(z)]\, n_B(z_j)\, e^{z_j \tau} \end{aligned}}$$

Boson.

4.102

① Bruus and Flensberg;

Many-body Quantum Theory in Condensed Matter physics;

② G. Rickayzen; [superconductor] ☆

Green's function and condensed matter;

③ Akkermans and Montambaux; ☆

Mesoscopic physics of electrons and photons,

④ Richard D. Mattuck ☆

A Guide to Feynman Diagrams in the Many-body problem.

⑤ AGD

Wick Theorem:

n-particle Green's function.

$$\mathcal{G}_0^{(n)}(\nu_1\tau_1, \cdots, \nu_n\tau_n; \nu_1'\tau_1', \cdots, \nu_n'\tau_n')$$

non-interacting H_0

$$= (-1)^n \langle T_\tau [\hat{C}_{\nu}(\tau_1) \cdots \hat{C}_{\nu_n}(\tau_n) C^{+}_{\nu_n'}(\tau_n') \cdots C^{+}_{\nu_1'}(\tau_1')] \rangle_0$$

$$\mathcal{G}_0^{(n)}(1, \cdots, n; 1', \cdots, n') = \begin{vmatrix} \mathcal{G}_0(1,1') & \cdots & \mathcal{G}_0(1,n') \\ \vdots & & \vdots \\ \mathcal{G}_0(n,1') & \cdots & \mathcal{G}_0(n,n') \end{vmatrix}_{B,F}$$

Polarizability of free electrons

$$\hat{\rho}(q) = \sum_{k\sigma} c^{+}_{k\sigma} c_{k+q,\sigma}$$

$$\chi_0(\vec{q},\tau) = -\frac{1}{V} \sum_{kk'\sigma\sigma'} \langle T_\tau (\underbrace{c^{+}_{k\sigma}(\tau)\, c_{k+q\sigma}(\tau)}_{\hat{\rho}(q,\tau)}\, \underbrace{c^{+}_{k'\sigma'}\, c_{k'-q\,\sigma'}}_{\hat{\rho}(-q)}) \rangle_0$$

$q \neq 0$

$$\chi_0(\vec{q},\tau) = \frac{1}{V}\Big[\sum_{kk'\sigma\sigma'} \langle T_\tau\, c_{k'-q\sigma'}\, c^{+}_{k\sigma}(\tau) \rangle_0 \langle T_\tau\, c_{k+q\sigma}(\tau)\, c^{+}_{k'\sigma'} \rangle_0$$

4.105

$$- \underbrace{\langle \hat{\rho}(q) \rangle_0}_{=0} \underbrace{\langle \hat{\rho}(-q) \rangle_0}_{=0} \Big]$$

$q \neq 0$

$$= \frac{1}{V} \sum_{k\sigma} g_0(k+q,\sigma,\tau)\, g_0(k\sigma,-\tau)$$

$$\chi_0(\vec{q}, iq_n) = \frac{1}{\beta}\sum_{ik_n} \frac{1}{V} \sum_{k\sigma} g_0(k+q,\sigma, ik_n + iq_n) \cdot g_0(k\sigma, ik_n)$$

① ②

$$g_0(k\sigma, ik_n) = \frac{1}{ik_n - \xi_k}$$

$$g_0(k\sigma, z) = \frac{1}{z - \xi_k} \Rightarrow z_1 = \xi_k \quad \text{Pole}$$

$$g_0(k+q, z + iq_n) = \frac{1}{z + iq_n - \xi_{k+q}} \Rightarrow z_2 = \xi_{k+q} - iq_n \quad \text{Pole}$$

momentum   Imaginary frequency

$$\chi_0(\vec{q}, iq_n) = \frac{1}{V}\sum_{k\sigma} \Big\{ n_F(\xi_k)\, g_0(k+q,\sigma, \xi_k + iq_n) + n_F(\xi_{k+q} - iq_n)\, g_0(k\sigma, \xi_{k+q} - iq_n) \Big\}$$

$$= \frac{1}{V}\sum_{k\sigma} \frac{n_F(\xi_k) - n_F(\xi_{k+q})}{iq_n + \xi_k - \xi_{k+q}}$$

$n_F(\xi_{k+q})$

4.106

Summary of Matasubara Green's function.

$$\mathcal{G}_0(\nu, i\omega_n) = \frac{1}{i\omega_n - \xi_\nu}, \quad \text{free particle}$$

Matasubara Sum

$$S^{F/B}(\tau) = \frac{1}{\beta}\sum_{ik_n} g_0(ik_n)\, e^{ik_n\tau} = \pm\sum_j \operatorname{Res}[g_0(z_j)]\, n_{F/B}(z_j)\, e^{\tau_j z}$$

Wick Theorem

$$\mathcal{G}_0^{(n)}(1,\cdots,n;1',\cdots n') = \begin{vmatrix} g_0(1,1') & \cdots & g_0(1,n') \\ \vdots & & \vdots \\ g_0(n,1') & \cdots & g_0(n,n') \end{vmatrix}_{B,F}$$

$$i \equiv (\nu_i, \tau_i)$$

Feynman diagrams in disorder system.

$$H = H_0 + V = \sum_\sigma \int dr\, \Psi_\sigma^\dagger(r) H_0(r)\, \Psi_\sigma(r) \qquad \underline{4.107}$$

$$+\sum_\sigma \int dr\, \Psi_\sigma^\dagger(r)\, V_\sigma(r)\, \Psi_\sigma(r)$$

full green's function:

$$\mathcal{G}(b,a) = -\langle T_\tau\, \Psi(b)\, \Psi^\dagger(a)\rangle$$

$$1 \equiv (r_1, \sigma_1, \tau_1)$$

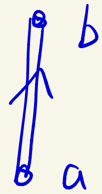
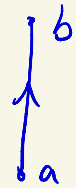
bare green's function

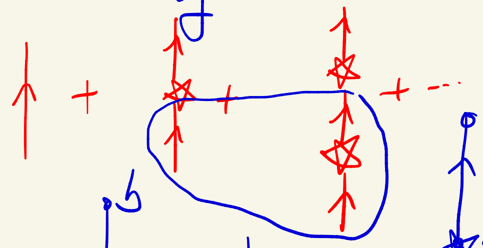
$$\mathcal{G}^0(b,a) = -\langle T_\tau\, \Psi(b)\, \Psi^\dagger(a)\rangle_0$$

$$\int d1 = \sum_{\sigma_1}\int dr_1 \int d\tau_1 \qquad \boxed{4.108}$$

Dyson equation:

$$\mathcal{G}(b,a) = \mathcal{G}^0(b,a) + \int d1\, \mathcal{G}(b,1)\, V(1)\, \mathcal{G}^0(1,a)$$

$g(b,a) =$  $\quad g^0(b,a) =$  $\quad \int d1\, V(1) = ☆\ 1$

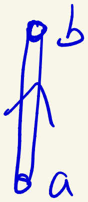
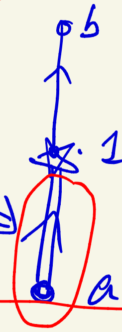


$0.\dot{8}\dot{8} = x$

$100x = 88.\dot{8}\dot{8} = 88 + x$

$x = \frac{88}{99}$

4.109

full  $=$ bare  $+$ 

Elastic Scattering, Matasubera Green's function.

$$g(r\tau, r'\tau') = \frac{1}{\beta}\sum_n g(r, r'; ik_n)\, e^{-ik_n(\tau-\tau')}$$

$$g(r, r'; ik_n) = \int_0^\beta d(\tau-\tau')\, g(r\tau, r'\tau')\, e^{ik_n(\tau-\tau')}$$

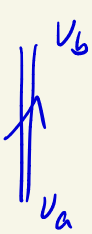
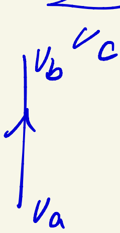
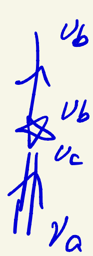
$$g(r_b, r_a; ik_n) = g^0(r_b, r_a; ik_n) + \int dr_1\, g^0(r_b, r_1; ik_n)\, V(1)\, g(r_1, r_a; ik_n)$$

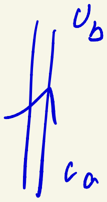
$\Downarrow$

general representation

$$g^0_{\nu\nu'}(ik_n) = \frac{1}{ik_n - \xi_\nu}\, \delta_{\nu\nu'}$$

$$g(\nu_b \nu_a; ik_n) = \delta_{\nu_b \nu_a}\, g^0(\nu_b \nu_a; ik_n) + \sum g^0(\nu_b \nu_b; ik_n)\, V_{\nu_b \nu_c}\, g(\nu_c \nu_a; ik_n) \quad 4.111$$

 $=$  $+$  $\quad$ 4.112

 $= g_{v_b v_a}$  $= g^0_{v_b v_a} = \dfrac{\delta_{v_b v_a}}{ik_n - \xi_{v_b}}$;  $V_{v_b v_c}$

4.113

$$V(\vec{r}) = \sum_{j=1}^{N_{imp}} u(\vec{r} - \vec{P}_j) \qquad P_j \text{ random}$$

Weak disorder $\qquad \dfrac{n_{imp}}{n_{el}} \ll 1$

ℓ mean-free path $\qquad d_{imp} \gg \lambda$

$k_F \ell \gg 1$

☆ strong disorder

↳ high order Green's function calculation

↳ numerics ; (transfer matrix)

Feynmann diagrams in Momentum Representation.

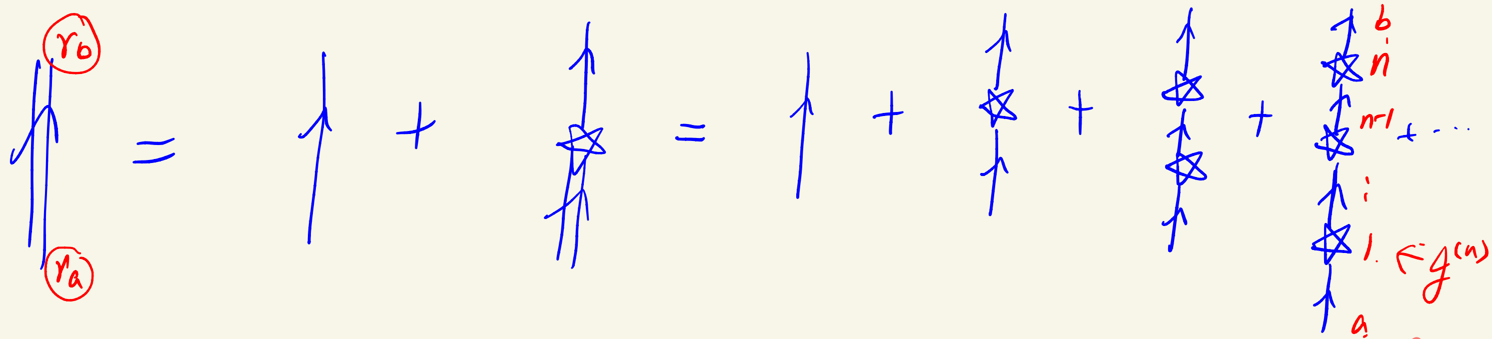
translational invariant.

$$g(r_b, r_a; ik_n) = g^0(r_b - r_a; ik_n) + \sum_{j=1}^{N_{imp}} g^0(r_b - r_1; ik_n)\, u(r_1 - P_j) \cdot g(r_1, r_a; ik_n)$$

$$g(r_b, r_a; ik_n) = \sum_{n=0}^{\infty} g^{(n)}(r_b, r_a; ik_n)$$

4.115

$$g^{(n)}(r_b, r_a; ik_n) = \sum_{j_1}^{N_{imp}} \cdots \sum_{j_n}^{N_{imp}} \int dr_1 \cdots \int dr_n \cdot g^0(r_b - r_n)\, u(r_n - P_{j_n}) \cdots u(r_2 - P_{j_2})\, g^0(r_2 - r_1)\, u(r_1 - P_{j_1})\, g^0(r_1 - r_a)$$



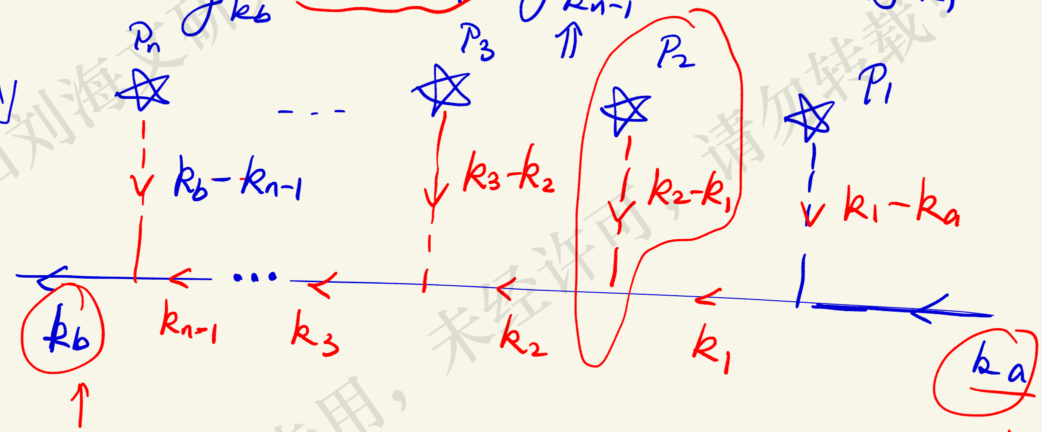
$$g^{(n)}(r_b, r_a) = \frac{1}{V^2}\sum_{k_a k_b} e^{ik_b r_b - ik_a r_a}\, g^{(n)}_{k_b k_a} \qquad (4.116)$$

$$g^{(n)}_{k_b k_a} = \sum_{j_1\cdots j_n}^{N_{imp}} \frac{1}{V^{n-1}} \sum_{k_1\cdots k_{n-1}} e^{-i[(k_b-k_{n-1})P_{j_n} + \cdots + (k_1-k_a)P_{j_1}]} \qquad (4.117)$$

Potential

$$\times\, g^0_{k_b}\, u_{k_b-k_{n-1}}\, g^0_{k_{n-1}} \cdots u_{k_2-k_1}\, g^0_{k_1} \cdots u_{k_1-k_a}\, g^0_{k_a}$$

$$g^{(n)}_{k_b k_a} =$$

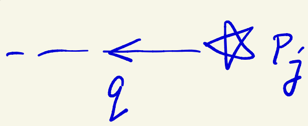


(4.118)

Feynman rules:

① 

g^0_k (4.119)

② 

$u_q \cdot e^{-iqP_j}$

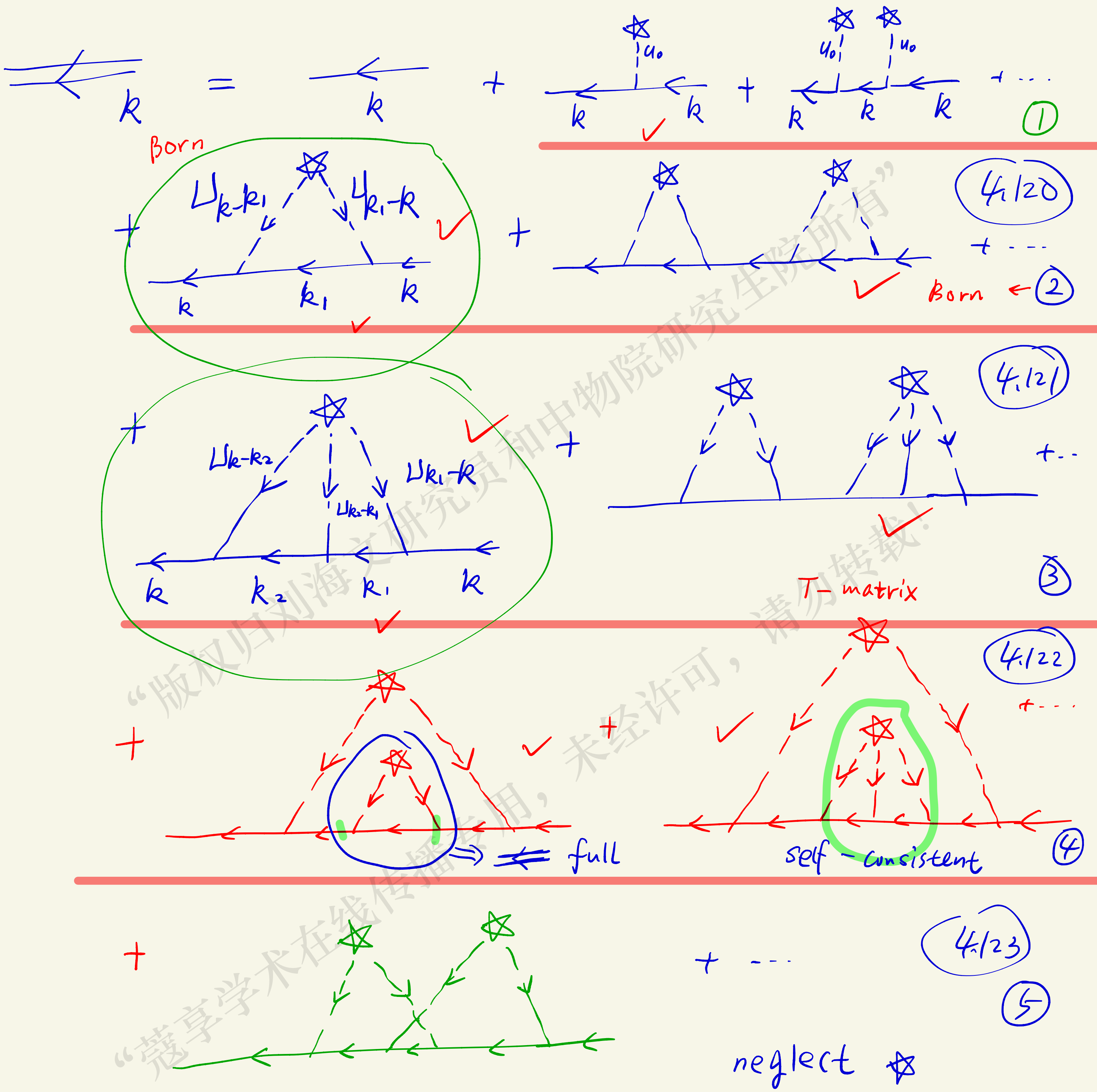
③ $\frac{1}{V}\sum_{k_j}$ summation.

$$u_q \equiv \int u(r)\, e^{-iqr}\, dr$$

Impurity average. ⟺ average "translational invariance"

$\langle g_k \rangle_{imp}$

Dyson

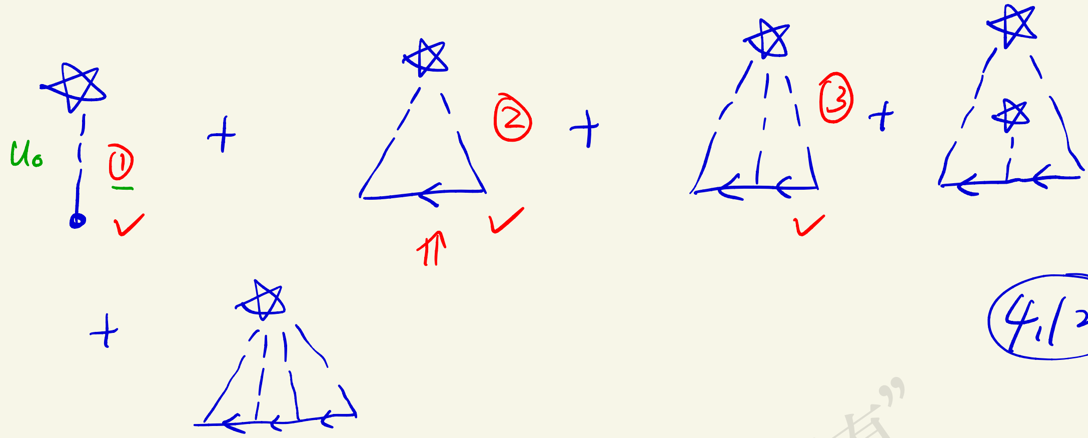


$\langle g_k \rangle_{imp}$ = bare + bard full

full

$$\langle g_k \rangle_{imp} = g_k^0 + g_k^0 \Sigma_k \cdot \langle g_k \rangle_{imp}$$

$\Sigma_k =$



(4.124)

$$\langle g_k \rangle_{imp} = \frac{g_k^0}{1 - g_k^0 \Sigma_k} = \frac{1}{(g_k^0)^{-1} - \Sigma_k}$$

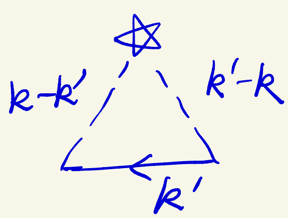
$$= \frac{1}{ik_n - \xi_k - \Sigma_k(ik_n)} \qquad (4.125)$$

lowest-order approximation. $\quad u_q = \int dr\, u(r)\, e^{-iqr}$

$\Sigma_k^{①}(ik_n) =$  $= n_{imp} \cdot u_0 = n_{imp} \int u(r)\, dr$

$$g_k^{①}(ik_n) = \frac{1}{ik_n - \xi_k - n_{imp} u_0} \qquad (4.126)$$

first-order Born approximation.

$\Sigma_k^{1BA}(ik_n) \equiv$ 

$= \frac{n_{imp}}{V} \sum_{k'} \frac{|U_{k'-k}|^2}{ik_n - \xi_{k'}}$

$\underline{\xi_k = \xi_{k'}}$ Imaginary part of self-energy. $\underline{ik_n \sim \omega \simeq \xi_k}$

$$\Sigma_k^{1BA}(ik_n) \simeq -i\pi\, \mathrm{sgn}(k_n) \frac{n_{imp}}{V} \sum_{k'} |U_{k'-k}|^2 \delta(\xi_{k'} - \xi_k)$$

$$\star\quad 2\pi \frac{n_{imp}}{V}\sum_{k'} \left|U_{k'-k}\right|^2 \delta(\xi_k-\xi_{k'}) \equiv \frac{1}{\tau_k} \qquad (4.129)$$

Scattering time τ_k ; $\frac{1}{\tau_k}$ scattering rate

$$\Sigma_k^{1BA}(ik_n) = -i\,\mathrm{sgn}(k_n)\cdot\frac{1}{2\tau_k}$$

first order BA

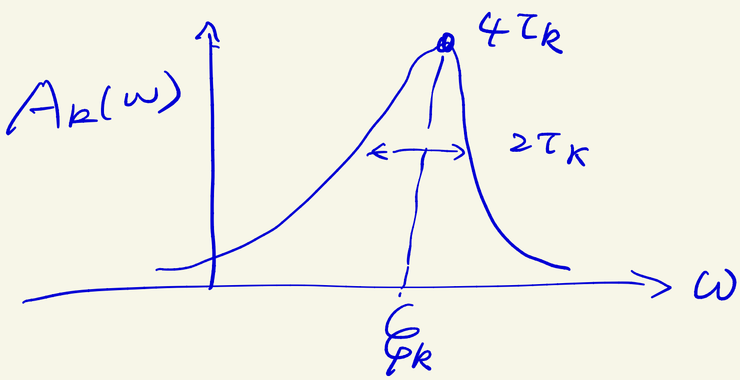
$$g_k^{1BA}(ik_n) = \frac{1}{ik_n - \xi_k + i\,\mathrm{sgn}(k_n)\cdot\frac{1}{2\tau_k}} \qquad (4.130)$$

$$g_k^{R,1BA}(\omega) = \frac{1}{\omega-\xi_k + i/2\tau_k} \qquad (4.131)$$

$$g_k^{R,1BA}(t) = \int \frac{d\omega}{2\pi}\,\frac{e^{-i(\omega+i\eta)t}}{\omega-\xi_k+\frac{i}{2\tau_k}} = -i\,\theta(t)\,e^{-i\xi_k t}\cdot e^{-t/2\tau_k}$$

$$A_k^{1BA}(\omega) = -2\,\mathrm{Im}\, g_k^{1BA}(\omega+i\eta) = \frac{1/\tau_k}{(\omega-\xi_k)^2+\frac{1}{4\tau_k^2}}$$

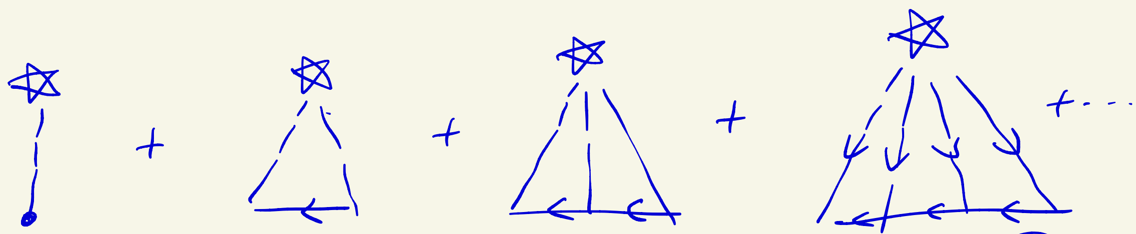
Lorentz shape / wigner-Bret shape (4.132)



T- matrix approximation. ⇐ Single impurity scattering

Full- Born approximation

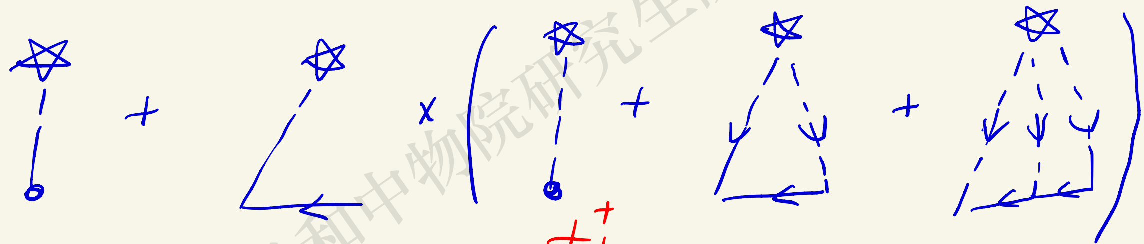
$\Sigma_k^{TMA} \equiv$



(4.133)

Bound state ⟷

$t_{k_1 k_2}(ik_n) =$



t_k^+

$$= n_{imp} u_0 \delta_{k_1 k_2} + \underline{\sum_{k'} U_{k_1 - k'}\, g^0_{k'}\, t_{k' k_2}} \quad (4.134)$$

$$t = u + u g^0 t = u[1 + g^0 u + g^0 u g^0 u + \cdots] \quad (4.135)$$

$$= \frac{u}{1 - g^0 u} \leftarrow \text{Pole}$$

$$u = -t^+ (g^0)^+ u + t^+$$

$$t = u + t^+ g^0 t - t^+ (g^0)^+ u\, g^0 t \quad (4.136)$$

$$\mathrm{Im}\, t = \mathrm{Im}(t^+ g^0 t)$$

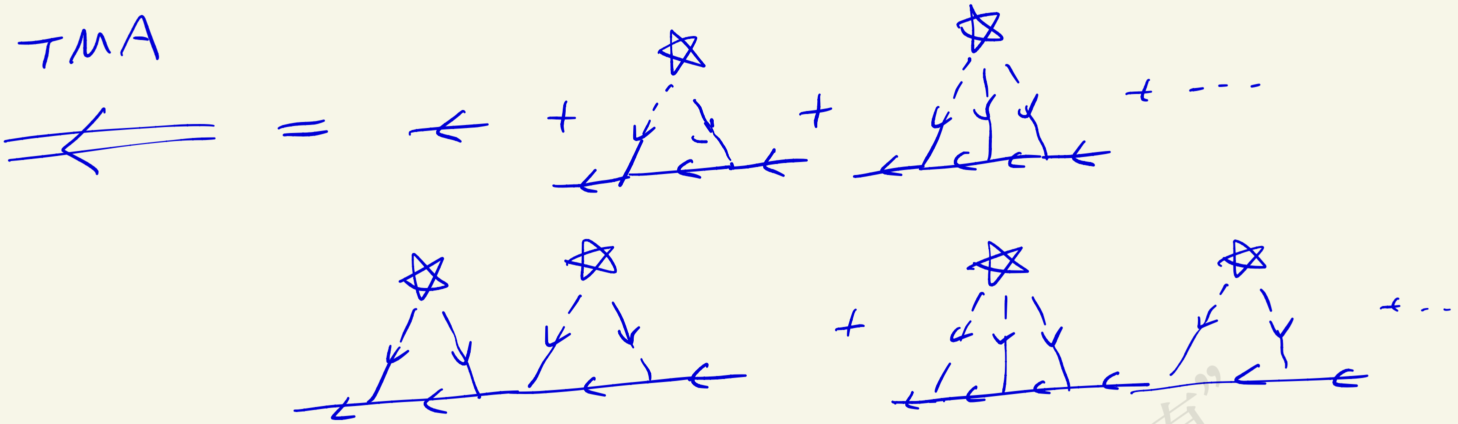
QD + Lead

$$\mathrm{Im}\, \Sigma_k^{TMA}(ik_n) = \mathrm{Im} \sum_{k'} \frac{|H_{k,k'}|^2}{ik_n - \epsilon_{k'}} \quad (4.137)$$

$$\Sigma_k^{TMA}(ik_n) = -i\, \mathrm{sgn}(k_n) \frac{1}{2\tau_k} \quad (4.138)$$

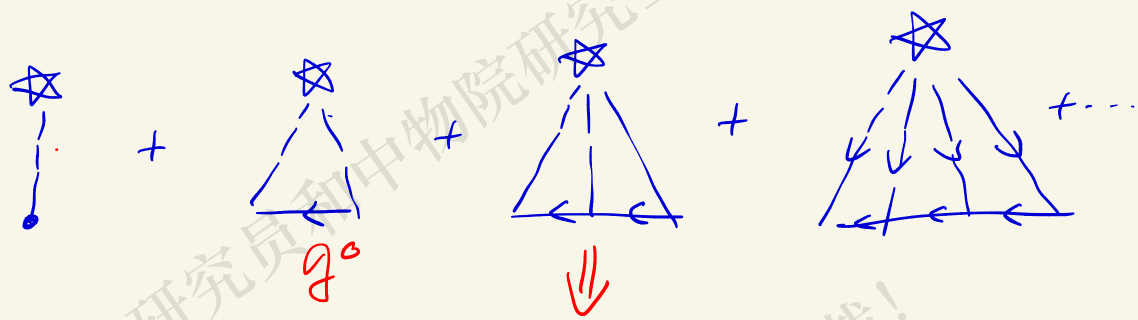
$$\frac{1}{\tau_k} = 2\pi \sum_{k'} |t_{k'k}|^2 \delta(\epsilon_k - \epsilon_{k'})$$

TMA



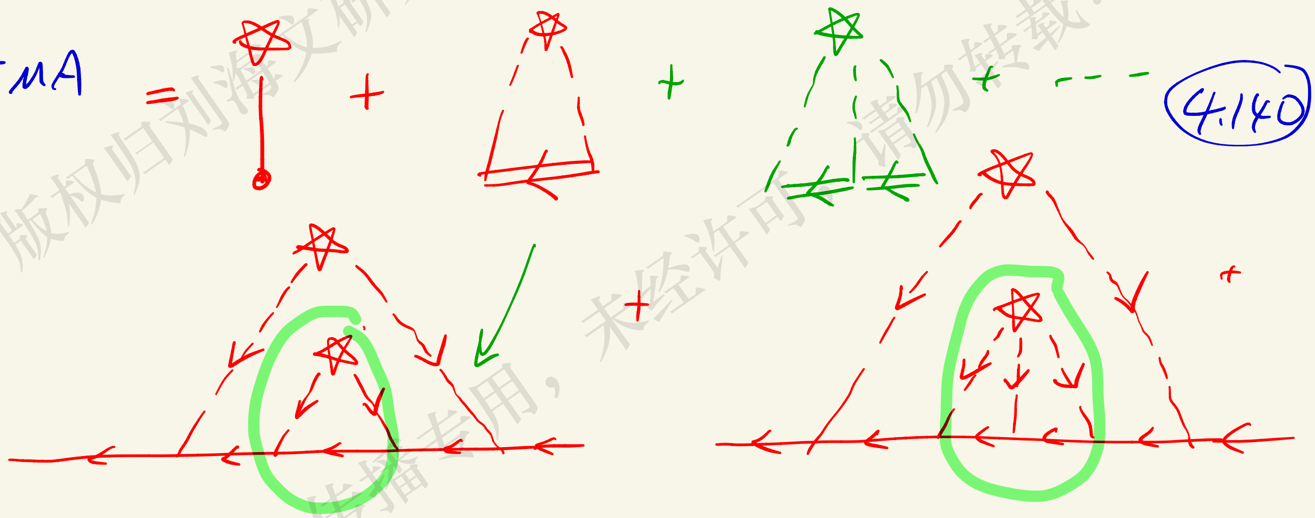
Self-consistent TMA.

$t^{TMA}_{kk} =$



$\Downarrow$

$t^{scTMA}_{kk} =$



(4.140)

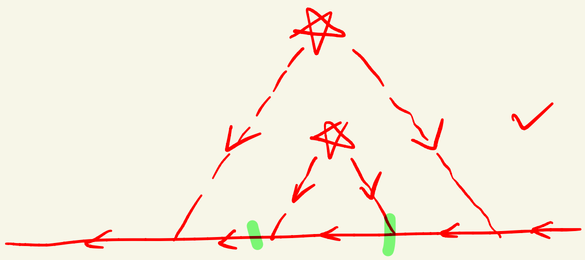
$$= n_{imp} u_0 \delta_{kk'} + \sum_{k'} u_{kk'} g_{k'} t^{scTMA}_{k'k}$$

$$\boxed{\Sigma^{scTMA}_{k} = \mathrm{Im} \sum_{k'} \frac{|t^{scTMA}_{kk'}|^2}{ik_n - \xi_{k'} - \Sigma^{scTMA}_{k}}} \quad (4.141)$$

2 3 1 2

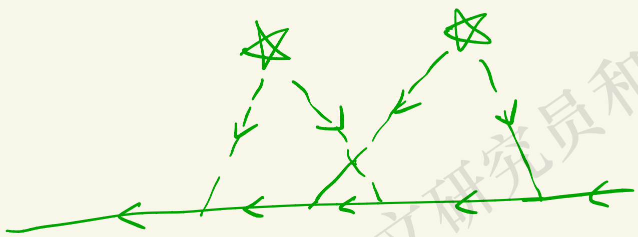
$$\Sigma_k^{SCBA} = \mathrm{Im} \sum_{k'} \frac{|U_{kk'}^{SCBA}|^2}{ik_n - \xi_{k'} - \Sigma_k^{SCBA}} \quad (4.142)$$

$l \gg \lambda \leftarrow$ wave length l mean free path.



$k_F l \gg 1$ weak disorder

✓ dominant



$k_F \underline{\underline{l}} \ll 1$ dominant

$\boxed{l \ll \lambda}$ Strong disorder.

$l \ll \frac{1}{k_F} \sim \lambda$
↑ wave length

✡ hydrodynamics ✡

$$\frac{1}{\tau_k} = 2\pi \sum_{k'} |t_{k,k'}|^2 \, \delta(\xi_k - \xi_{k'})$$

$$A_k^{TMA} = \frac{1/\tau_k}{(\omega - \xi_k)^2 + \frac{1}{4\tau_k^2}}$$

Lesson 10

Feynmann diagrams for interacting Fermions.

$$H = H_0 + W \qquad H_0 = \sum_{\nu_1 \nu_2} C^\dagger_{\nu_1\sigma} h_{0,\nu_1\nu_2} C_{\nu_2\sigma}$$

$$W = \frac{1}{2} \sum_{\sigma_1\sigma_2} \int dr_1 dr_2 \, \Psi^\dagger(\sigma_1 r_1) \Psi^\dagger(\sigma_2, r_2) W(\sigma_2 r_2, \sigma_1 r_1) \Psi(\sigma_2 r_2) \Psi(\sigma_1 r_1)$$