review of Band structure

lesson 2

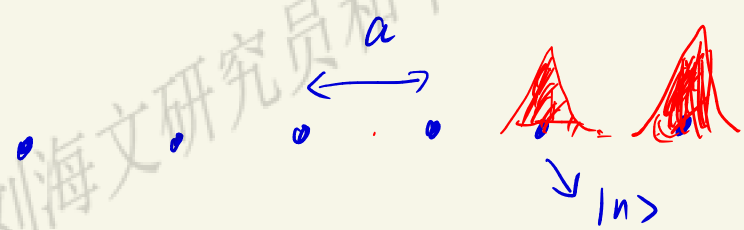
2020 02 26

Solids are collections of tightly bound atoms.

A: ① electrons moving in one dimension

1.1 tight-binding Model

$$\langle n | m \rangle = \delta_{nm}$$



Nearest neighbouring hopping.

$$\mathcal{H}_0 = E_0 \sum_n |n\rangle\langle n| - t \sum_n \left[|n\rangle\langle n+1| + |n+1\rangle\langle n| \right]$$

t : hopping parameter

$$\psi = \sum_m \psi_m |m\rangle$$

$$\mathcal{H}|\psi\rangle = E|\psi\rangle \Rightarrow$$

$$E_0 \sum_m \psi_m |m\rangle - t \sum_m \left[\psi_{m+1}|m\rangle + \psi_m |m+1\rangle \right] = E \sum_m \psi_m |m\rangle$$

overlap with $\langle n|$ $\Rightarrow E_0 \psi_n - t(\psi_{n+1} + \psi_{n-1}) = E\psi_n$.

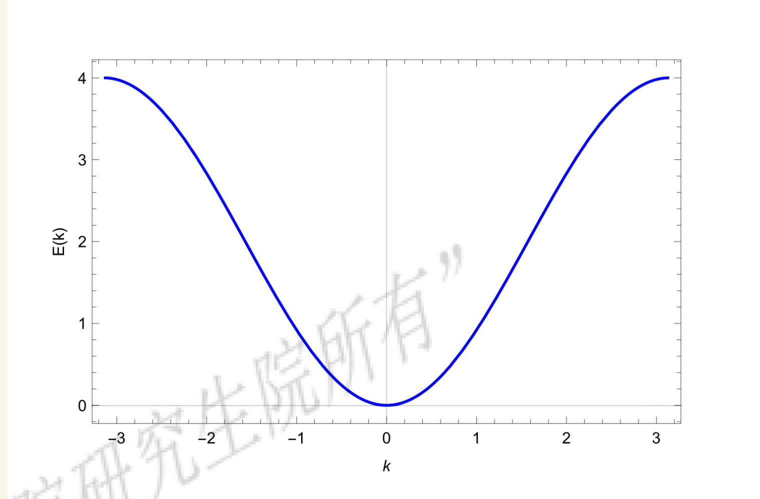
solved by the ansatz [Fourier transformation]

$\boxed{\Delta k \cdot a = 2\pi}$ $\qquad \psi_n = \frac{1}{\sqrt{N}} e^{ik\cdot na}$ $\qquad p = \hbar k$

$k \to k + \frac{2\pi}{a}$; $\quad k \in [-\frac{\pi}{a}, \frac{\pi}{a})$

$$\psi_{n\pm1} = e^{\pm ika}\,\psi_n$$

$$\Rightarrow E = E_0 - 2t\cos ka$$



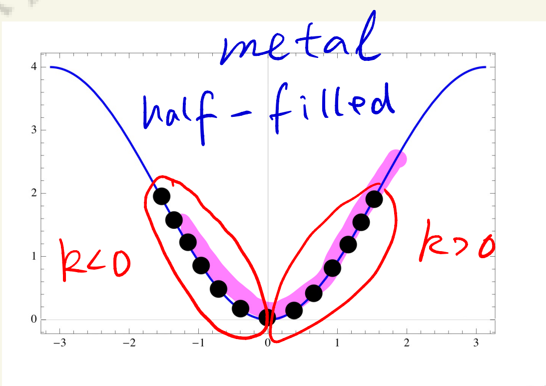
• $k \ll \frac{\pi}{a}$, Taylor expansion.

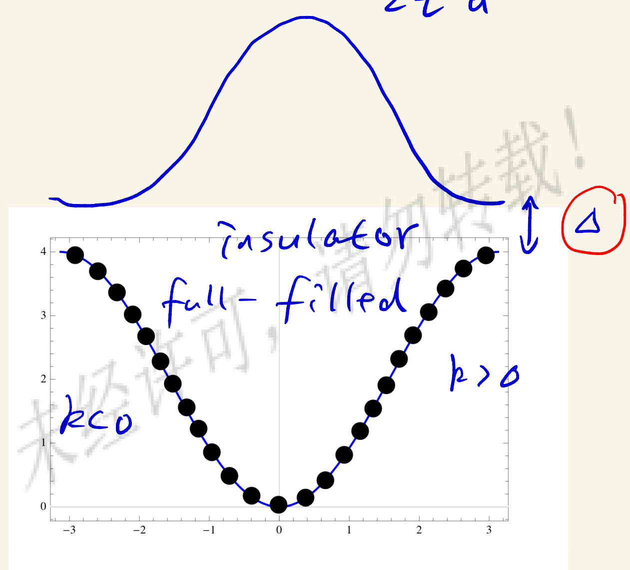
$$E(k) \simeq (E_0 - 2t) + ta^2k^2$$

$$E = \frac{\hbar^2k^2}{2m^*}$$

$$m^* = \frac{\hbar^2}{2ta^2}$$

metal versus insulator $\longrightarrow E$





inversion symmetry. no current.

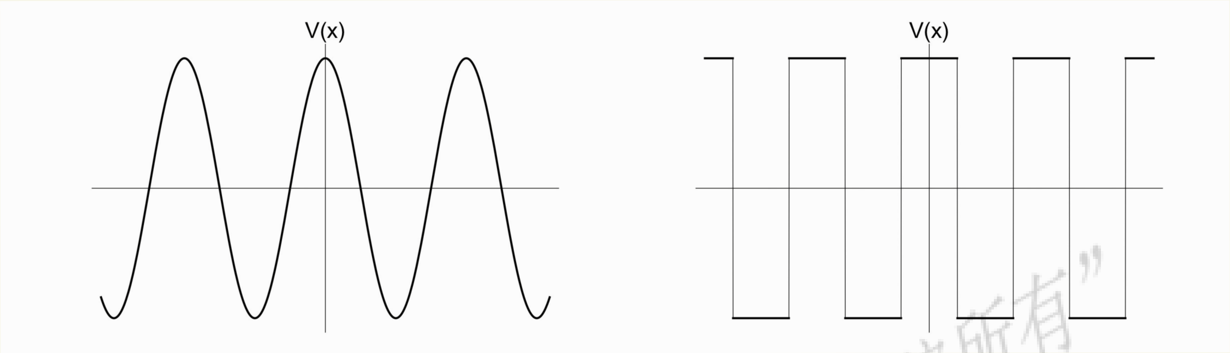
apply electric field, current.

apply a weak electric field
current = 0

1.2 Nearly Free electrons: neglect e-e interaction

$$\mathcal{H} = \frac{p^2}{2m} + V(x)$$

periodic potential $\quad V(x+a) = V(x)$



$$\psi_k(x) = \langle x | k \rangle = \frac{1}{\sqrt{L}} e^{ikx}$$

$$\langle k | k' \rangle = \frac{1}{L} \int dx\, e^{i(k-k')x} = \delta_{kk'}$$

$$E_0(k) = \frac{\hbar^2 k^2}{2m}$$

Perturbation : $E_0(k) = E_0(-k)$

$$\langle k | V | k' \rangle$$

$$V(x) = \sum_{n \in \mathbb{Z}} V_n \cdot e^{i \cdot 2\pi n x / a}, \quad V_n = V_{-n}^*$$

$$\langle k | V | k' \rangle = \frac{1}{L} \int dx \sum_{n \in \mathbb{Z}} V_n \cdot e^{i(k' - k + \frac{2\pi n}{a})x}$$

$$= \sum_{n \in \mathbb{Z}} V_n \, \delta_{k-k', \frac{2\pi \cdot n}{a}}$$

Coupling between k and k' if $k = k' + \frac{2\pi n}{a}$

$-k$

$$|k\rangle \longleftrightarrow |-k\rangle \qquad \boxed{k = \frac{\pi n}{a}}$$

V_0

$$E(k) = \frac{\hbar^2 k^2}{2m} + \langle k | V | k \rangle + \sum_{k' \neq k} \frac{|\langle k | V | k' \rangle|^2}{E_0(k) - E_0(k')}$$

At the edge Brillouin Zone $k = \frac{n\pi}{a}$

degenerate perturbation theory. $\alpha|k\rangle + \beta|k'\rangle$

$$\begin{bmatrix} \langle k|H|k\rangle & \overset{V_n}{\underbrace{\langle k|H|k'\rangle}} \\ \langle k'|H|k\rangle & \langle k'|H|k'\rangle \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = E \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

$$\star \quad \begin{bmatrix} E_0(k) + V_0 & V_n \\ V_n^* & E_0(k') + V_0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = E \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

$$E = \frac{\hbar^2}{2m} \frac{n^2\pi^2}{a^2} + V_0 \pm \underbrace{|V_n|}_{2|V_n|}$$

$$\uparrow \quad k = \frac{n\pi}{a}$$

Near the edge of Brillouin Zone.

$$k = \frac{n\pi}{a} + \delta \qquad k' = -\frac{n\pi}{a} + \delta$$

$$[E_0(k) + V_0 - E] \cdot [E_0(k') + V_0 - E] - |V_n|^2 = 0$$

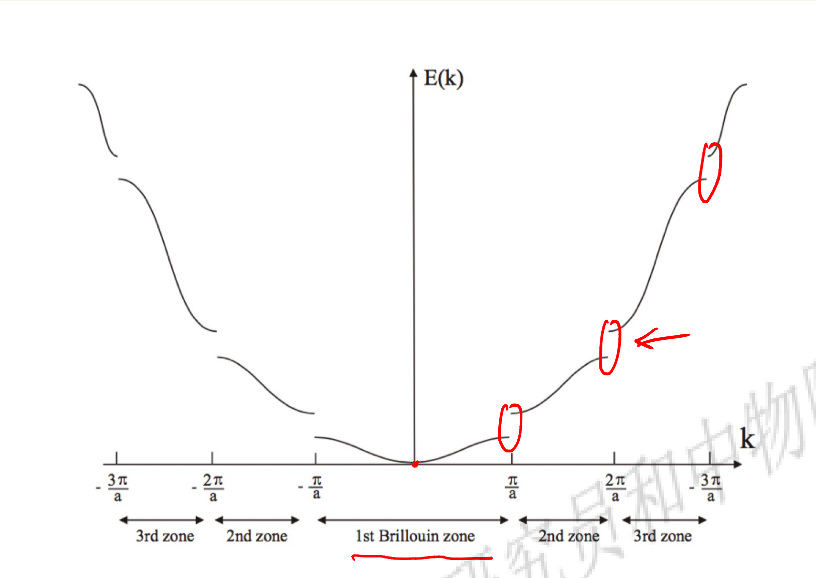
$$E_0(k) = \frac{\hbar^2}{2m}\left(\frac{n\pi}{a} + \delta\right)^2 \qquad E_0(k') = \frac{\hbar^2}{2m}\left(\frac{n\pi}{a} - \delta\right)^2$$

$$\star \quad E_\pm = \frac{\hbar^2}{2m}\left(\frac{n^2\pi^2}{a^2} + \delta^2\right) + V_0 \pm \sqrt{|V_n|^2 + \left(\frac{\hbar^2}{2m}\frac{2n\pi\delta}{a}\right)^2}$$

$$\Updownarrow$$

2nd perturb $E_\pm = E_0(n\pi a \pm \delta) + V_0 \pm \dfrac{|V_n|^2}{E_0(n\pi/a + \delta) - E_0(n\pi/a - \delta)}$

Check when $|V_n|^2 \ll \left(\frac{\hbar^2}{2m}\frac{2n\pi\delta}{a}\right)^2$



$$E_\pm \simeq \frac{\hbar^2}{2m}\frac{n^2\pi^2}{a^2} + V_0$$
$$\pm |V_n| + \frac{\hbar^2}{2m}\left(1 \pm \frac{1}{|V_n|}\frac{n^2\hbar^2\pi^2}{ma^2}\right)\delta^2$$

Band structure:

① $k \ll \frac{\pi}{a}$, spectrum unchanged

② $k = \frac{n\pi}{a}$, gap $\to 2|V_n|$

Bloch Theorem in 1D $\quad V(x) = V(x+a)$

$$\psi_k(x) = e^{ikx} u_k(x)$$

periodic function; Bloch function

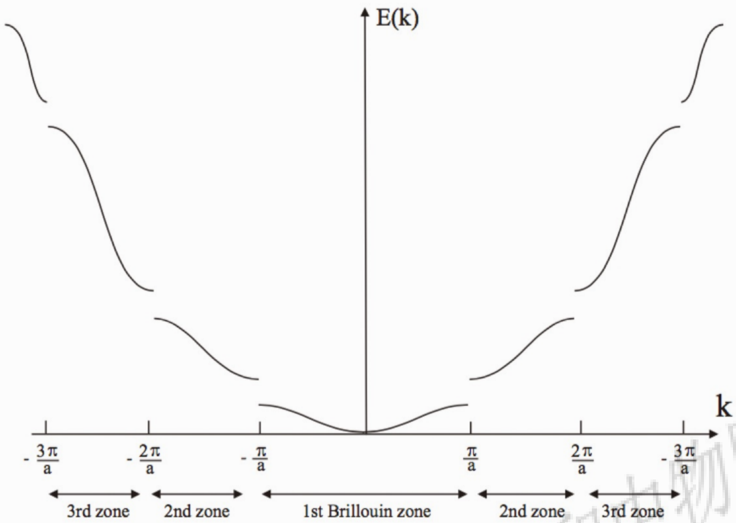
$$u_k(x) = u_k(x+a)$$

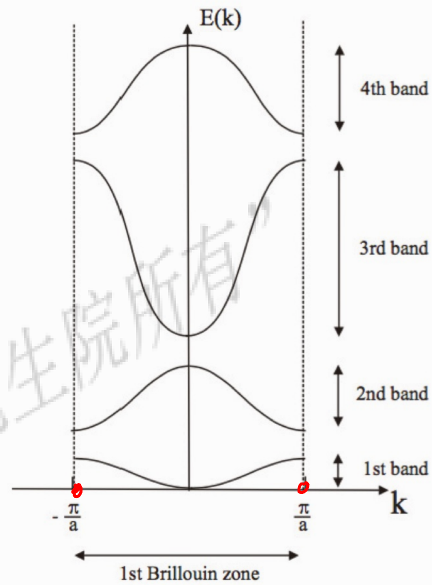
Proof: translational operator $\underline{\underline{T_a}}$

$$T_a \psi_k(x) = \psi_k(x+a)$$

$$\underline{\psi_k(x+a)} = e^{ika}\underline{\psi_k(x)}$$

$$\underline{u_k(x+a)} = e^{-ik(x+a)}\psi_k(x+a) = e^{-ikx}\psi_k(x)$$
$$= \underline{u_k(x)}$$



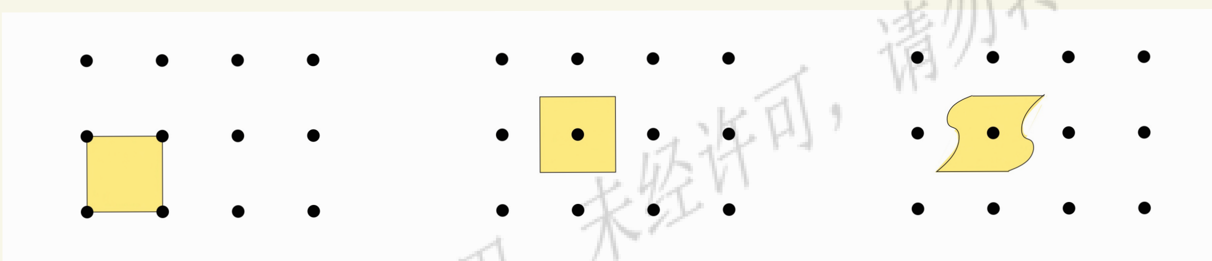


A-2 Lattice

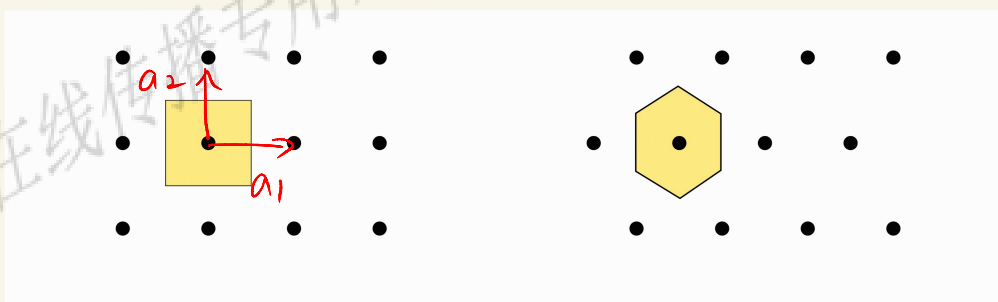
2.1 Bravais lattice

$$\vec{r} = n_1\vec{a_1} + n_2\vec{a_2} + n_3\vec{a_3} \; ; \quad n_i \in \mathbb{Z}$$

$$V = |\vec{a_1}\cdot(\vec{a_2}\times\vec{a_3})|$$



Wigner-Seitz Cells

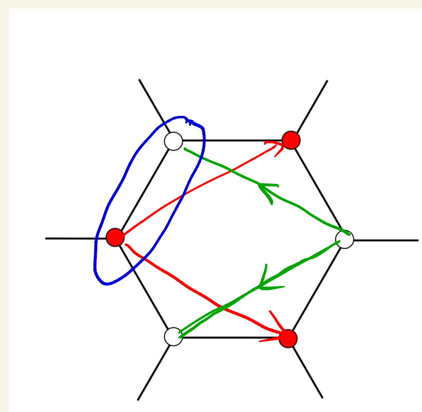


2D Bravis lattice

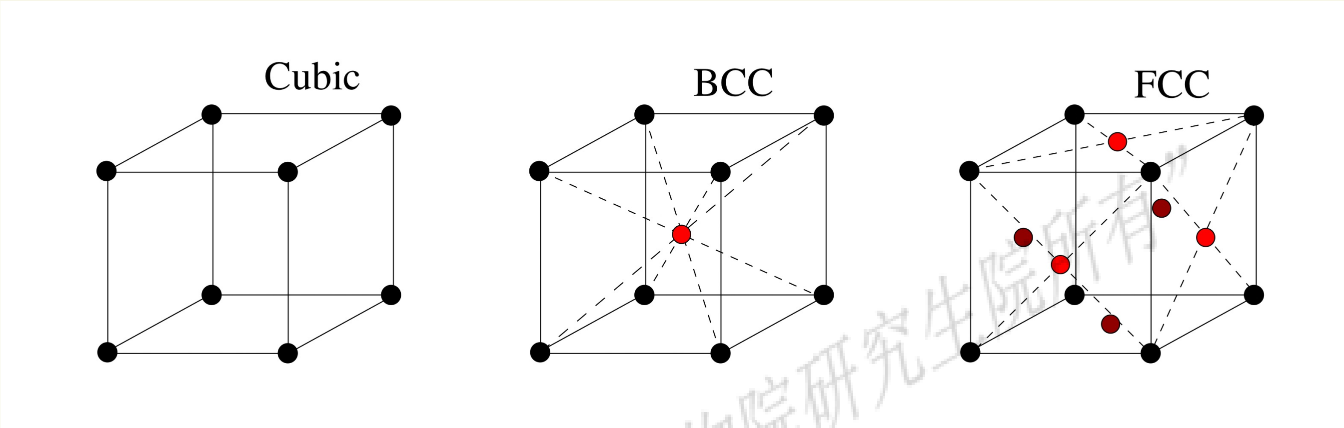
- Square, Triangular, oblique, ...

六角格子 Bi-Particle.

$$a_1 = \frac{\sqrt{3}a}{2}(\sqrt{3}, 1) \; ; \quad a_2 = \frac{\sqrt{3}a}{2}(\sqrt{3}, -1)$$



Bravais Lattice in 3D.



2.2 Reciprical Lattice.

$$\vec{k} = \sum_i n_i \vec{b_i} \quad ; \qquad n_i \in \mathbb{Z}$$

$$* \boxed{\vec{a_i} \cdot \vec{b_j} = \delta_{ij}} \cdot 2\pi$$

$$V^* = |\vec{b_1} \cdot (\vec{b_2} \times \vec{b_3})| = \frac{(2\pi)^3}{V}$$

Fourier transform in Lattice.

考虑 $f(x) = f(x+r)$

$$\tilde{f}(k) = \int d^3x \, e^{-i\vec{k}\cdot\vec{x}} f(x)$$

$\Lambda = \{\vec{r} \mid \vec{r} = n_1\vec{a_1} + n_2\vec{a_2} + n_3\vec{a_3},\ n_i \in \mathbb{Z}\}$

$\Lambda^* = \{\vec{k} \mid \vec{k} = n_1\vec{b_1} + n_2\vec{b_2} + n_3\vec{b_3},\ n_i \in \mathbb{Z}\}$

$$= \sum_{\vec{r}\in\Lambda} \int_\Gamma d^3x \, e^{-i\vec{k}(x+\vec{r})} f(x+r)$$

$\Gamma \leftarrow$ Wigner-Seitz cell Γ

$$= \underbrace{\sum_{\vec{r}\in\Lambda} e^{-ikr}} \underbrace{\int_\Gamma d^3x \, e^{-ikx} f(x)}$$

$$\Delta(k) = \sum_{\vec{r}\in\Lambda} e^{-i\vec{k}\cdot\vec{r}}$$

$\vec{k} \in \Lambda^*$, $e^{-i\vec{k}\cdot\vec{r}} = 1$

if $\vec{k} \notin \Lambda^*$, $\Delta(\vec{k}) \to 0$

$$S(k) = \int_\Gamma d^3x \, e^{-ikx} f(x) \leftarrow \underline{\underline{\text{static}}} \text{ structure factor}$$

$\uparrow$ X-ray scattering.

* Wigner - Seitz cell of reciprocal lattice is called Brillouin zone *

A.3 Band structure

3.1 Bloch Theorem.

$V(x+r) = V(x)$ for all $r \in \Lambda$

$$\psi_k(x) = e^{ikx} \cdot u_k(x), \quad u_k(x+r) = u_k(x); \quad r \in \Lambda$$

translational operators

$$T_r T_{r'} = T_{r+r'} \quad \text{Abelian Group.}$$

$$[H, T_r] = 0 \quad T_r \text{ imply a phase factor}$$

$$T_{a_i} \psi(x) = \psi(x + a_i) = e^{i\theta_i} \psi(x)$$

$$\Rightarrow \underline{r} = \sum_i n_i \vec{a}_i \qquad \underset{\uparrow}{\underline{\vec{k}}} \cdot \underline{\underline{\vec{a}_i}} = \theta_i$$

$$T_r \psi(x) = \psi(x+r) = e^{i\sum_i n_i \theta_i} \psi(x) = e^{i\vec{k}\cdot\vec{r}} \psi(x)$$

T_r eigenvalues is labeled by $\vec{k}$

$$T_r \psi_k(x) = \psi_k(x+r) = e^{i\vec{k}\cdot\vec{r}}\psi_k(x)$$

$$\Rightarrow \quad u_k(x) = e^{-ikx}\psi_k(x)$$

$$\underline{u_k(x+r)} = e^{-ik(x+r)}\psi_k(x+r)$$

$$= e^{-ikx}\, \underbrace{e^{-i\vec{k}\vec{r}}\psi_k(x+r)}$$

$$= e^{-ikx}\psi_k(x) = \underline{u_k(x)}$$

$$r \in \Lambda$$

$k' = \underline{\underline{k}} + q$; $\underline{\underline{q \in \Lambda^*}}$ reciprocal lattice

$$T_r \psi_k(x) = e^{ik\cdot r}\psi_k(x) \qquad e^{iqr} = 1$$

$$T_r \psi_{k'}(x) = e^{i(k+q)r}\psi_{k'}(x) = e^{ikr}\psi_{k'}(x)$$

<u>crystal momentum</u> : reduced Brillouin zone.

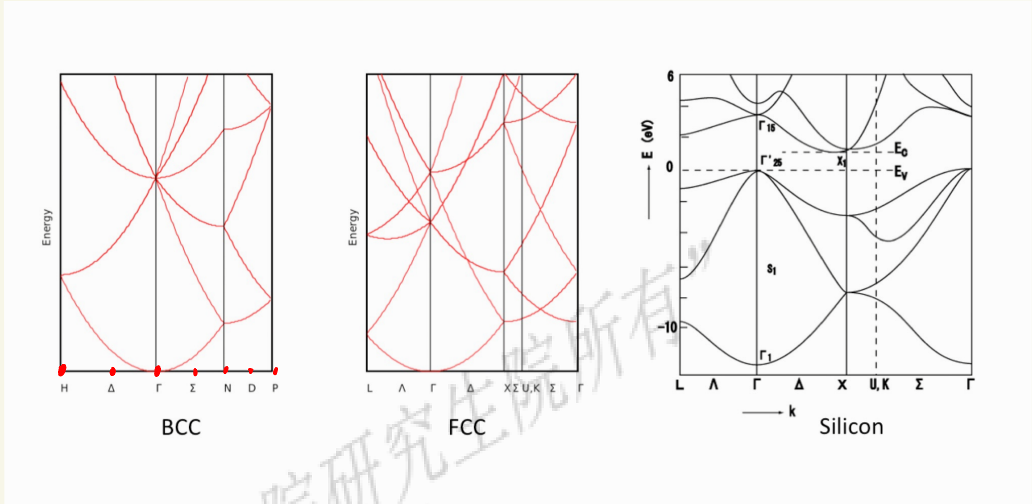
momentum conversion : $k - k' = q$, $q \in \Lambda^*$

3.2 Band structure in 3D near boundary of BZ

$E_0(k) = \frac{k^2\hbar^2}{2m}$

$$E_0(\vec{k}) = E_0(\vec{k}+\vec{q}) \Rightarrow k^2 = (\vec{k}+\vec{q})^2$$

$$\vec{q} \in \Lambda^* \Rightarrow \boxed{2\vec{k}\cdot\vec{q} + q^2 = 0};$$



3.3 Wannier functions

Bloch wave: $\psi_k(x) = e^{ikx} \cdot u_k(x)$

$k \in$ 1st BZ $\quad u_k(x)$ periodic in $r \in \Lambda$

Bloch wave $\longrightarrow$ sum of localized states

$$W_{\vec{r}}(x) = \frac{1}{\sqrt{N}} \sum_{\vec{k}} e^{-i\vec{k}\cdot\vec{r}} \psi_k(x)$$

Lattice

sum of $\vec{k} \in$ 1st BZ

$W_{\vec{r}}(x)$ localized around $\vec{r}$; $\vec{r} \in \Lambda$.

$$W_{\vec{r}+\vec{r}'}(x+\vec{r}') = \frac{1}{\sqrt{N}} \sum_{\vec{k}} e^{-i\vec{k}(r+\vec{r}')} \psi_{\vec{k}}(x+\vec{r}')$$

$$= \frac{1}{\sqrt{N}} \sum_{\vec{k}} e^{-i\vec{k}(\vec{r}+\vec{r}')} e^{i\vec{k}\vec{r}'} \psi_{\vec{k}}(\vec{x})$$

$$= W_{\vec{r}}(\vec{x})$$

$$\boxed{W_{\vec{r}}(\vec{x}) = W(\vec{r}-\vec{x})}$$

Wannier function

B. 1 Graphene [single layer]

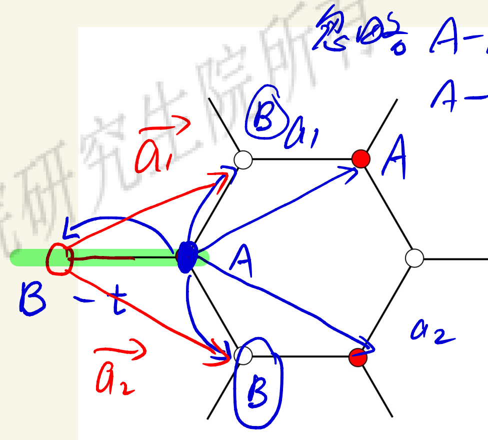
Andre Geim, Konstantin Novoselov, 2004

2010 Nobel prize

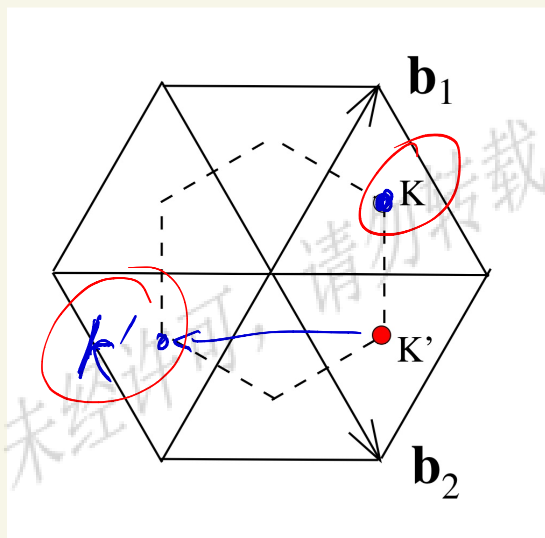
忽略 A-A 次近邻

A-B 最近邻

$$\begin{cases} \vec{a}_1 = \frac{\sqrt{3}}{2} a (\sqrt{3}, 1) \\ \vec{a}_2 = \frac{\sqrt{3}\, a}{2} (\sqrt{3}, -1) \end{cases}$$



$$\begin{cases} \vec{b}_1 = \frac{2\pi}{3a} (1, \sqrt{3}) \\ \vec{b}_2 = \frac{2\pi}{3a} (1, -\sqrt{3}) \end{cases}$$



$$\underline{\underline{K}} = \frac{1}{3}(2\vec{b}_1 + \vec{b}_2) = \frac{2\pi}{3a}\left(1, \frac{1}{\sqrt{3}}\right)$$

$$\underline{\underline{K'}} = \frac{1}{3}(\vec{b}_1 + 2\vec{b}_2) = \frac{2\pi}{3a}\left(1, \frac{-1}{\sqrt{3}}\right)$$

$\boxed{\text{Dirac Points}}$ two

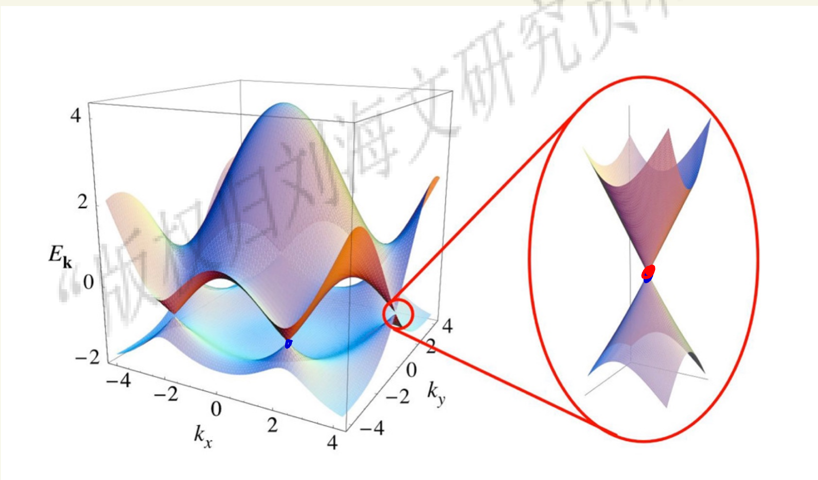
$$H = -t \sum_{r \in \Lambda} \Big[|r, A\rangle\langle r, B| + |r, A\rangle\langle r + \vec{a}_1, B| + |r, A\rangle\langle r + \vec{a}_2, B| + h.c. \Big]$$

We make the ansatz

$$\psi(\vec{k}) = \frac{1}{\sqrt{2N}} \sum_{r \in \Lambda} e^{ikr} \left[C_A |r, A\rangle + C_B |r, B\rangle \right]$$

$$\begin{pmatrix} 0 & \underbrace{-t\left(1+e^{i\vec{k}\cdot\vec{a_1}}+e^{i\vec{k}\cdot\vec{a_2}}\right)}_{\gamma(\vec{k})} \\ \gamma^*(\vec{k}) & 0 \end{pmatrix} \begin{pmatrix} C_A \\ C_B \end{pmatrix} = E(k) \begin{pmatrix} C_A \\ C_B \end{pmatrix}$$

$$E(k) = \pm t \sqrt{1 + 4\cos\left(\frac{3k_x a}{2}\right)\cos\left(\frac{\sqrt{3}k_y}{2}a\right) + 4\cos^2\left(\frac{\sqrt{3}k_y a}{2}\right)}$$



$$\vec{k} = \vec{K} + \vec{q}$$

Taylor expansion

$$|q| \ll 1$$

$$\gamma(\vec{k}) = -t\left[1 + e^{i\vec{k}\cdot\vec{a_1}} + e^{i\vec{k}\cdot\vec{a_2}}\right)$$

$$= -t\left[1 - 2e^{3i\cdot q_x a/2}\cos\left(\frac{\pi}{3} + \frac{\sqrt{3}}{2}q_y a\right)\right]$$

$$\simeq \frac{3ta}{2}(iq_x - q_y) \qquad \frac{3ta}{2} = \hbar v_F$$

$$a = 0.14\ nm$$

$$\gamma(\vec{k}) = \hbar v_F (iq_x - q_y) \qquad t = 2.7\ eV$$

$$v_F \simeq 10^6\ m/s$$

$$\mathcal{H}_K(\vec{q}) = \hbar v_F \begin{pmatrix} 0 & iq_x - q_y \\ -iq_x - q_y & 0 \end{pmatrix} = -\hbar v_F(\sigma_x q_y + \sigma_y q_x)$$

$\sigma_x = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}$ $\sigma_y = \begin{pmatrix} & -i \\ i & \end{pmatrix}$; redefinition

$H_K(\vec{q}) = \hbar v_F (\sigma_x q_x + \sigma_y q_y)$

① 2D massless "Dirac equation"

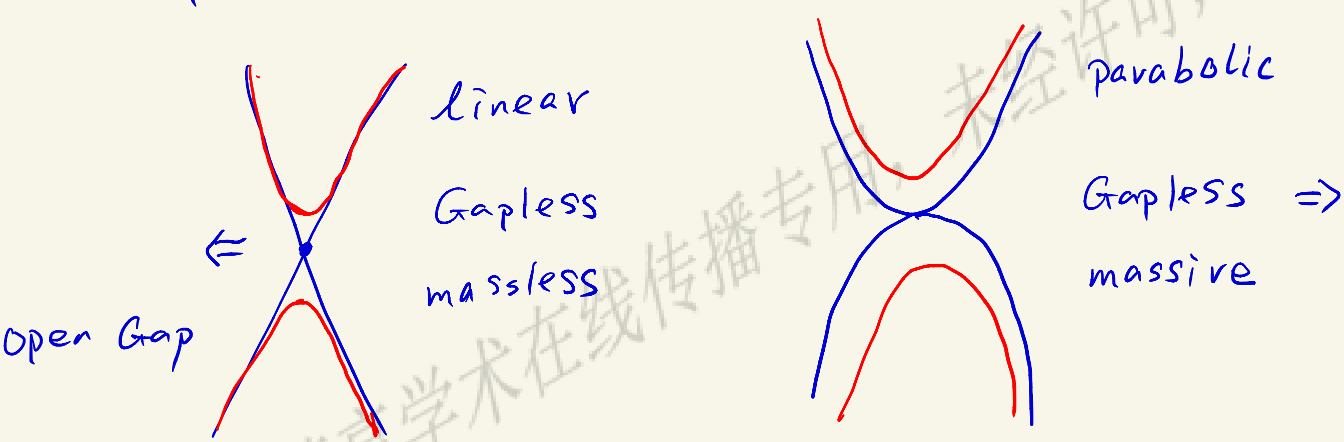
$\boxed{\text{Dirac equation}}$

$$H = \begin{bmatrix} m & \vec{\sigma}\cdot\vec{k} \\ \vec{\sigma}\cdot\vec{k} & -m \end{bmatrix} \qquad \vec{\sigma}\cdot\vec{k} = \sigma_x k_x + \sigma_y k_y + \sigma_z k_z$$

$m = 0$ Dirac equation : Weyl equation.

$H_{K'}(\vec{q}) = H^*_K(-\vec{q})$ ⇐ krammers Theorem

Gapless massless ; parabolic linear



* monolayer Graphene — A-B stacking Bilayer Graphene
* reletiviristic — * non-reletivistic
* Graphene $v_F = 10^6 m/s$
  $C = 3\times10^8 m/s$

$\boxed{\text{most simplest band structure}}$

* twisted bilayer / multi-layer Graphene * Band structure

B.2 Dynamics of Bloch electrons

2.1 Velocity:

$$\boxed{\vec{v} = \frac{1}{\hbar}\frac{\partial E}{\partial \vec{k}}}$$

$$\Rightarrow \quad \vec{v} = \boxed{\frac{1}{m}\langle \psi | -i\hbar\nabla | \psi \rangle} \leftarrow$$

$$\psi_k(x) = e^{ikx} u_k(x)$$

We have:

$$\mathcal{H}\psi_k(x) = E(k)\psi_k(x) \qquad \mathcal{H}_k = \frac{\hbar^2}{2m}(-i\nabla + \vec{k})^2 + V(x)$$

$$\Downarrow$$

$$\mathcal{H}_k u_k(x) = E(k) u_k(x)$$

$$\mathcal{H}_{k+q} = \mathcal{H}_k + \underline{\frac{\partial \mathcal{H}_k}{\partial \vec{k}} \cdot \vec{q}} + \frac{1}{2}\frac{\partial^2 \mathcal{H}_k}{\partial k^i \partial k^j} q^i q^j$$

$$\Delta E = \langle u_k | \frac{\partial \mathcal{H}_k}{\partial \vec{k}} \cdot \vec{q} | u_k \rangle$$

$$\Delta E = E(k+q) - E(k) = \frac{\partial E}{\partial \vec{k}} \cdot \vec{q}$$

$$\therefore \quad \langle u_k | \frac{\partial \mathcal{H}_k}{\partial \vec{k}} | u_k \rangle = \frac{\partial E}{\partial \vec{k}}$$

$$\frac{\hbar^2}{m}\langle u_k | -i\nabla + \vec{k} | u_k \rangle = \frac{\hbar}{m}\langle \psi_k | -i\nabla\hbar | \psi_k \rangle$$

$$= \hbar \vec{v} \quad \Longrightarrow \quad \boxed{\vec{v} = \frac{1}{\hbar}\frac{\partial E}{\partial \vec{k}}}$$

<u>semiclassical</u> filled band No current, No energy flow

Ziman Theory of solids ☆☆

Semiclassical electron dynamics

$$\mathcal{H}_\uparrow(k) = \mathcal{H}_\downarrow^*(-\vec{k})$$ Time reversal symmetry.

no magnetic field, no magetization, no magnetic impurity.

C, P, T; symmetry: Sakurai Mordern QM: ☆☆

B.3 Bloch electrons under magnetic field.

$$\hbar \frac{d\vec{k}}{dt} = -e\,\vec{v} \times \vec{B} \quad \leftarrow \text{particle}$$

$$\vec{v} = \frac{1}{\hbar}\frac{\partial E}{\partial \vec{k}} \quad \leftarrow \text{Band}$$

① $\frac{d}{dt}[\vec{k}\cdot\vec{B}] = 0$ $\quad \vec{k} \parallel \vec{B}$ no force

② $\frac{dE}{dt} = \frac{\partial E}{\partial k}\frac{\partial k}{\partial t} = -e\vec{v}\cdot(\vec{v}\times\vec{B}) = 0$ $\quad$ constant energy.

$\boxed{\text{Orbits}}$ in real space: cross product

$$\hat{B} \times \hbar\dot{\vec{k}} = -e\,\hat{B} \times (\dot{\vec{r}} \times \vec{B}) = -eB\,\dot{\vec{r}}_\perp$$

$\vec{r}_\perp$ is projection onto a plane $\perp \vec{B}$

$$\vec{r}_\perp = \vec{r} - (\hat{B}\cdot\vec{r})\,\hat{B}$$

$$\vec{r}_\perp(t) = \vec{r}_\perp(0) - \frac{\hbar}{eB}\hat{B}\times\left[\vec{k}(t) - \vec{k}(0)\right]$$

Onsager – Bohr – Sommerfeld quantization rule

$\star$ $\frac{1}{2\pi}\oint \vec{p}\cdot d\vec{r} = \hbar(n+\gamma)$ $n\in\mathbb{Z}$

γ: geometric factor

* Generalized WKB approximation

* semi classical analysis * Berry;

$$\frac{1}{2\pi}\oint \vec{p}\, d\vec{r} = \frac{\hbar}{2\pi}\oint \vec{k}\, d\vec{r} = \frac{\hbar^2}{2\pi e B}\underbrace{\oint \vec{k}\cdot(d\vec{k}\times\hat{B})}_{\text{loop}}$$

E_F, A_n

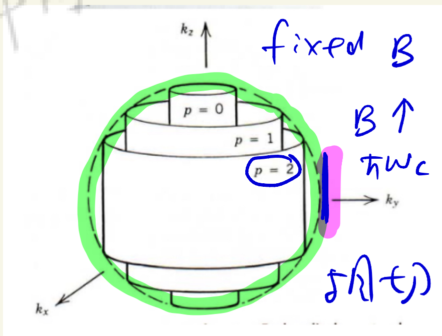
$\oint \vec{k}\cdot(d\vec{k}\times\vec{B}) \equiv A_n$ denotes the area of cross-section $\perp B$

for Fermi surface

$$\boxed{\frac{1}{B} = \frac{2\pi e}{\hbar A_n}(n+\gamma)}$$

area $\rightarrow$ $$\boxed{A_n = \frac{2\pi e B}{\hbar}(n+\gamma)}$$

$\omega_c = \frac{eB}{m}$

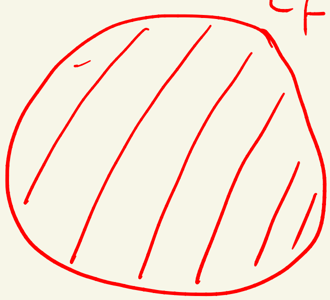


Landau Level: $\vec{B} \parallel z$

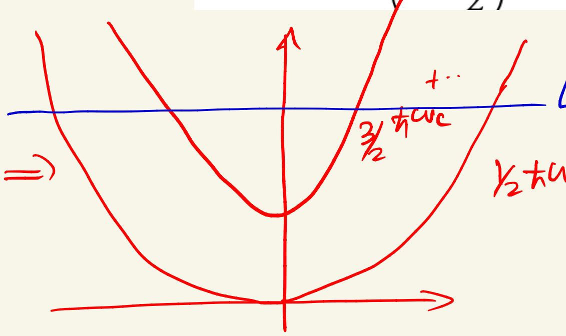
• Notice that the k_z direction is not quantized

$$\varepsilon_{n,k_z} = \left(n+\frac{1}{2}\right)\hbar\omega_c + \frac{\hbar^2 k_z^2}{2m}$$

$E_F = \frac{\hbar^2 k_F^2}{2m}$

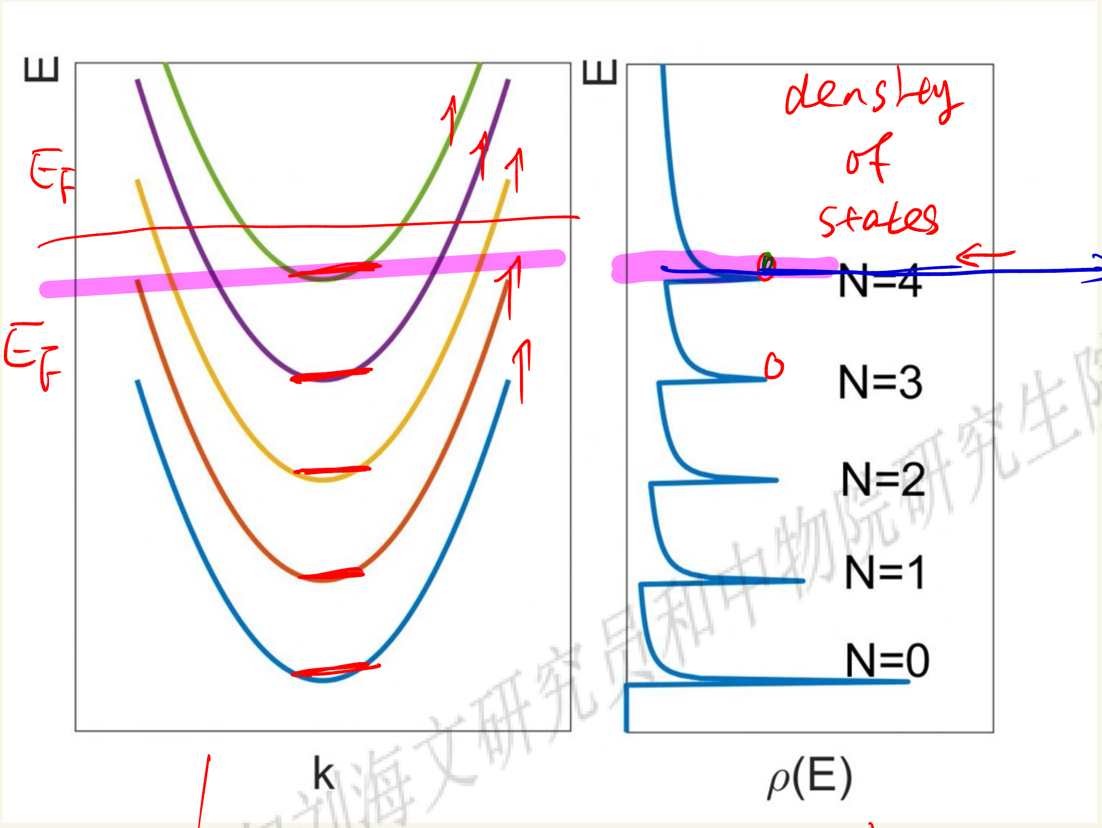


$\Rightarrow$



$B\uparrow$

$\omega_c\uparrow$



density of states divergent

$B \uparrow$

$\hbar\omega_c \uparrow$

$B \uparrow$

$\hbar\omega_c \cdot N \simeq E_F$

$N \simeq \dfrac{E_F \cdot m}{\hbar e B}$

$E = (n+\frac{1}{2})\hbar\omega_c + \dfrac{\hbar^2 k_z^2}{2m}$ $\omega_c = \dfrac{eB}{m}$

Fermi surfaces and metals

- construction of Fermi surface
- semiclassical electron dynamics
- de Haas-van Alphen effect ✓
- experimental determination of Fermi surface

Quantum Oscillations.

Resistivity $\quad A_n = \frac{2\pi e B}{\hbar}(n+\gamma)$ $\qquad \Delta\left(\frac{1}{B}\right) = \frac{2\pi e}{\hbar A_n}\cdot \Delta n$

$\boxed{\text{density of states}}$ $\quad \Delta\left(\frac{1}{B}\right) = \frac{2\pi e}{\hbar A_n}.$ $\qquad \Delta n = 1$

① Shubnikov- de Hass effect ✓ Resistivity.

de Hass - Van Alfen effect ✓ Magentic susepetibity

measuring the Fermi Surface 1950

☆ Brian Pippard : ☆

Real space orbits

$$S = \left(\frac{\hbar}{eB}\right)^2 \cdot A_n$$

area of real space orbits

area of momentum space orbits.

$$\Phi_n = B\cdot S = B\left(\frac{\hbar}{eB}\right)^2 \cdot \frac{2\pi e B}{\hbar}(n+\gamma)$$

$$= \frac{2\pi\hbar}{e}(n+\gamma) = \Phi_0\cdot(n+\gamma)$$

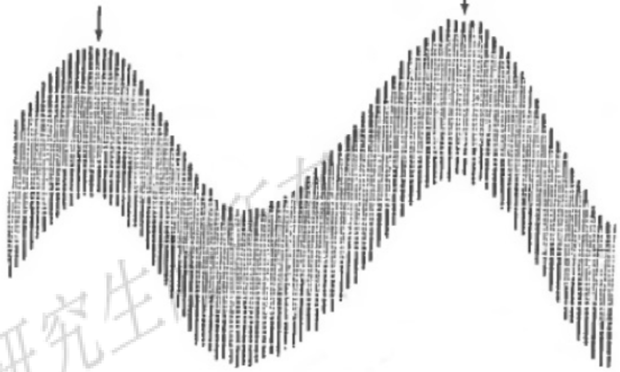
$\Phi_0 = \frac{2\pi\hbar}{e}$ flux quantum.

degeneracy : $\quad N = \frac{\Phi_n}{\Phi_0}$ $\leftarrow$ landau Level label.

De Haas-van Alphen effect (1930)

In a high magnetic field, the magnetization of a crystal oscillates as the magnetic field increases

Silver

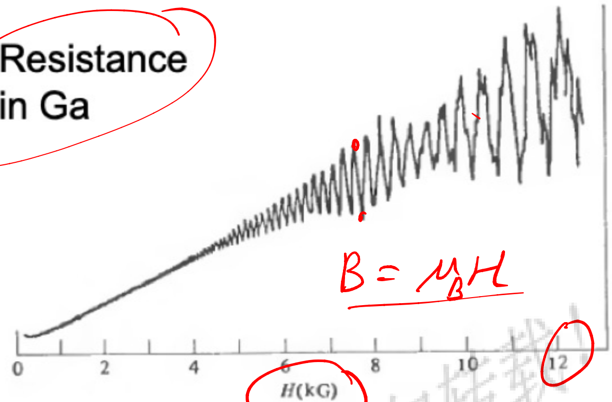


Similar oscillations are observed in other physical quantities, such as

- magnetoresistivity (Shubnikov-de Haas effect, 1930)
- specific heat
- sound attenuation

… etc

Resistance in Ga



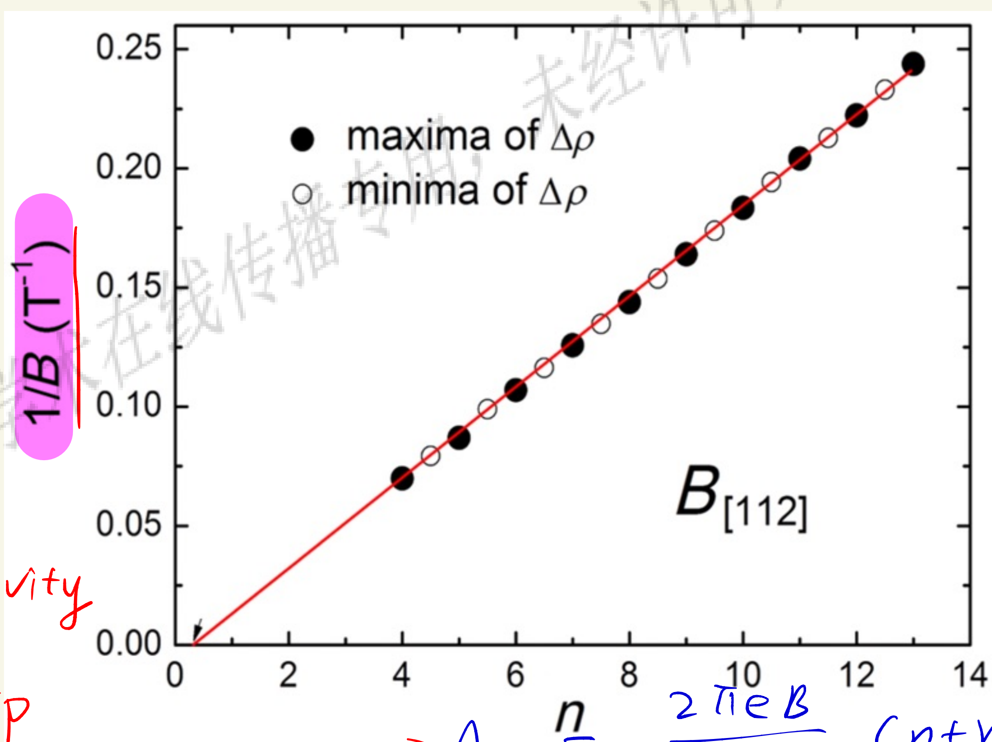
$B = \mu_B H$

1 kG = 0.1 T

Basically, they are all due to the quantization of electron energy levels in a magnetic field (Landau levels, 1930)

frequency.

FFT

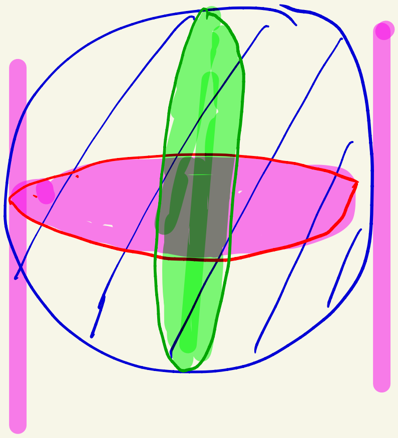


$\frac{1}{B} \propto n$

$B_n \leftarrow$ Resistivity peak or dip

$$\Rightarrow A_n \overset{?}{=} \frac{2\pi e B}{\hbar}(n+\gamma)$$

$$☆ \quad \Delta\left(\frac{1}{B}\right) = \frac{2\pi e}{\hbar A_n}$$

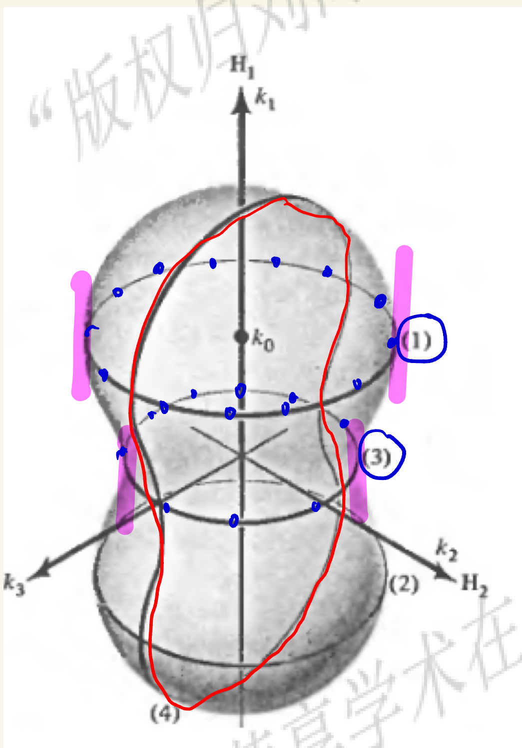


$\uparrow B_z$

$A_n \perp \vec{B}$ cross-section of Fermi surface.

$\longrightarrow B_x$ $A_n' \perp B_x$ Pippard

3D Fermi surface 3D tilting magnetic field



① H_2 single cross-section

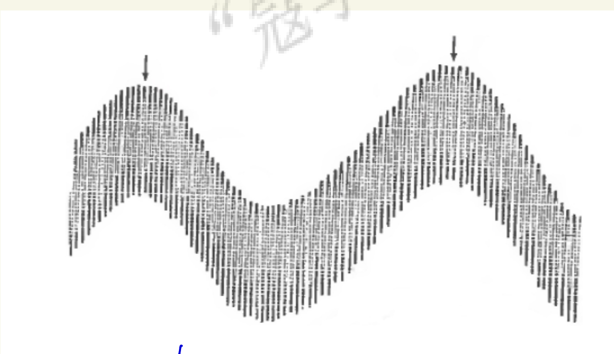
H_1 cross-section ① ③

$A_{①}$ $A_{③}$ two different periods:
beating pattern

② Fermi surface 孔闭合.

Weyl semimetal PRL 2017

Pippard : magnetoresistance in metals

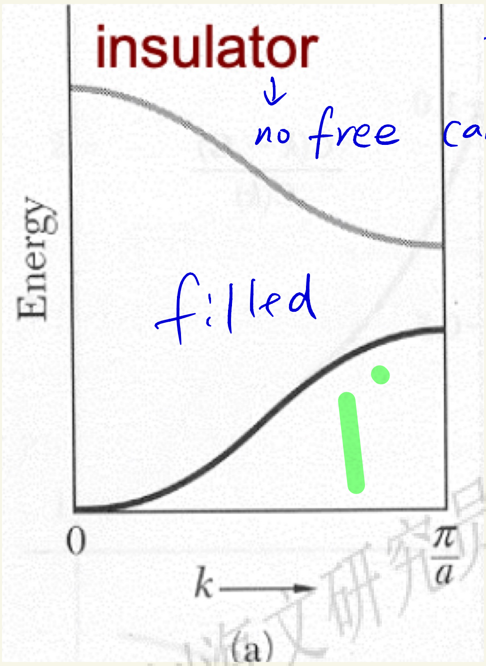


beating pattern

* magnetic breakdown *

* tunelling *

Insulator:

① 

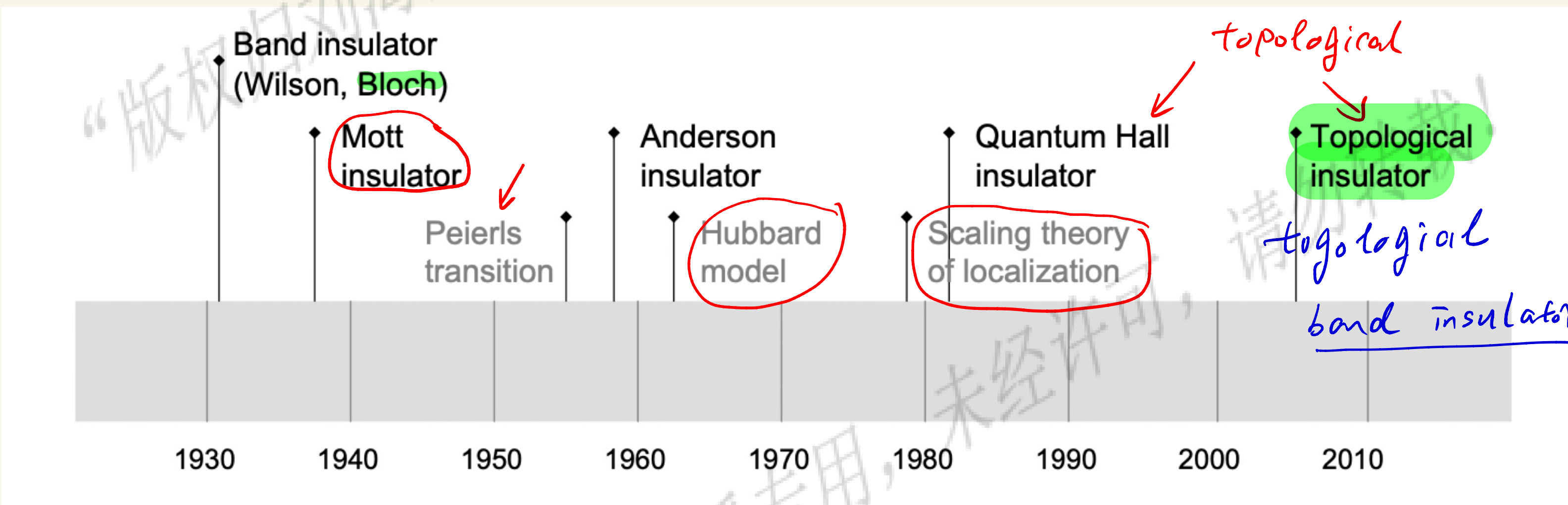
② mott insulator

e-e interaction > kinetic energy

③ Anderson insulator

disorder strength W

$W > W_c \quad \sigma \to 0$



* Kondo Insulator *

① Lattice ; Neutron Scattering.

② 2nd quantization ;