Topological order and its shadow – generalized symmetry

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Quantum matter (at $T = 0$ temperature)

- **Three kinds of quantum states of matter**

- Gapped $\rightarrow$ no excitations
  Band insulators, FQH states

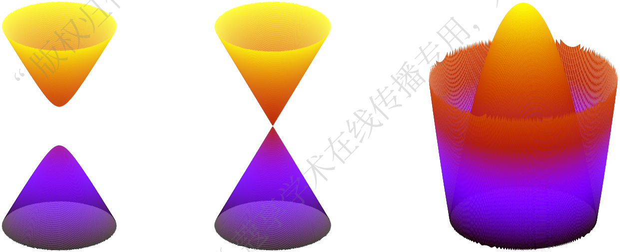
- Gapless (liquid) $\rightarrow$ few excitations, within QFT
  Dirac/Weyl semimetal, superfluid, critical point at continuous phase transition

- Gapless (non-liquid) $\rightarrow$ many excitations, beyond QFT
  Fermi metal, Bose metal, *etc*

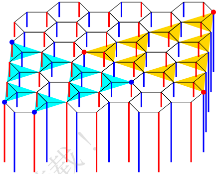
- The above states of matter can be divided into two classes:
  weekly correlated and **strongly correlated**
  (or **weekly interacting** and **strongly interacting**)

Weakly correlated quantum matter

- Weakly correlated **gapped states**:
  General theory: Band theory, K-theory for topological insulators
- Weakly correlated **gapless liquid states**:
  General theory: Band theory with Dirac points for fermion systems. Boson condensation for boson systems. Quantum field theory.
- Weakly correlated **gapless non-liquid states**:
  General theory: Fermi "liquid" theory for fermion systems. Weakly correlated boson systems have no gapless non-liquid states (?)



Strongly correlated quantum matter



- Strongly correlated **gapped states**:
General theory: Group theory and Ginzburg-Landau theory for symmetry breaking states. (Higher) category theory and topological quantum field theory for highly entangled liquid states (*ie* **topological order**, such as FQH states, spin liquids). Network formed by topological orders give rise to highly entangled non-liquid states (fracton states)

- Strongly correlated **gapless liquid states**:
General theory: 1+1D conformal field theory (CFT) for critical points at continuous phase transitions. No general theory for higher dimensions.

- Strongly correlated **gapless non-liquid states**:
General theory: No general theory (? Beyond quantum field theory). Some example: strongly correlated boson systems may have emergent fermionic quasiparticles with Fermi surface.

Gapless **liquid** states and emergent symmetries

How to develop a general theory of the **gapless liquid states**?

Find labels (*ie* invariants) that fully characterize the gapless liquid states, but not the labels like phase-A, phases-B, *etc* .

- A gapless state has more symmetry at low energies than the original lattice Hamiltonian, which is called **emergent symmetry**.
  Maybe the emergent symmetry is a good label to characterize gapless state.
- We find that the emergent symmetry can be very rich: symmetry, anomalous symmetry, higher symmetry, anomalous higher symmetry, algebraic higher symmetry, anomalous algebraic higher symmetry, ...
  algebraic (higher) symmetry = non-invertible (higher) symmetry = fusion (higher) category symmetry = ...

Those symmetries are called **generalized symmetries**.

Generalized symmetries and gapless states

- Maybe the emergent generalized symmetry can fully characterize gapless states. **Each gapless state may has its own distinct characteristic emergent (generalized) symmetry.**
  Use emergent (generalized) symmetry as a starting point to develop a general theory of gapless states
- The ordinary symmetry is described group theory. What is the unified theory that describes generalized symmetry, which include symmetry, anomalous symmetry, higher symmetry, anomalous higher symmetry, algebraic higher symmetry, anomalous algebraic higher symmetry, ...

  We find that **generalized symmetry $\sim$ non-invertible gravitational anomaly = topological order in one higher dimension**
- **A general theory for gapless liquid state as a boundary theory of topological order in one higher dimension.**

The plan

- Microscopic models of topological order, and the associated higher symmetries.
- Macroscopic theory of topological order (an introduction of category theory).
- Topological order in one higher dimension (braided fusion higher category) as a unified theory for generalized symmetry. (A pleasant surprise)
  We call "topological order in one higher dimension" as **categorical symmetry** to stress its connection to symmetry.
- From emergent generalized symmetry to gapless liquid state (a number theoretical approach). (A wish)