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ANALYSIS OF A PROPOSED BISTABLE INJECTION LASER*

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Abstract—The operation of a proposed bistable injection laser is analysed. The device consists of a Fabry–Pérot injection laser whose plated *p*-contact is divided into two electrically isolated portions by a slit parallel to the light reflecting sides of the crystal. An injection current is passed through one contact. The other is biased so that a negligible injection current flows through the junction beneath it, which then acts as a nonlinear absorber of light. Under proper conditions, the laser will have two states: in one, it emits coherent light and in the other, only spontaneous light, both being stable for the same injection current. Switching between states is possible with electrical pulses or incident light. The analysis is done for liquid helium temperatures and for the recombining electrons lying in an exponential conduction band tail, and for liquid nitrogen temperature and parabolic valence and conduction bands. In both cases, the analysis predicts bistable operation for certain conditions and favors a ratio of absorbing contact length to emitting contact length greater than unity.

Résumé—L'opération d'un laser d'injection à deux états stables est analysée. Le dispositif consiste d'un laser d'injection Fabry–Pérot dont le contact plaqué *p* est divisé en deux portions électriquement isolées par une fente parallèle aux côtés du cristal réfléchissant la lumière. Un courant d'injection traverse un contact. L'autre est polarisé de telle façon à ce qu'un courant d'injection négligeable traverse la jonction au-dessous dont le comportement est celui d'un absorbant non-linéaire de lumière. Sous des conditions appropriées, le laser aura deux états: dans un, il émet une lumière cohérente et dans l'autre une lumière spontanée seulement, les deux étant stables pour le même courant d'injection. La commutation entre deux états est possible avec des impulsions électriques ou une lumière incidente. L'analyse est faite à des températures d'hélium liquide et ayant des électrons de recombinaison situés dans la fin de la bande de conduction exponentielle et à des temperatures de nitrogène liquide ayant des bandes de conduction et de valence paraboliques. Dans les deux cas, l'analyse prédit une opération à deux états stables sous certaines conditions et favorise un rapport de la longueur du contact absorbant à la longueur du contact émetteur supérieur à un.

Zusammenfassung—Die Arbeitsweise eines bistabilen Injektionslasers wird analysiert. Das Gerät besteht aus einem Fabry-Pérot Injektionslaser, dessen plattierter *p*-Kontakt durch einen zu den lichtreflektierenden Seiten des Kristalls parallelen Spalt in zwei elektrisch voneinander isolierte Teile aufgeteilt ist. Durch den einen Kontakt fliesst ein Injektionsstrom. Am anderen liegt eine Vorspannung, so dass ein vernachlässigbarer Injektionsstrom durch den darunterliegenden Übergang fliesst, der als nicht-linearer Lichtabsorber wirkt. Unter geeigneten Bedingungen hat der Laser zwei Zustände; im einen emittiert er kohärentes Licht, im andern nur spontanes Licht. Beide sind für denselben Injektions strom stabil. Umschaltung vom einen auf den anderen Zustand ist mit elektrischen Impulsen oder einfallendem Licht möglich. Die Analyse wird durchgeführt für die Temperatur von flüssigem Helium und für eine Lage der rekombinierenden Elektronen in einem exponentiellen Leitungsbandschwanz sowie für die Temperatur des flüssigen Stickstoffs mit parabolischen Valenz- und Leitungsbändern. In beiden Fällen sagt die Analyse unter gewissen Bedingungen bistabile Arbeitsweise voraus, und der günstigste Wert für das Verhältnis der Länge des absorbierenden Kontakts zu der des emittierenden Kontakts ist grösser als Eins.

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1. INTRODUCTION AND SUMMARY

IN THIS paper we analyse the possibility of constructing a bistable injection laser from a single semiconducting crystal diode. By bistable we mean that it has two states for the same current level. In the on state, the device emits coherent light, and in the off state, it does not. The device may be switched from one state to the other by suitable current pulses added to the steady injection current or applied to an auxiliary contact. In principle it could also be switched by suitable light pulses.

In our analysis, we will particularly have in mind a GaAs diode which emits recombination radiation from electrons injected into an active layer on the *p*-side of the junction, but the principle should apply to all injection lasers. The bistability depends upon nonlinear absorption of light[1] by a portion of the *p*-side of the junction. Its operation is somewhat analagous to a bistable microwave maser recently constructed by GERRITSEN.[2]

The analysis is done for the liquid helium temperatures assuming the recombining electrons are in the exponential tail of the conduction band.[3] Bistability is predicted by assuming either a constant or a square root dependence of the density of states in the valence band on energy. For liquid nitrogen temperature the numerical results of LASHER and STERN[4] which refer to parabolic conduction and valence bands are used. For certain parameter values our results also predict bistability for this case.

2. QUALITATIVE DISCUSSION

Figure 1 is a sketch of an idealized bistable injection laser. It consists of a rectangular crystal with Fabry–Pérot optical modes; that is, two opposite sides of the four sides which intersect the *p–n* junction are roughened and the other two

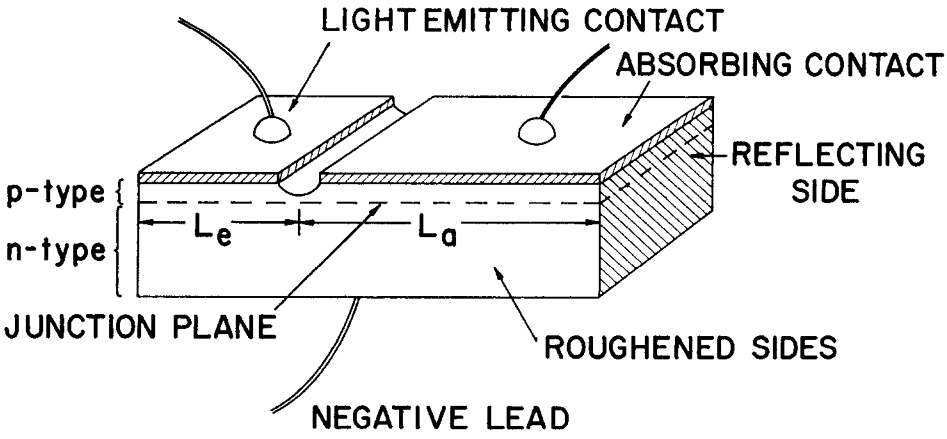


FIG. 1. Idealised bistable injection laser. An electron injecting current is passed through the light emitting contact. The light absorbing contact is biased so that no injection current passes through the portion of the junction beneath it. The coherent Fabry–Pérot modes are generated in an active layer on the *P* side of the junction plane.

are optically smooth and preferably are coated for high optical reflectivity. Its distinctive feature is the division of its plated positive contact into two electrically isolated sections. The slit separating these is parallel to the smooth sides of the crystal. One of these sections is electrically biased to cause the injection of electrons into the portion of the *p*-layer directly beneath it and acts as the light-emitting section. The other has a somewhat smaller voltage so that a negligible injection current is present in that portion and acts as the light-absorbing section. The difference in potential may be as little as a tenth of a volt for GaAs and the necessity of dissipating the heat generated by the current passing between the two positive contacts determines the minimum resistance between them.

It is crucial to the operation of the device that the highest quality optical modes pass through both the emitting and absorbing regions of the junction and that the light in these modes is not scattered or otherwise interrupted in passing between the two regions. It is also necessary that the dimensions and bulk resistivity of the n and p regions are such that no sizable injection current flows through the absorbing region. The ratio $\gamma = L_a/L_e$ of the length of the light absorbing region to that of the emitting region is an important parameter of the device.

When the injection current is first turned on, the device will be in its off state. A light quantum emitted by the recombination of electrons and holes in the emitting region and travelling into the absorbing region will be absorbed there, forming a new electron-hole pair. The recombination of this pair will emit light in a random direction. Thus the absorbing region is a poor reflector, and there will be a very high threshold for lasing. If, however, a sufficient number of conduction band electrons are formed in the absorbing region, the light from the emitting region will not be so strongly absorbed and some will be reflected back by the reflecting side. The lasing threshold will then be reduced. A momentary increase in the number of electrons in the absorbing region might be caused by a current pulse through the absorbing contact, by an external pulse of light entering the absorbing region, or, indirectly, by an additional pulse of current through the emitting contact. Whether or not both states are stable for some single value of current through the emitting contact can only be decided by analysis of some detailed mathematical model.

3. THE RATE EQUATIONS

The variables in our problem are the assumed uniform densities of electrons, n_e and n_a, in the active layers of the light emitting and absorbing regions, and the number of photons, $S\omega$, in the Fabry–Pérot models of frequency, ω. These quantities satisfy rate equations of the form:

$$\frac{dn_e}{dt} = j/d - n_e/\tau - \sum_{\omega} S_\omega g(\omega, n_e) \tag{1a}$$

$$\frac{dn_a}{dt} = -n_a/\tau - \sum_{\omega} S_\omega g(\omega, n_a) \tag{1b}$$

$$\frac{dS_\omega}{dt} = V_E S_\omega g(\omega, n_e) + \gamma V_E S_\omega g(\omega, n_a) - S_\omega/\tau_\omega \tag{1c}$$

where j is the injection current per unit area; d is the thickness of the active layer of the diode; τ is the electron recombination lifetime; $g(\omega,n)$ is the volume rate of stimulated emission into the ω mode for an electron density, n; V_ω is the volume of the active layer of the light emitting region; γ is the length ratio defined above; and τ_ω the characteristic damping time of the ω optical mode. The fringing of the light in the Fabry–Pérot modes is neglected so that we can consider $g(\omega, n)$to be the bulk gain in the active layer. In equation (1c) the effect of spontaneous emission is ignored.

In the off state all the S_ω's will be small compared to unity, n_e will have the approximate value, $j\tau/d$, and n_a will be much less than n_e. In the on state one S_ω will be large compared to unity, the remaining photon populations will still be small, n_e will be decreased from its value in the off state and n_a increased. Our procedure to demonstrate the existence of these solutions is to set the right-hand side of equations (1a) and (1b) to zero and solve for the quantity:

$$\begin{aligned} R_\omega(S_\omega) &= V_E S_\omega[g(\omega,n_e) + \gamma g(m,n_a)] \\ &= V_E(j/d - n_e/\tau - n_a/\tau) \end{aligned} \tag{2}$$

assuming that only one S_ω differs from zero. This quantity is the rate of stimulated emission into the ω mode when there are S_ω photons in that mode and none in the other modes. It is the rate of stimulated emission which gives zero rates of change of n_e and n_a, equations (1a) and (1b). If it equals S_ω/τ_ω, then equation (1c) will give zero for dS_ω/dt as well and we will have a steady solution of all the rate equations for an on state.

In Fig. 2, we have sketched some fictitious stimulated emission functions in order to discuss these solutions. If a function has the shape labeled ω_0 and two intersections with the straight line $R = S/\tau_\omega$ then that mode is bistable. When S_{ω_0} has an instantaneous value between zero and the value S_a at the first intersection, the rate of stimulated emission is less than the rate of loss, and S_{ω_0} will decay to zero. This is the off state. When S_{ω_0} lies between S_a and S_b, the rate of stimulated emission exceeds the loss rate and the population of the ω_0 mode will relax to the value S_b. This is the on state. It is clearly necessary that the stimulated emission function be concave upward for small values of S_ω. For the device to be stable in the off condition, it is necessary that the stimulated emission function for all modes lie below the 'load line' $R = S/\tau_\omega$ for small values of S_ω. One way to determine this is to draw the envelope of the $R_\omega(S_\omega)$ curves for all ω and see if it has two intersections with the line $R = S/\tau_\omega$. In this discussion, we have ignored the possibility of n_e and n_a having non-zero rates of change. This would mean that the on state may not be stable and the device may have continuous relaxation oscillations. It is easily shown, however, that our conditions for the stability of the off state are sufficient even when all three variables are considered.

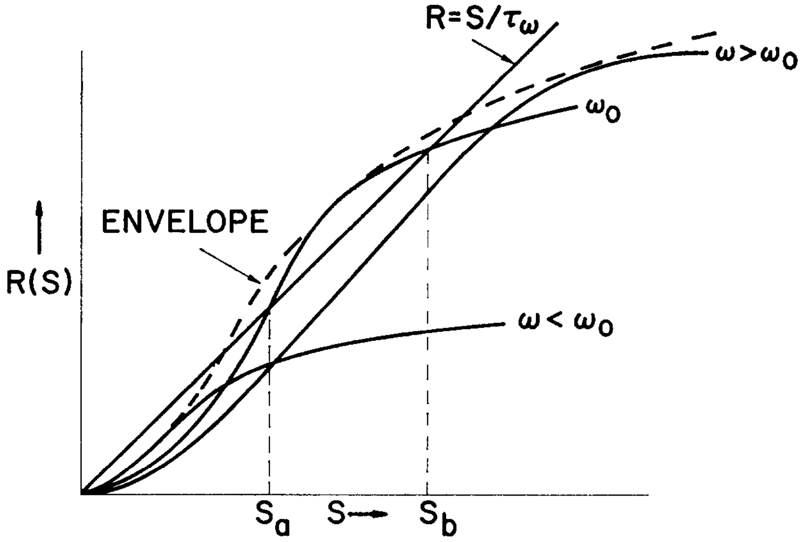


FIG. 2. A representative set of stimulated emission functions. The ordinate is the net rate of stimulated emission (emission from the emitting region minus absorption in the absorbing region) vs. the photon population of a single mode. The straight line $R = S/\tau_\omega$ give the loss of photons due to damping of the optical mode. The curve labeled ω_0 shows that this mode is bistable; the stable values of S_{ω_0} being zero and S_b. The value S_a is an unstable solution. The dashed line is the envelope of the stimulated emission functions for all modes; a device with such an envelope would be bistable.

4. THE LOW TEMPERATURE ANALYSIS

For sufficiently low temperatures analytic treatments of our problem become feasible. In the region of liquid helium temperatures the electron occupation probability of states below the quasi-Fermi levels are close to unity, and above the quasi-Fermi levels they are close to zero. We will also assume simple shapes for the density of state functions for both bands and take the recombination probability of holes and electrons to be independent of this energy. This latter approximation is discussed in LASHER and STERN.[4]

In this limit of low temperature, a simple argument indicates that the off condition of the device is not likely to be stable if the conduction band has a sharp band edge. The minimum frequency for the emission of recombination radiation is the effective band gap, and therefore the gain function in the emitting region has the shape of the curve labeled g_e in Fig. 3. As shown in Lasher and Stern, this curve goes through zero for frequencies corresponding to the difference between the quasi-Fermi levels and becomes negative, indicating absorption, for greater frequencies. The minimum frequency for absorption when there are no electrons present, however, is equal to the band density of states for the n-doping greater than 3×10^{18} cm^{-3}.(3) When this is the case, the gain functions do not have sharp cutoffs at low frequencies, and we may achieve a stable off state by having sufficient length of the absorbing region. In section VII of Lasher and Stern, a comparison of theory and experiment is made for diodes of

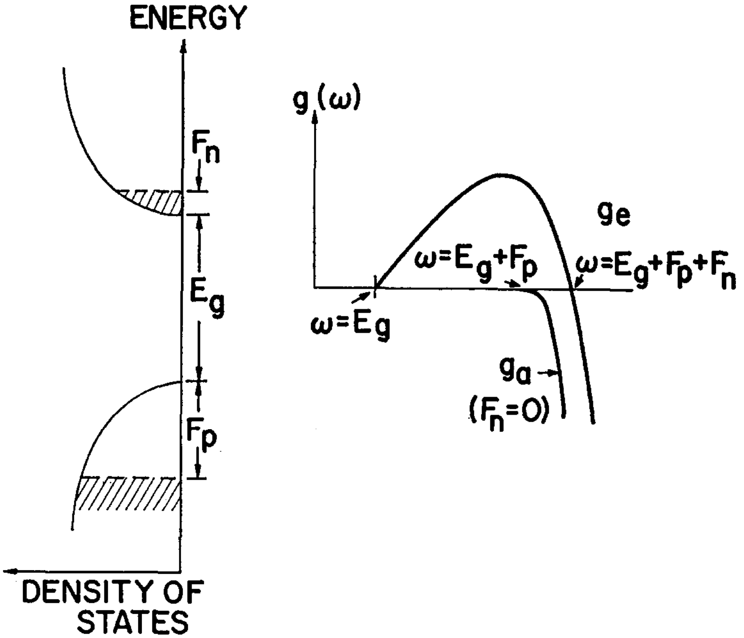


FIG. 3. Density of states and stimulated emission spectral functions for low temperature and sharp effective band edges. The figure illustrates the argument in the text that it is difficult to achieve bistability under these conditions.

effective gap plus the penetration of the quasi-Fermi level into the valence band as shown by the curve g_a. The result is that all the modes between the frequencies E_g and $E_g + F_p$ are amplified in the emitting region, but not absorbed in the absorbing region, and these modes will turn on easily, preventing the device from being stable in the off condition.

It is known, however, that in GaAs there is a prominent exponential tail to the conduction this type. The density of states in the conduction band may be fit by:

$$\rho_c(E) = \rho_0 \exp(E/E_0) \tag{3}$$

with E_0 in the range 8–15 MeV and valence band quasi-Fermi levels in the same range.

In our simplest model we assume a constant density of states, ρ_v, in the valence band and, following Lasher and Stern, obtain for the gain function:

$$g(\omega,n) = K\rho_o\rho_v \int^{Fv} dE\exp[(\omega-E)/E_0]$$

(the lower limit of the integral is $\omega - F_n$ or 0; whichever is greater). (4a)

$$g(\omega,n) = \begin{cases} E_0K\rho_0\rho_v\{\exp F_n/E_0 - \exp[(\omega-F_v)/E_0]\} \ ; \ \omega > F_n & (4b) \\ E_0K\rho_0\rho_v \exp(\omega/E_0)[1-\exp(-F_v/E_0)] \ ; \ \omega < F_n & (4c) \end{cases}$$

where K is a constant proportional to recombination probability. The concentration of electrons is:

$$n = \rho_0 \int_{-\infty}^{F_n} dE\exp(E/E_0) = \rho_0 E_0 \exp(F_n/E_0). \quad (5)$$

To simplify our analysis, we introduce a new set of quantities; a variable monotonic with the optical frequency:

$$v = \rho_0 E_0 \exp(\omega/E_0) \quad (6a)$$

a variable proportional to the photon mode population:

$$S_v = \tau K \rho_v S_\omega; \quad (6b)$$

a parameter proportional to the injection current:

$$J = j\tau/d; \quad (6c)$$

and a parameter:

$$f = \exp(-F_v/E_0). \quad (6d)$$

In these terms the gain function is:

$$g(v,n) = \begin{Bmatrix} K\rho_v(n-fv) \ ; \ v>n \\ K\rho_v v(1-f) \ ; \ v<n \end{Bmatrix}. \quad (7)$$

This function can be substituted into the rate equations, (1a) and (1b) with zero time derivatives, and the results for n_e and n_a used in equation (2) for the rate of stimulated emission:

$$\frac{\tau}{V_E}R_v(S_v) = \begin{cases} \dfrac{S_v}{1+S_v}\left[J-(\gamma+1)fv\right]; \ v > \dfrac{J}{1+(1-f)S_v} & (8a) \\ \dfrac{vS_v}{1+S_v}\left[1-(\gamma+1)f+(1-f)S_v\right]; \ v < \dfrac{J}{1+(1-f)S_v}. & (8b) \end{cases}$$

The envelope of these functions is obtained by maximising them with respect to v for given values of S:

$$\frac{\tau}{V_E}R_{\max}(S) = \begin{cases} \dfrac{S[(1-f)S+1-(\gamma+1)f]}{(1+S)[1+(1-f)S]}J; \ S > \dfrac{(\gamma+1)f-1}{1-f} \\ 0 \qquad ; \ S < \dfrac{(\gamma+1)f-1}{1-f}. \end{cases} \quad (9)$$

This envelope of the stimulated emission functions contains the injection current parameter only as a proportionality factor and, therefore, our bistability criterion will be independent of the current or the damping of the optical modes, τ_ω, as long as the device turns on while the electron fermi level lies in the exponential conduction band tail. This feature can be shown to result from assuming an exponential dependence of state density in the conduction band and will also hold for the other low temperature cases. The shape of the envelope depends on two parameters, γ and f. If the length ratio γ is chosen to be:

$$\gamma_0 = (1-f)/f \quad (10)$$

the envelope will have zero slope with respect S at $S = 0$, and in fact this is the smallest value of γ for which this is true. The physical significance of γ_0 is that for this value of the length ratio, the gain in the emitting region is exactly balanced by the absorption in the absorbing region at low light levels. Figure 3 is a plot of the envelopes for $f = 1/2$, that is, $F_v/E_0 = \ln 2$, and $\gamma = \gamma_0 = 1$; and $\gamma = 2\gamma_0 = 2$. Both envelopes have the proper shape to have two intersections with a straight line through the origin and therefore predict bistability for these conditions. The envelope for $\gamma = 1$ predicts bistability only by virtue of the small region of upward curvature near the origin. This slight curvature might be removed by some of the physical effects we have neglected, and the prediction of bistability for a length ratio of two is certainly more convincing.

In this case, there is an absolute minimum of the length ratio, γ, consistent with bistability. We obtain this critical value of γ by setting the second derivative of the envelope equation (9) equal zero for $S = 0$ and solve for γ:

$$\gamma_c = \frac{(1-f)^2}{f(2-f)} = \frac{1-f}{2-f}\gamma_0. \qquad (11)$$

For $f = 1/2$, the value in Fig. 4, $\gamma_c = 1/3$.

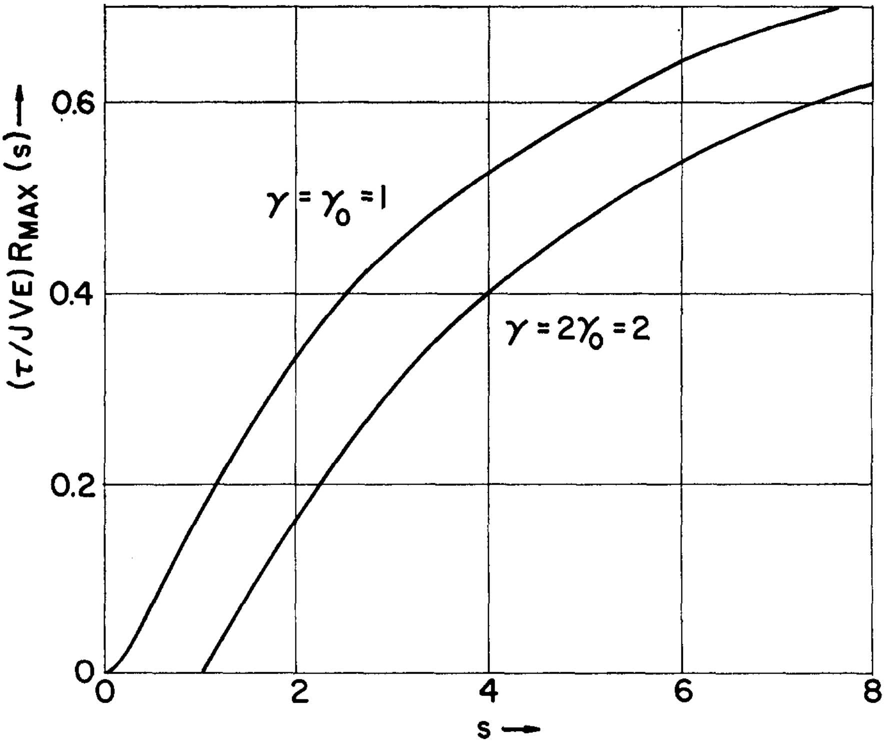


FIG. 4. Envelopes of the stimulated emission curves at low temperature for an exponential energy dependence of the conduction band density of states and a constant valence band density of states. The two envelopes are for two values of γ, the ratio of absorbing region length to emitting region length.

The assumption of a constant density of states in the valence band is a very special one, and we have therefore also applied our criterion for low temperature bistability using a density of states varying with the square root of the energy. The integral equivalent to equation (4a) for the gain function now yields an error function, and we have resorted to numerical procedure to determine the rate of stimulated emission. Figure 5 shows the result for F_v equal to E_0 and a length ratio γ equal to the value $\gamma_0 = 1{\cdot}41$ which balances gain and absorption for low light levels. The rate of stimulated emission is plotted for four mode frequencies, and each curve is labeled by its value of $\exp[(\omega - F_0)/E_0]$ where F_0 is the energy difference between the electron quasi-Fermi level in the emitting region with all $S_\omega = 0$ and the edge of the valence band. From these curves we conclude that this envelope has the proper shape for bistability.

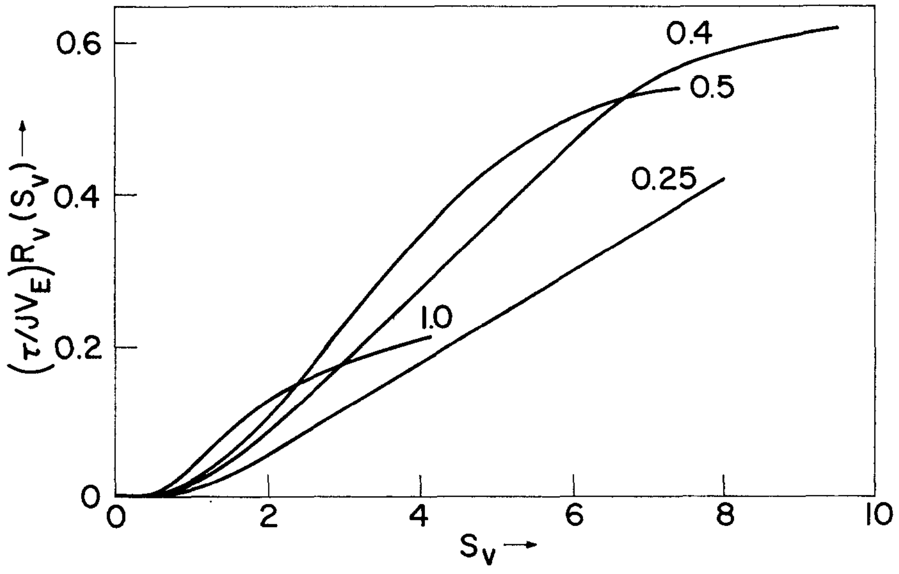
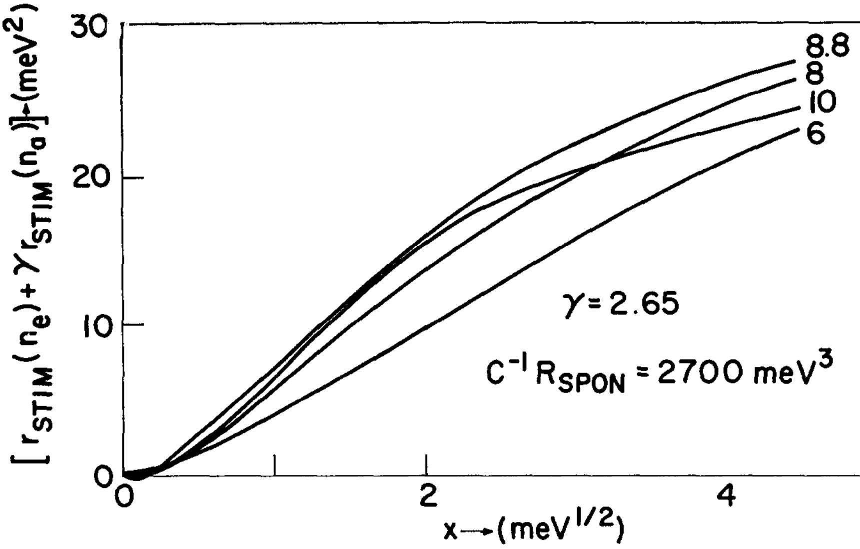


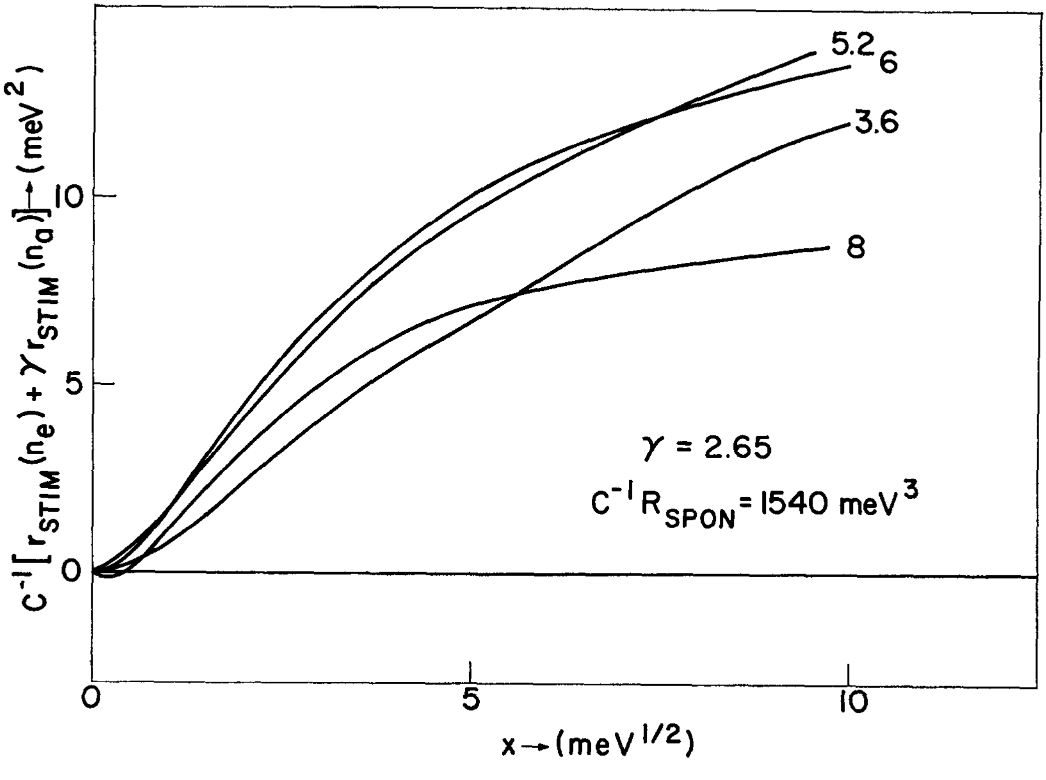
FIG. 5. Stimulated emission rates at low temperatures for an exponential energy dependence of the conduction band density of states and a square root dependence of the valence band density of states vs. the photon population of a single mode. The various curves are labeled by their values of a quantity $\exp[(\omega - F_0)/E_0]$ which specifies the optical mode frequency, ω. F_0 is the energy difference between the electron quasi-Fermi level in the emitting region for all $S_v = 0$ and the top of the valence band. A length ratio $\gamma = 1{\cdot}41$ was assumed.

HIGHER TEMPERATURE ANALYSIS

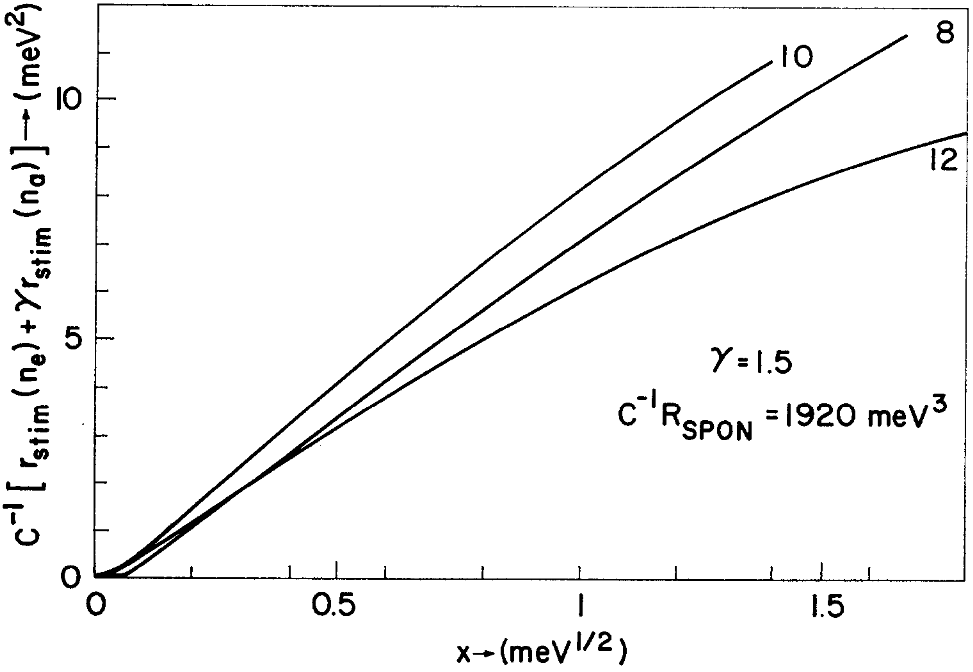
Simple analytic expressions for the gain function do not exist for temperatures comparable to the electron or hole degeneracy temperatures. We therefore use the numerical results of Lasher and Stern for parabolic bands at 80°K and a valence band quasi-Fermi level of 11·8 MeV. The procedure is of necessity entirely numerical, and the stimulated emission functions for certain length ratios, γ and currents are given in Figs. 6(*a*), 6(*b*) and 6(*c*). It is no longer true that the shape of these curves is independent of current and therefore these results only apply if the mode decay time, τ_ω, and other parameters have the proper values to make the linear rate of photon escape have two intersections with the envelope. The curves of 6(*a*), 6(*b*) and 6(*c*) all indicate bistability, with as before, a more clear-cut prediction for larger length ratio, γ. The argument given for lack of bistability with a sharp effective band edge does not apply here because the Fermi distribution function is smooth and does not cut off the absorption sharply at low frequencies.


30
20
10
0
0
2
4
8.8
8
10
6
γ = 2.65
C⁻¹ R_SPON = 2700 meV³
[r_STIM(n_e) + γ r_STIM(n_a)] → (meV²)
x → (meV^1/2)

(a)


10
5
0
0
5
10
5.2
6
3.6
8
γ = 2.65
C⁻¹ R_SPON = 1540 meV³
C⁻¹[r_STIM(n_e) + γ r_STIM(n_a)] → (meV²)
x → (meV^1/2)

(b)



(c)

FIG. 6. Stimulated emission rates at 80°K for parabolic conduction and valence bands vs. a quantity proportional to the photon population of a single optical mode. They were computed from the numerical data of LASHER and STERN.[4] Each set of curves *a*, *b* and *c* are for a different length ratio, γ, and injection currents specified by $C^{-1}R_{\text{spon}}$ defined in Section 2 of Lasher and Stern. The curve is labeled by the energy of the mode above the effective band gap in MeV.

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