Nonlocal Thermoelastic Damping in Microelectromechanical Resonators

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Abstract: The evaluation of loss mechanisms in microscale mechanical resonators is addressed. Among various dissipation causes, thermoelastic loss is considered as a fundamental dissipation mechanism in microbeam resonators packed in a near-vacuum environment. However, the standard thermoelastic analysis is unable to interpret the size effect experimentally evidenced in resonators when the dimensions become very small, below several microns. In this paper we propose an enhanced nonlocal thermoelastic model, based on a thermodynamical formulation, which incorporates internal characteristic material lengths. Analytical results obtained with this nonlocal theory are compared with experimental results reported in the literature. It is shown how nonlocality can better interpret the observed behavior, at least in a certain range of resonators dimensions.

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Introduction

The efficiency of microelectromechanical systems with vibrating parts, such as high-frequency resonators, is usually quantified by means of the so-called *quality factor Q*, which is proportional to the ratio between the resonant frequency for the undamped system and the damping factor. If the damping is not excessive (i.e., Q is larger than a certain threshold), the inverse of the quality factor can be physically interpreted as the fraction of energy lost per radian. Although strong miniaturization allows the designer to increase the resonant frequency and thus the sensitivity of the resonators, it is difficult to increase the quality factor, as evidenced by several experimental studies (Yasumura et al. 2000; Yang et al. 2002).

The sources of dissipation can be roughly divided into two categories: the first one ("fluid damping") embraces phenomena related to the interaction of the solid parts with the surrounding gases; the second is connected to loss mechanisms in the solid material, and for this reason it is often called "intrinsic damping" or "solid damping."

Fluid damping (studied in Frangi et al. 2006) has been recognized as a major source of dissipation in case of moderately high gas pressure. Its importance is, however, dramatically reduced when the micromechanical devices are packed in a near-vacuum environment; moreover, it has been shown that, in the low pressure regime, the high frequency resonators are less susceptible to gas friction rather than low frequency ones (Mohanty et al. 2002). On the other hand, solid damping is induced by a lot of physical and chemical processes and many of them are still needing further investigation.

Among them, thermoelastic loss is considered as a fundamental dissipation mechanism in microbeam bending resonators (Lifshitz and Roukes 2000). The dissipation is caused by the complex interaction of acoustic modes with the thermally excited modes in the crystalline lattice. It is common practice to evaluate the thermoelastic quality factor by means of simplified formulas, such as the well-known Zener's expression (1937).

The Q factor predicted by a thermoelastic analysis is in good agreement with the quality factor experimentally measured on several silicon microresonators (Roszhart 1990).

However, the classical local thermoelastic analysis is unable to interpret the size effect recently evidenced in resonators when the dimensions become very small, below several microns (see, e.g., Le Foulgoc et al. 2006; Yasumura et al. 2000). Several causes of additional dissipation, not yet exhaustively investigated, come into play. Intrinsic dissipation of the superficial oxide layer, surface loss, and support loss may become relevant at very small scales, as discussed by Le Foulgoc et al. (2006). Each mechanism is connected to an amount of dissipated energy and, consequently, to a value Q_j. The overall quality factor can therefore be expressed as

$$Q = \left(\sum_j \frac{1}{Q_j} \right)^{-1} \tag{1}$$

In this work we focus on the thermoelastic dissipation only and we investigate the possibility to enhance the thermoelastic prediction by using a nonlocal model. In fact, as it has been evidenced in other contexts, the discrepancy between theory predictions and experimental results when the scale of the problem decreases, suggests that some internal characteristic length is not sufficiently small as compared to the external scales of the problem to justify the use of continuum local theory. To extend the domain of applicability of continuum theories at small scale problems nonlocal elasticity models were formulated by Eringen (1972) and Eringen and Edelen (1972) and then applied

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in many areas including nanotechnology (see, e.g., Eringen 1987; Peddieson et al. 2003; Ece and Aydogdu 2007).

Following the thermodynamic formulation developed by Polizzotto (2001) for nonlocal elasticity, in this paper we propose an enhanced nonlocal coupled thermoelastic model. Nonlocal total strains and temperature are introduced, depending on two internal characteristic material lengths.

The nonlocal model is then applied to evaluate thermoelastic loss in microbeam resonators and a refined Zener's type formula is obtained, accounting for nonlocality.

Parametric studies show that nonlocality can increase the thermoelastic damping, resulting in a lower quality factor, which can better interpret the observed behavior, at least in a certain range of resonators dimensions.

Nonlocal Thermoelastic Model

Consider a body of volume V in the three-dimensional Euclidean space, with a reference frame $\mathbf{x}=(x_1,x_2,x_3)$. Assume that this body elastically deforms in the small strain regime, subject to external loads and temperature changes. The local thermoelastic constitutive law is usually obtained in a thermodynamic framework by defining pointwise, at each point $\mathbf{x}$, a free-energy potential, which is a function of the local values of the state variables total strain $\boldsymbol{\varepsilon}$ and temperature T. The nonlocal thermoelastic constitutive law here developed requires instead the definition of a free energy potential ψ depending on the state variables fields at all points of the body.

According to Eringen (1972), the effect of the material microstructure can be included in the continuum model by introducing a nonlocal stress–strain relation in which the stress field at a point depends on the strain field in the whole body. In the linear elastic case this results in the definition of a nonlocal strain variable

$$\bar{\boldsymbol{\varepsilon}}(\mathbf{x}) = \Re_l(\boldsymbol{\varepsilon}) \equiv \int_V W_l(\mathbf{x}-\mathbf{s})\boldsymbol{\varepsilon}(\mathbf{s})d\mathbf{s} \quad \forall\, \mathbf{x} \in V \tag{2}$$

$\Re_l(\cdot)$ denotes an integral self-adjointed nonlocal operator and $W_l(\mathbf{x}-\mathbf{s})$ represents the scalar attenuation (or weight) function which depends on an internal characteristic length l of the material. We also introduce a nonlocal temperature variable defined as

$$\bar{T}(\mathbf{x}) = \Re_\lambda(T) \equiv \int_V W_\lambda(\mathbf{x}-\mathbf{s})T(\mathbf{s})d\mathbf{s} \quad \forall\, \mathbf{x} \in V \tag{3}$$

where $W_\lambda(\mathbf{x}-\mathbf{s})$ represents the scalar attenuation function, depending on another internal length λ. Different choices can be done for the attenuation functions. For isotropic solids they should depend only on the distance $r=\|\mathbf{x}-\mathbf{s}\|$ between points $\mathbf{x}$ and $\mathbf{s}$ and they should tend to zero as the ratio r/l or r/λ become large. In the following we will adopt the Gaussian attenuation function

$$W_l(\mathbf{x}-\mathbf{s}) = \frac{1}{W_{0l}}\exp\left[-\frac{\|\mathbf{x}-\mathbf{s}\|^2}{l^2}\right] \tag{4a}$$

$$W_\lambda(\mathbf{x}-\mathbf{s}) = \frac{1}{W_{0\lambda}}\exp\left[-\frac{\|\mathbf{x}-\mathbf{s}\|^2}{\lambda^2}\right] \tag{4b}$$

where W_{0l} and $W_{0\lambda}$=constant values introduced in order to fulfill the following normalization conditions:

$$\int_{V_\infty} W_l(\mathbf{x}-\mathbf{s})dV = \int_{V_\infty} W_\lambda(\mathbf{x}-\mathbf{s})dV = 1 \tag{5}$$

where V_∞ represents the infinite Euclidean space. When the internal lengths tend to zero, the attenuation functions tend to the δ-Dirac function and the local model is recovered. We will call l the *mechanical internal length* and λ the *thermal internal length*. Both are assumed to be material-dependent characteristic lengths related to the material microstructure. The consequences of different assumptions on these parameters will be discussed later.

We assume the following quadratic form of the free energy for a nonlocal thermoelastic body, which extends the one proposed in Polizzotto (2001) to the nonisothermal case

$$\psi = \frac{1}{2}\boldsymbol{\varepsilon}:\mathbf{D}:\bar{\boldsymbol{\varepsilon}} - \boldsymbol{\varepsilon}:\mathbf{D}:[\alpha\mathbf{I}(T-T_0)] - \frac{\rho C}{2T_0}T\bar{T} \tag{6}$$

where $\mathbf{D}$=elastic moduli tensor; $\mathbf{I}$=second order identity tensor; α=thermal expansion coefficient; T_0=reference temperature; ρ=material density; and C=specific heat. Although strain energy and thermal contribution in Eq. (6) are affected by nonlocality, the thermoelastic coupling term is considered local. This simplifying assumption leads to a simple nonlocal model, which can be employed to obtain semianalytical solutions for the beam resonator.

The state equations defining stresses σ and entropy density s are obtained taken the derivatives of ψ with respect to the state variables $(\boldsymbol{\varepsilon},T)$. As the nonlocal operators in Eqs. (2) and (3) are self-adjoint one has

$$\boldsymbol{\sigma} = \frac{\partial\psi}{\partial\boldsymbol{\varepsilon}} + \Re_l\left(\frac{\partial\psi}{\partial\bar{\boldsymbol{\varepsilon}}}\right) = \mathbf{D}:\bar{\boldsymbol{\varepsilon}} - \alpha\mathbf{D}:\mathbf{I}(T-T_0) \tag{7a}$$

$$s = -\frac{\partial\psi}{\partial T} - \Re_\lambda\left(\frac{\partial\psi}{\partial\bar{T}}\right) = \alpha\boldsymbol{\varepsilon}:\mathbf{D}:\mathbf{I} + \frac{\rho C}{T_0}\bar{T} \tag{7b}$$

The nonlocal linear thermoelastic problem is defined by the above-mentioned constitutive laws and by equilibrium and heat conduction equations. Denoting by $\mathbf{b}$, the body force vector, and by $\ddot{\mathbf{u}}$, the acceleration vector, dynamic equilibrium reads

$$\mathrm{div}\,\boldsymbol{\sigma} + \mathbf{b} = \rho\ddot{\mathbf{u}} \tag{8}$$

Heat conduction is expressed by the Fourier heat transfer equation, modified with the introduction of a heat source due to thermomechanical coupling and accounting for nonlocality, Eq. (7*b*)

$$\rho C\dot{\bar{T}} = -\,\mathrm{div}(-\,\mathbf{k}\cdot\mathrm{grad}\,T) - \alpha T_0\dot{\boldsymbol{\varepsilon}}:\mathbf{D}:\mathbf{I} \tag{9}$$

where $\mathbf{k}=k\,\mathbf{I}$=(isotropic) conductivity tensor.

Remark 1: If the definition in Eq. (2) is adopted, as already proposed by Eringen (1972), the nonlocal strain field is nonuniform also when the local strain field is uniform. This fact does not appear to be a drawback for the microscale problems considered here; on the contrary, it introduces a sort of "boundary layer effect," which increases the nonlocal effect in small scale structures, consistently with experimental data, as it will be shown in section entitled "Discussion of the Results and Experimental Validation."

To recover a uniform nonlocal strain field one can introduce a normalization factor in the weight functions, which accounts for the presence of a finite volume body V (see, e.g., Comi 2001)

$$W_a^*(\mathbf{x}-\mathbf{s}) = \frac{W_{a0}}{\int_V \exp\left[-\frac{\|\mathbf{x}-\mathbf{s}\|^2}{a^2}\right] d\mathbf{s}} W_a(\mathbf{x}-\mathbf{s}), \quad a = l \text{ or } \lambda \tag{10}$$

However, the above-mentioned normalization produces nonsymmetric operators. Alternative proposals of nonlocal symmetric formulations can be found in Polizzotto et al. (2006).

Remark 2: The nonlocal formulation can be extended to include anisotropic solids (Eringen 1972, 1987).

The nonlocal elastic stress–strain relation requires the definition of a nonlocal elastic tensor $\bar{\mathbf{D}}$ depending on the difference of coordinates **x-s** between two points (and not only on their distance). This tensor should have the same group symmetry of the local one **D** and, in general, depends on three internal lengths. The nonlocal state equation for isothermal transformations reads

$$\boldsymbol{\sigma}(\mathbf{x}) = \int_V \bar{\mathbf{D}}(\mathbf{x}-\mathbf{s}):\boldsymbol{\varepsilon}(\mathbf{s})d\mathbf{s} \tag{11}$$

This expression could be easily generalized to the nonisothermal case.

The anisotropic formulation represented by Eq. (11) should be considered when dealing with resonators made of polysilicon with columnar structure (see, e.g., Mariani et al. 2007)

Extension of Zener's Model

In thermoelastic solids the coupling of the strain field to the temperature field induces the irreversible flow of heat driven by temperature gradient. This process of energy dissipation is called thermoelastic damping (TED) and introduces an upper limit to the quality factor of resonators.

The modal analysis of a thermoelastic solid yields eigenvalues ζ, which are generally complex; the imaginary part $\mathrm{Im}(\zeta)$ represents the oscillatory part of free-vibration thermoelastic response and the real part $\mathrm{Re}(\zeta)$ influences the vibration decay. The thermoelastic quality factor Q is defined by

$$Q = \frac{|\mathrm{Im}(\zeta)|}{2|\mathrm{Re}(\zeta)|} \tag{12}$$

which is the fraction of energy lost per radian.

Zener (1937) studied the special case of a thermoelastic vibrating beam. In this case, the usual Bernoulli–Euler kinematic hypothesis is assumed and the heat conduction is limited to the transverse direction. In the limits of validity of such hypotheses, an approximated closed-form solution for the quality factor can be obtained

$$Q = \frac{\rho C}{E\alpha^2 T_0}\frac{1+(\omega\tau_z)^2}{\omega\tau_z} \tag{13}$$

where E=material Young's modulus; T_0=reference temperature; ω=natural frequency of the beam; and τ_z=relaxation time, defined as follows in terms of beam thickness h and thermal diffusivity $k/\rho C$

$$\tau_z = \frac{h^2}{\pi^2(k/\rho C)} \tag{14}$$

The Zener's solution Eq. (13) is referred to the first vibration mode of the thermoelastic beam; it is worth mentioning that: (1) for real-life material properties, the first thermoelastic natural frequency is very similar to the purely mechanical one; and (2) in case of rectangular beams the thermoelastic dissipation is essentially dominated by the first mode (Lifshitz and Roukes 2000).

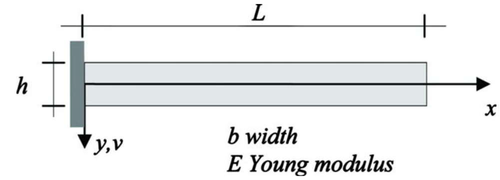

Fig. 1. Schematic representation of the beam's geometry

The Zener's formula is now extended to the case of nonlocal thermoelastic behavior. Consider a thin beam schematically representing a resonator, shown in Fig. 1. In order to obtain an analytical solution for the nonlocal thermoelastic beam, we assume that nonlocality manifests only in the transversal (y) direction. This hypothesis is justified by the fact that, although the height h of the beam is very small, and thus nonlocality is expected to have relevance, the beam is usually long enough to justify the use of a local model.

According to the Bernoulli–Euler hypothesis axial strain is expressed in terms of beam curvature by

$$\varepsilon = -\frac{d^2 v}{dx^2}y \tag{15}$$

where v=transversal displacement of the cross-section centroid. The nonlocal strain is then

$$\bar{\varepsilon} = -\frac{d^2 v}{dx^2}\int_{-h/2}^{h/2} \eta W_l(y-\eta)d\eta \tag{16}$$

Using the thermoelastic model in Eq. (7*a*), the bending moment can be computed as follows:

$$M = -E\bar{I}\frac{d^2 v}{dx^2} - bE\alpha\int_{-h/2}^{h/2}(T-T_0)y\,dy \tag{17}$$

having defined a "nonlocal moment of inertia" $\bar{I}$ in the following form:

$$\bar{I} \equiv b\int_{-h/2}^{h/2}\int_{-h/2}^{h/2} \eta W_l(y-\eta)d\eta\, y\,dy \tag{18}$$

By considering the expression of the attenuation function W_l, Eq. (4*a*), it is possible to demonstrate that the nonlocal moment of inertia is always smaller than the "standard" value $I \equiv bh^3/12$.

According to Eq. (3), the nonlocal temperature time rate is expressed as follows:

$$\dot{\bar{T}} = \int_{-h/2}^{h/2} \dot{T}(\eta)W_\lambda(y-\eta)d\eta \tag{19}$$

If one neglects heat conduction along the longitudinal direction of the beam, the governing equations for thermoelastic vibration are

$$\rho h b\ddot{v} - \frac{d^2}{dx^2}\left[-E\bar{I}\frac{d^2 v}{dx^2} - bE\alpha\int_{-h/2}^{h/2}(T-T_0)y\,dy\right] = 0 \tag{20}$$

$$\rho C\int_{-h/2}^{h/2} \dot{T}(\eta)W_\lambda(y-\eta)d\eta = \alpha T_0 E\frac{d^2\dot{v}}{dx^2}y + k\frac{\partial^2 T}{\partial y^2} \tag{21}$$

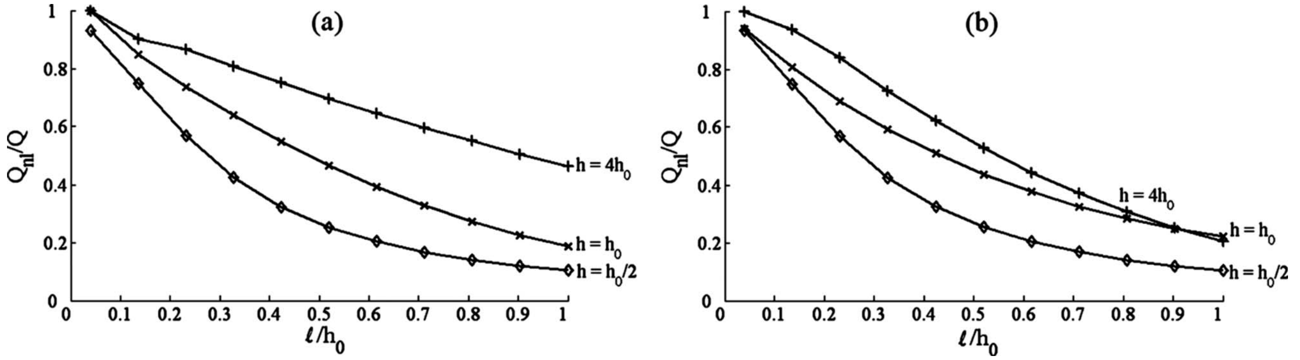

Fig. 2. Nonlocal quality factor versus characteristic length for different values of beam thickness h: (a) purely mechanical nonlocal model, $\lambda=0$; (b) thermal and mechanical nonlocal model with $\lambda=l$

The coupled thermoelastic problem is solved for the case of harmonic vibrations of the form

$$v(x,t)=V(x)e^{i\omega t} \tag{22}$$

Assuming adiabatic boundary conditions at $y=\pm h/2$, the solution in terms of temperature is sought in the form of the Fourier series

$$T=T_0+\sum_{n=0}^{\infty} G_n e^{i\omega t}\sin\left[(2n+1)\pi\frac{y}{h}\right] \tag{23}$$

Substituting into the heat equation, after some algebra, one obtains

$$\rho C i\omega\sum_{n=0}^{\infty} G_n\int_{-h/2}^{h/2}\sin\left[(2n+1)\pi\frac{\eta}{h}\right]W_\lambda(y-\eta)d\eta + k\sum_{n=0}^{\infty}\left[(2n+1)\frac{\pi}{h}\right]^2 G_n\sin\left[(2n+1)\pi\frac{y}{h}\right]=\alpha T_0 E i\omega\frac{d^2V}{dx^2}y \tag{24}$$

Multiplication of Eq. (24) by $\sin[(2m+1)\pi y/h]$ and subsequent integration over the beam height yield the following set of equations:

$$i\omega\sum_{n=0}^{\infty} G_n\kappa_{mn}+G_m\mu_m=i\omega\beta\frac{d^2V}{dx^2}\gamma_m,\quad m=0,1,\ldots \tag{25}$$

where

$$\kappa_{mn}=\frac{2}{h}\int_{-h/2}^{h/2}\sin\left[(2m+1)\pi\frac{y}{h}\right]\times\int_{-h/2}^{h/2}\sin\left[(2n+1)\pi\frac{\eta}{h}\right]W_\lambda(y-\eta)d\eta dy \tag{26}$$

$$\beta=\frac{E\alpha T_0}{\rho C};\quad \mu_m=\frac{(2m+1)^2\pi^2}{h^2}\frac{k}{\rho C};\quad \gamma_m=\frac{2h}{\pi^2(4m^2+4m+1)}2(-1)^m \tag{27}$$

It is worth noting that the coefficients κ_{mn}, which include the effects of nonlocality, are always smaller than unity, for the chosen attenuation function W_λ Eq. (4b).

The solution of the fully coupled linear system in Eq. (25) gives the temperature field. It is possible to demonstrate numerically that: (1) good accuracy is obtained even by considering a small number of harmonics; and (2) the coupling terms κ_{mn}, $m\neq n$, are negligible if compared to the diagonal values κ_n in the case of small n. For these reasons, the linear system in Eq. (25) can be artificially decoupled by ignoring the coupling coefficients. This operation, which entails no significant errors in the final solution, allows one to achieve the temperature field in the following form:

$$T-T_0=\frac{d^2V}{dx^2}\sum_{n=0}^{\infty}\frac{i\omega\beta\gamma_n}{i\omega\kappa_n+\mu_n}e^{i\omega t}\sin\left[(2n+1)\pi\frac{y}{h}\right] \tag{28}$$

Introducing Eq. (28) into the dynamic equilibrium equation, one obtains the following fourth-order differential equation:

$$\rho hb\omega^2V+i\left(Ehb\frac{\alpha\beta}{2}\sum_n\frac{\omega\mu_n\gamma_n^2}{\omega^2\kappa_n^2+\mu_n^2}\right)\frac{d^4V}{dx^4}+\left(E\bar{I}+Ehb\frac{\alpha\beta}{2}\sum_n\frac{\omega^2\kappa_n\gamma_n^2}{\omega^2\kappa_n^2+\mu_n^2}\right)\frac{d^4V}{dx^4}=0 \tag{29}$$

In the previous equation, due to thermal coupling, the second dissipative term arises. As shown by Zener (1937), the quality factor, formally defined by Eq. (12), can be numerically evaluated as the ratio between the elastic deformation energy, proportional to the last real term in Eq. (29), and the dissipated energy, proportional to the imaginary term in Eq. (29)

$$Q_{nl}=\frac{\bar{I}+hb\frac{\alpha\beta}{2}\sum_n\frac{\omega^2\kappa_n\gamma_n^2}{\omega^2\kappa_n^2+\mu_n^2}}{hb\frac{\alpha\beta}{2}\sum_n\frac{\omega\mu_n\gamma_n^2}{\omega^2\kappa_n^2+\mu_n^2}} \tag{30}$$

The expression of the quality factor is very similar to the one derived by Zener (1937) for thermoelastic damping of thin beams; the nonlocal effect here considered modifies the quality factor through the nonlocal moment of inertia $\bar{I}$, which incorporates the mechanical internal characteristic length l, and through the coefficients κ_n, which contain the thermal internal characteristic length λ.

Discussion of the Results and Experimental Validation

The quality factor Q_{nl} computed in the previous section depends on the characteristic lengths of the nonlocal model and is always

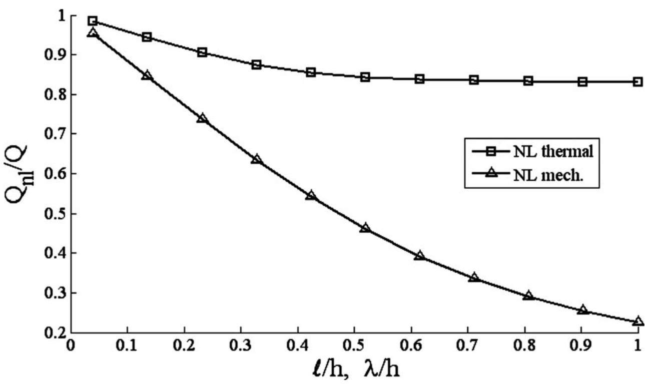

Fig. 3. Nonlocal quality factor versus characteristic length. The two curves are referred to a purely mechanical nonlocal model ($\lambda \to 0$) and to a purely thermal one ($l \to 0$).

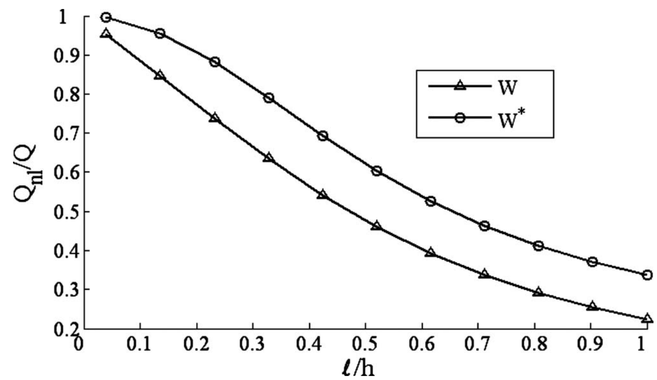

Fig. 5. Nonlocal quality factor versus characteristic length obtained with a symmetric attenuation function (W) and with a nonsymmetric version (W^*)

smaller than the local quality factor Q. This means that thermoelastic dissipation of the enhanced model is higher than the one of the local model. In qualitative agreement with experimental observation, this difference becomes important for very small beams height h. In particular, Fig. 2 shows the evolution of the ratio Q_{nl}/Q with the nondimensional characteristic length for three different beam height. Fig. 2(a) refers to the nonlocal mechanical model (with $\lambda=0$), whereas Fig. 2(b) refers to the nonlocal model with $\lambda=l$. In all cases the normalized quality factor decreases as the internal length increases; the lower curve, i.e., the one with the greater effect of nonlocality, corresponds to the thinner beam. The value h_0 is an arbitrary reference beam height, here introduced in order to make all lengths nondimensional and to emphasize the effect of the internal length on beams of different thickness.

Fig. 3 shows the evolution of the ratio Q_{nl}/Q with the ratio between the characteristic length and the beam height. The two curves correspond to the limit cases of zero interaction length for the temperature ($\lambda=0$ and varying l, triangle symbols) and zero interaction length for the strain ($l=0$ and varying λ, square symbols). For fixed height h the ratio decreases as the material lengths increase. This effect is more important for the model with nonlocal strain definition; for the model with nonlocal temperature the curve for increasing λ tends to an asymptotic value. The contribution of nonlocality on the thermal part with respect to the mechanical nonlocality is evidenced in Fig. 4, where different values of λ are considered. Only for small ratios l/h the thermal nonlocal contribution plays a significant role.

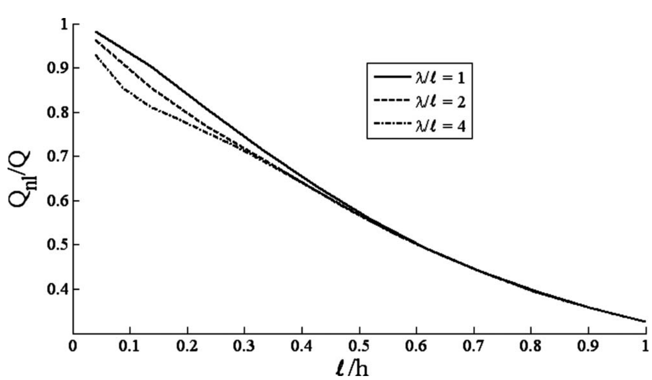

Fig. 4. Nonlocal quality factor versus characteristic length for a thermal and mechanical nonlocal model with various λ/l values

Fig. 5 compares the predictions obtained with the adopted attenuation function W, Eq. (4) to those obtained with the normalized attenuation function W^*, Eq. (10). The boundary layer effect introduced by W results in a lower quality factor, which could better fit the experimental results.

The quality factor computed with the proposed model has been compared with experimental values measured on microresonators reported in the literature. It is worth noting that many experimental results evidence a size effect on the damping behavior of resonators when their height becomes very small; such an effect cannot be reproduced by a local thermoelastic analysis. In fact it is possible to show, on an experimental bases, that only for "thicker" beams the dissipation level agrees with the classical thermoelastic computations. Fig. 6(a) reports the experimental results published by various authors concerning the quality factor of resonators of different heights: the circles refer to data by Le Foulgoc et al. (2006), $h=30$ μm; the asterisks and diamonds to Yasumura et al. (2000), $h=2.3$ and 1.2 μm; and the squares to Yang et al. (2002), $h=0.5$ μm. It has been possible to plot all the data (referred to different material) in the same graph by considering dimensionless variables. The independent variable is represented by the ratio between the beam frequency $f=\omega/2\pi$ and the characteristic frequency f_0, which can be correlated to the relaxation time τ_z, Eq. (14), as follows:

$$f_0 = \frac{1}{2\pi\tau_z} \tag{31}$$

The quality factor is normalized by considering the material-related quantity Ψ

$$\Psi = \frac{1}{2}\frac{\rho C}{E\alpha^2 T_0} \tag{32}$$

By considering Fig. 6(a), it is evident that only the thicker beams (thickness $h=30$ μm) show a quality factor in sufficient agreement with the Zener's curve. The thinner structures, even if characterized by similar relative frequencies, exhibit a markedly different behavior, with small values of the quality factor. This effect could in part be explained by the nonlocal model. In fact, by keeping a fixed value of the internal characteristic length, the ratio l/h increases as the thickness decreases. In this situation, as demonstrated by Fig. 6(b) (which is referred to a purely mechanical nonlocal model), the nonlocal quality factor turns out to be smaller than the standard values.

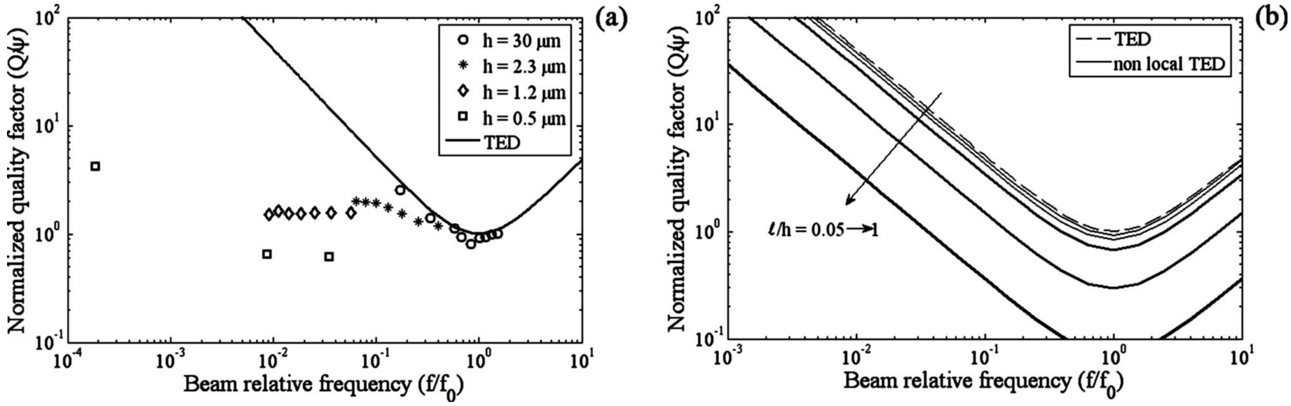

Fig. 6. Experimental evidences of dimensional effects and possible interpretation by the nonlocal thermoelastic model: (a) normalized quality factors resulting by various experiments are plotted versus beam relative frequency and compared to the Zener's curve's curve; (b) a similar reduction of Q is achieved by nonlocal computations for different l/h ratios [data adapted from Le Foulgoc et al. (2006), Yang et al. (2002), and Yasumura et al. (2000)]

The nonlocal predictions could better fit the experimental results. However, it should be noted that, for a reasonable value of the internal length (see, e.g., Eringen 1987 for a discussion), nonlocal thermoelastic dissipation is quantitatively not sufficient to explain the quality factor drop. Other dissipation sources, related to surface loss and support loss, also mentioned in Le Foulgoc et al. (2006), should be included.

In order to provide a quantitative assessment of the nonlocal thermoelastic model, the aforementioned experimental outcomes have been compared to the numerical simulations, in which the thermoelastic dissipation is computed by considering a nonlocal model with the same mechanical and thermal internal length $\lambda = l$. This material properties is a priori unknown and should be identified for each material; in this paper, for the sake of illustration, we have considered four different values of the internal length, namely 100, 250, 500, and 1,000 nm. Keeping constant these values, we have considered the effect of nonlocality for various beam's height.

We have considered first the tests of Le Foulgoc et al. (2006) on clamped-free single-crystal silicon resonators. The quality factor has been measured for different beam lengths (ranging from 280 to 1,200 μm). The mechanical and thermal properties of single-crystal silicon are summarized in Table 1. Fig. 7 displays the quality factor versus relative frequency for 30 μm thick resonators. The symbols represent the experimental values, the dotted line corresponds to thermoelastic dissipation of a standard local model and solid lines correspond to nonlocal TED. The ratios l/h for the nonlocal curves are recalled in the Fig. 7 (the line referred to $l = 100$ nm is totally superposed to the Zener's curve). For this case of large dimensions, the nonlocal effect practically vanishes and the nonlocal quality factor tends to the local one, which correctly predicts experimental results.

We have also considered the experimental tests performed by Yang et al. (2002) on silicon submicron beam resonators. In particular, Fig. 8 shows the experimental results and the numerical outcomes (in terms of quality factor versus relative frequency f/f_0) for a series of 500 nm thick beams with various length (ranging from 6 to 82 μm). The material properties are summa-

Table 1. Thermal and Mechanical Properties of Single-Crystal Silicon

Property	Value	Property	Value
E	150 GPa	ρ	2,330 kg/m^3
v	0.2	C	700 J/(kg K)
α	2.6×10^{-6}	k	148 W/(m K)

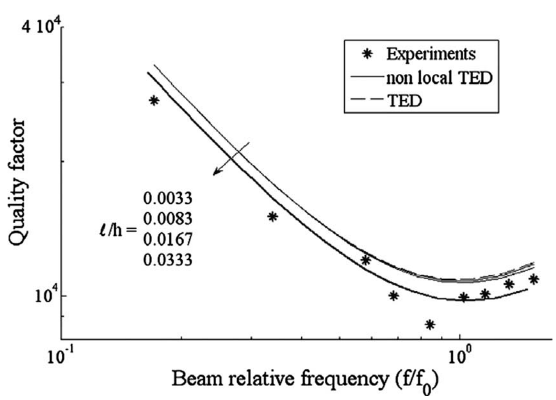

Fig. 7. Experimental validation of the nonlocal thermoelastic model for single-crystal Si beams. Quality factor versus beam relative frequency; beam thickness $h = 30$ μm, beam depth $b = 15$ μm [data adapted from Le Foulgoc et al. (2006)].

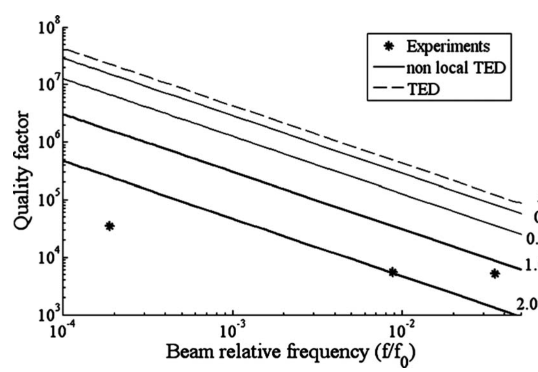

Fig. 8. Experimental validation of the nonlocal thermoelastic model for single-crystal Si beams. Quality factor versus beam relative frequency; beam thickness $h = 0.5$ μm [data adapted from Yang et al. (2002)].

Table 2. Thermal and Mechanical Properties of Silicon Nitride (SiN)

Property	Value	Property	Value
E	126 GPa	ρ	3,440 kg/m^3
υ	0.2	C	710 J/(kg K)
α	3.0×10^{-6}	k	3.2 W/(m K)

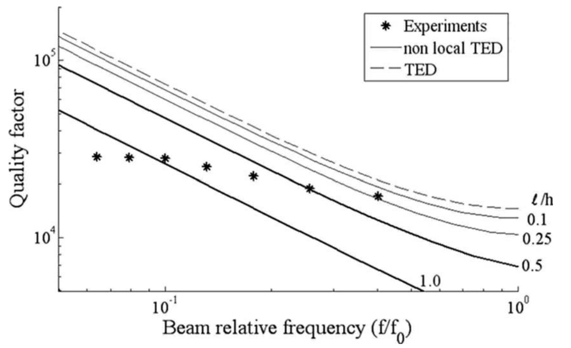

Fig. 9. Experimental validation of the nonlocal thermoelastic model for SiN beams. Quality factor versus beam relative frequency; beam thickness $h=2.3$ μm, beam depth $b=10$ μm [data adapted from Yasumura et al. (2000)].

rized in Table 1. The nonlocal model provides an improvement of the thermoelastic prediction, even though in this case, account taken of the very small thickness, the other dissipation sources (surface losses above all) play a more significant role.

Finally, we have computed the quality factor for the silicon–nitride 2.3 μm thick resonators tested by Yasumura et al. (2000). The mechanical and thermal properties for such material are reported in Table 2. Fig. 9 compares the local and nonlocal quality factors with the experimental values, for beam lengths ranging from 90 to 300 μm. The aforementioned effects of nonlocal thermoelasticity are therefore confirmed also for this different material.

Conclusions

The aim of this work is to improve the estimate of the quality factor related to thermoelastic dissipation by enriching the continuum model. An integral nonlocal model is proposed to take into account, in a phenomenological way, the effect of the microstructure on the dynamic response of resonant microelectromechanical systems (MEMS). The influence of a finite material length scale manifests itself when the dimensions of the resonators becomes extremely small. The quality factor computed with the nonlocal model better interprets experimental results, even though other dissipation phenomena, mainly related to surface loss, should also be included.

In order to obtain semianalytical results the nonlocal variables are computed by averaging only in the transversal direction of the beam. The average in the longitudinal direction can be easily included in a finite element (FE) formulation, currently under development. The numerical FE model would allow one to consider different model geometries, other than the rectangular beam, e.g., as the resonators considered by Abdolvand et al. (2003).

In this work the thermoelastic stress–strain relation and the entropy are defined starting from a free energy involving averages of the state variables, strains and temperature, whereas the dissipation potential and hence of the conduction term in the heat equation are defined in the standard local form. The consequences of different assumptions will be discussed in a forthcoming paper.

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References

Abdolvand, R., Ho, G. K., Erbil, A., and Ayazi, F. (2003). "Thermoelastic damping in trench-refilled polysilicon resonators." *Proc., Transducers '03*, IEEE, Boston, 324–327.

Comi, C. (2001). "A nonlocal model with tension and compression damage mechanisms." *Eur. J. Mech. A/Solids*, 20(11), 1–22.

Ece, M. C., and Aydogdu, M. (2007). "Nonlocal elasticity effect on vibration of in-plane loaded double-walled carbon nano-tubes." *Acta Mech.*, 190(1–4), 185–195.

Eringen, A. C. (1972). "Linear theory of nonlocal elasticity and dispersion of plane waves." *Int. J. Eng. Sci.*, 10(5), 425–435.

Eringen, A. C. (1987). "Theory of nonlocal elasticity and some applications." *Res. Mech.*, 21(4), 313–342.

Eringen, A. C., and Edelen, D. G. B. (1972). "On nonlocal elasticity." *Int. J. Eng. Sci.*, 10(3), 233–248.

Frangi, A., Spinola, G., and Vigna, B. (2006). "On the evaluation of damping in MEMS in the slip-flow regime." *Int. J. Numer. Methods Eng.*, 68(10), 1031–1051.

Le Foulgoc, B., Bourouina, T., Le Traon, O., Bosseboeuf, A., Marty, F., Breluzeau, C., Grandchamp, J.-P., and Masson, S. (2006). "Highly decoupled single-crystal silicon resonators: An approach for the intrinsic quality factor." *J. Micromech. Microeng.*, 16(6), S45–S53.

Lifshitz, R., and Roukes, M. L. (2000). "Thermoelastic damping in micro- and nanomechanical systems." *Phys. Rev. B*, 61(8), 5600–5609.

Mariani, S., Ghisi, A., Corigliano, A., and Zerbini, S. (2007). "Multiscale analysis of MEMS sensors subject to drop impacts." *Sensors*, 7(9), 1817–1833.

Mohanty, P., Harrington, D. A., Ekinci, K. L., Yang, Y. T., Murphy, M. J., and Roukes, M. L. (2002). "Intrinsic dissipation in high frequency micromechanical resonators." *Phys. Rev. B*, 66(8), 085416/1–085416/15.

Peddieson, J., Buchanan, G. R., and McNitt, R. P. (2003). "Application of nonlocal continuum models to nanotechnology." *Int. J. Eng. Sci.*, 41(3–5), 305–312.

Polizzotto, C. (2001). "Nonlocal elasticity and related variation principles." *Int. J. Solids Struct.*, 38(42–43), 7359–7380.

Polizzotto, C., Fuschi, P., and Pisano, A. A. (2006). "A nonhomogeneous nonlocal elasticity model." *Eur. J. Mech. A/Solids*, 25(2), 308–333.

Roszhart, T. V. (1990). "The effect of the thermoelastic internal friction on the Q of micromachined silicon resonators." *Proc., Tech. Dig. Solid-State Sens. Actuators Workshop*, IEEE, Hilton Head, S.C., 13–16.

Yang, J. L., Ono, T., and Esashi, M. (2002). "Energy dissipation in submicrometer thick single-crystal silicon cantilevers." *J. Microelectromech. Syst.*, 11(6), 775–783.

Yasumura, K. Y., Stowe, D. T., Chow, E. M., Pfafman, T., Kenny, T. W., Stipe, B. C., and Rugar, D. (2000). "Quality factor in micron- and submicron-thick cantilevers." *J. Microelectromech. Syst.*, 9(1), 117–125.

Zener, C. (1937). "Internal friction in solids. I: Theory of internal friction in reeds." *Phys. Rev.*, 52(3), 230–235.