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MAGNETICALLY INDUCED IMPURITY BANDING IN *n*-InSb

R. J. SLADEK

Westinghouse Research Laboratories, Pittsburgh 35, Pennsylvania

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Abstract—A detailed study of the effect of a strong magnetic field on donor levels in *n*-InSb has been made by means of Hall-effect and resistivity measurements down to 1·5°K, using field strengths up to 28,000 G.

The results of these measurements indicate that donor levels which are split off from the conduction band under the influence of the magnetic field form a narrow impurity band with finite mobility.

The position of the impurity band (i.e. donor ionization energy) and the mobility of electrons in the impurity band have been determined as a function of magnetic field strength.

The impurity band mobility can be understood in terms of quantum-mechanical resonance jumping of electrons between donors by taking into account the dependence of the donor wave function on magnetic field strength, although there are some quantitative difficulties when the magnetic field is parallel to the current.

Experimental confirmation of our impurity banding model is provided by non-ohmic effects which indicate the occurrence of impact ionization of donors in electric fields of the order of 1 V/cm.

1. INTRODUCTION

In even the purest *n*-type InSb available, donor energy levels are merged with the conduction band under ordinary circumstances. However, these levels may lie below the conduction band when a strong magnetic field is present.[1,2] Such behavior is a consequence of the small effective electron mass,[3] which causes electronic wave functions centered on different donor atoms to overlap at a small enough impurity concentration so that experimentally available magnetic fields can shrink the donor wave functions sufficiently to remove the overlap.[4]

With the disappearance of overlap, electronic orbitals due to donors become more or less localized in the vicinity of donor ions. This effect depends on electron motion being quantized by the magnetic field in the plane perpendicular to the direction of the field. An estimate of the field strength required may be obtained by noting when the radius of the cyclotron motion of conduction-band electrons ($r_c = m^*v/eB$, where v is the electron velocity, e the electronic charge, and B the magnetic field) becomes as small as the distance between neighboring donors. Thus, the smaller the effective mass, the smaller the magnetic field needed for the disappearance of overlap.

Due to its influence on the conduction-band orbitals, a quantizing magnetic field causes the lowest state in the conduction band to be raised by an energy of $\frac{1}{2}\hbar\omega$ (where $\hbar$ is Planck's constant divided by 2π and ω is the cyclotron angular frequency eB/m^*). In addition, due to its influence on the donor wave functions, the field narrows the previously spread-out donor levels into an impurity band and raises them by an amount less than $\frac{1}{2}\hbar\omega$. Thus, when the field is strong enough, some donor levels lie below the conduction band, and a non-zero donor ionization energy exists.

The ionization energy increases as the magnetic field is increased and may exceed 1 Ry (the ionization energy given by the hydrogen atom model of an impurity in a semiconductor, which for *n*-InSb is 0·00069 eV) considerably when the overlap between electronic wave functions centered on adjacent donors is small.[5] The existing theory of these states is not applicable to available *n*-InSb in weak magnetic fields. However, for strong enough fields the theory should at least indicate the qualitative behavior of the donor ionization energy in n-InSb as a function of magnetic field strength.

The initial purpose of the present work was to deduce from experimental measurements the donor ionization energy in n-InSb due to the presence of a quantizing magnetic field. Accordingly, we measured the Hall coefficient as a function of temperature and magnetic field strength at low temperatures for several n-InSb samples. Analysis of the Hall data permits a determination of the concentrations of electrons in the conduction band and in donor levels and thereby the donor ionization energy.

Of course, in the event that electrons in donor levels are completely immobile, the Hall coefficient is simply inversely proportional to the concentration of electrons in the conduction band. The first experiments performed, involving the "freeze-out" of electrons into donor levels in the presence of a strong magnetic field, were in fact analyzed on such a basis.[1]

Subsequent Hall-effect data showed that electrons localized around donor ions have non-zero mobility, and hence the donor levels must form a band of finite width. Thus we extended our investigation to study the mobility of electrons in this impurity band as a function of magnetic field strength.

To test the two-band-model interpretation of our Hall data, we have also made Hall-effect and resistivity measurements as a function of *electric* field strength above the Ohm's law region. Under proper conditions we expected that the electric field would accelerate conduction-band electrons enough to ionize donors by impact, thus raising electrons from the donor impurity band into the conduction band. This effect would presumably be reflected as a decrease in the Hall coefficient as the electric field is increased until the exhaustion value was reached. Further increases in electric field strength should produce no further change in Hall coefficient. Our experiments confirm these expectations in detail.

Our method of presentation will be as follows. First, we shall describe the two-band model and the theory used to analyze our Hall data in Section 2(a). Next, the method used to determine the impurity band mobility from resistivity measurements is indicated in Section 2(b). Then, following a short description of experimental details in Section 3, the results of our experiments will be presented and discussed in Section 4. Finally, the most important conclusions will be summarized in Section 5.

2. THEORY AND METHOD OF ANALYSIS OF DATA

(a) *Donor Ionization Energy and Number of Impurity-Band Levels*

Our method for determining the donor ionization energy in a high magnetic field involves measuring the Hall coefficient over a range of temperatures including those at which the maximum and exhaustion values of R_H occur. To determine the influence of the magnetic field strength on this energy, the Hall coefficient versus temperature measurements are made at a number of different magnetic field strengths.

For analyzing our Hall data, we employed a model in which conduction takes place in a high-mobility conduction band and a low-mobility impurity band, the latter having been split off from the conduction band by a quantizing magnetic field. In its simplest form a two-band model applied to electrons in such bands gives the following expression for the Hall coefficient[6]

$$R_H = \frac{xb^2+(1-x)}{n_0e[xb+(1-x)]^2}, \tag{1}$$

where b is the ratio of the mobility of electrons in the conduction band (μ_c) to the mobility of electrons in the donor impurity band (μ_i), x and (1-x) are the fractional concentrations of electrons in the conduction band and in the impurity band, respectively, and n_0 is the total electron concentration, which equals the excess in the concentration of donors over that of acceptors.

Strictly speaking, equations (1) and (2), below, are valid as written only for magnetic field strengths such that $\omega\tau \ll 1$, where τ is the relaxation time of the charge carrier in question, and when, in addition, the ratio of Hall mobility to drift mobility for each type of carrier $= 1$. In our samples the latter condition is true for impurity-band electrons, and should be at least approximately true for conduction-band electrons, which incidentally are no longer in a degenerate distribution when high fields are present. However, since in our samples $\omega\tau$ for conduction-band electrons becomes of the order of unity at the largest fields employed, we derived a simple formula for R_H which takes this fact into account by assuming τ to be independent of energy. Applying this formula to analyze our data on one sample, we obtained values for x and b which were not sufficiently different from those obtained using equations (1) and (2) to affect the value of the donor ionization energy. Hence equations (1) and (2) are adequate as written for analyzing our Hall data.

When the Hall coefficient (for a fixed magnetic field) goes through a reasonably large maximum as a function of temperature, the mobility ratio, b, can be calculated by means of equation (1), using the values of R_H at maximum and at exhaustion, i.e. when all electrons are in the conduction band, provided that b is independent of temperature,[7] or at least only slightly temperature-dependent compared to x. Specifically, for a fixed magnetic field,

$$\frac{(R_H)_{\max}}{(R_H)_{\text{exh}}} = \frac{(b+1)^2}{4b}. \qquad (2)$$

Using the value of b obtained from equation (2), the exhaustion electron concentration, n_0, and the measured Hall coefficient at a given temperature, we can calculate x and hence the conduction electron concentration $n_c = xn_0$ for that temperature from equation (1).

Having determined n_c by using equations (1) and (2), we can obtain the Fermi energy, ζ, from the relation

$$n_c = 2\sqrt{2m^*kT}\frac{eB}{h^2}F_{-\frac{1}{2}}(\zeta/kT), \qquad (3)$$

where, in addition to the already defined symbols, T is the absolute temperature; h is Planck's constant; k is Boltzmann's constant; and $F_{-\frac{1}{2}}(\zeta/kT)$ is a Fermi–Dirac integral.[8] The $-\frac{1}{2}$ order of this integral results because of the one-dimensional nature of the conduction band in the direction parallel to the magnetic field. The zero of energy has been taken to be the lowest state in the conduction band in the presence of the magnetic field.

As written, equation (3) applies only when all conduction-band electrons are in the lowest oscillator state. For this to occur two conditions must be met. First, there must be a sufficient number of conduction-band energy states, N_c, associated with the lowest oscillator level to accommodate the electrons which are not bound to donors. Integration of the density of conduction-band energy states in the presence of a quantizing magnetic field up to $\hbar\omega_0$ reveals that $N_c \approx 10^{10}B^{\frac{3}{2}}\text{cm}^{-3}$, where B is in gauss. Since all our samples have total electron concentrations less that 10^{15}cm^{-3} (see Table 1), there are indeed enough states associated with the lowest oscillator level to accommodate all electrons which do not "freeze out".

Second, thermal energy must be too small to cause electrons to occupy higher oscillator states. This requires that $kT \ll \hbar\omega$. For n-InSb we note that at $B = 1600$ G, $\hbar\omega/k$ is already equal to 16°K. Since the pertinent measurements are made at liquid-helium temperature and higher field strengths, this second condition is also met.

The concentration of electrons in the donor impurity band, n_d, is related to ζ by the relation

$$n_d = \frac{N_B}{\frac{1}{2}\exp[(\epsilon_B-\zeta)/kT]+1} = n_0 - n_c, \qquad (4)$$

where N_B is the number of impurity-band (bound) states per cm³ and ϵ_B is the energy at which all these states are assumed to lie. Referred to the same zero of energy as that used in equation (3), ϵ_B is negative and equal to minus the donor ionization energy. For N_B and ϵ_B independent of temperature, we can calculate ϵ_B from equation (4) by using the values of n_d and ζ for two different temperatures, since

$$\frac{n_d(T_1)}{n_d(T_2)} = \frac{n_0-n_c(T_1)}{n_0-n_c(T_2)} = \frac{\frac{1}{2}\exp[(\epsilon_B-\zeta_2)/kT_2]+1}{\frac{1}{2}\exp[(\epsilon_B-\zeta_1)/kT]+1}. \qquad (5)$$

Although equation (5) has two solutions, only the larger value for $|\epsilon_B|$, or ionization energy, is consistent with a number of bound donor levels which is independent of temperature and is also of reasonable magnitude. The smaller value of $|\epsilon_B|$ would require a number of bound states over ten times the excess donor concentration. Indeed such a value would be greater than what we believe the total donor concentration to be. (For a discussion of various impurity concentrations see Section 4(d).)

To check the accuracy of the values of the ionization energy and N_B obtained for a given sample, data for several pairs of temperatures were used to calculate these quantities for a given magnetic field strength.

(b) *Impurity-Band Mobility*

In order to determine the mobility of electrons in the impurity band as a function of magnetic field strength, we measured the resistivity at a temperature far enough below the temperatures of the maxima in the Hall coefficient so that most of the conduction is due to the impurity-band electrons. Then we calculated the mobility from the data, using the relation

$$\mu_i = \frac{1}{\rho e n_0} \tag{6}$$

where ρ is the measured resistivity and n_0 is the total electron concentration arising from excess donors. Using n_0 to calculate μ_i means, of course, that we assume that all electrons from excess donors are in the impurity band.

3. EXPERIMENTAL DETAILS

(a) *Specimens*

The specimens were cut and lapped to size (approximately $10\times2{\cdot}5\times1$ mm) from single-crystal *n*-type indium antimonide provided through the courtesy of Dr. A. C. Beer of the Battelle Memorial Institute and Dr. H. P. R. Frederikse of the National Bureau of Standards, and from polycrystalline *n*-type indium antimonide made by the Ohio Semiconductor Company, which was supplied to us by Mr. O. Lindberg of the Materials Engineering Department, Westinghouse Electric Corporation.

Two current and four potential leads of No. 40 copper wire were attached to each specimen with tin solder. While most measurements were made on samples with lapped surfaces, some measurements were also made on samples with etched surfaces. No differences in electrical properties due to these two different types of surfaces were observed.

The specimens are identified in Table 1.

Table 1. Concentrations of electrons and impurities and donor spacing obtained from Hall and resistivity measurements

n_0 is the concentration of electrons in the conduction band when all excess donors are ionized and is equal to the excess-donor concentration. N_I is the total concentration of ionized impurities. r_s/a_0 is one-half the separation between, and N_D the concentration of, donors important for conduction by impurity-band electrons when a transverse magnetic field is present.

The sample letters stand for the source of the material as follows: N—National Bureau of Standards, Washington, D.C.; O—Ohio Semiconductor Company, Columbus, Ohio; and B—Battelle Memorial Institute, Columbus, Ohio.

n-InSb sample	n_0 (cm^{-3})	N_I (cm^{-3})	r_s/a_0	N_D (cm^{-3})
	$\times10^{14}$	$\times10^{15}$		$\times10^{14}$
N-4	0·30	2·2	1·33	3·5
O-1*	1·04	1·8	1·37	3·1
O-2*	0·99	1·8	1·35	3·3
B-1	1·4	3·0	1·15	5·3
N-3	3·3	1·4	1·14	5·5

* These samples were cut from the same slice of the same ingot and had very similar electrical properties.

(b) *Apparatus and technique*

The specimens were immersed directly in liquid helium, hydrogen, or nitrogen contained in a metal Dewar flask of the type described by other investigators.[9]

The temperature was controlled by keeping the vapor pressure of the refrigerant constant, using a large vacuum pump appropriately throttled. Mercury or Octoil-S manometers were used to measure the vapor pressure of the bath and thereby determine the temperature.

An Arthur D. Little electromagnet provided magnetic fields up to 28,000 G. The field was measured with the Rawson rotating coil fluxmeter supplied with the magnet. Control of the field to within ±2 per cent or better was provided manually by adjusting the excitation to the generator supplying the magnet current, while using the fluxmeter to monitor the field.

Specimen current and potentials were measured by means of a conventional d.c. potentiometer-galvanometer arrangement.

(c) *Experimental uncertainties*

At all but the lowest temperatures, our Hall and resistivity measurements have a precision of a few per cent, which is limited principally by how well the magnetic field is controlled. At the lowest temperatures and large magnetic fields, the resistance of some samples became large enough to seriously limit galvanometer sensitivity and hence the precision of our Hall-effect data. Such data are not reported here.

Our data indicate the presence of slight inhomogeneities in the impurity concentrations of the samples. We deduce the presence of such inhomogeneities from slight disagreements ($\leqslant$ 20 per cent) between the values of the Hall coefficient determined from two different sets of leads on the same sample. Samples showing greater inhomogeneity were not measured in detail and are not reported here. Considering the high degree of compensation which occurs in samples having the small excess donor content needed for our experiments,* we regard the inhomogeneities in the specimens reported on here as quite small.

4. RESULTS AND DISCUSSION

(a) *Hall Effect*

Hall-coefficient versus reciprocal-temperature data for three different *n*-InSb samples are presented in Figs. 1–3, respectively. Note that logarithmic scales are used for the Hall coefficient.

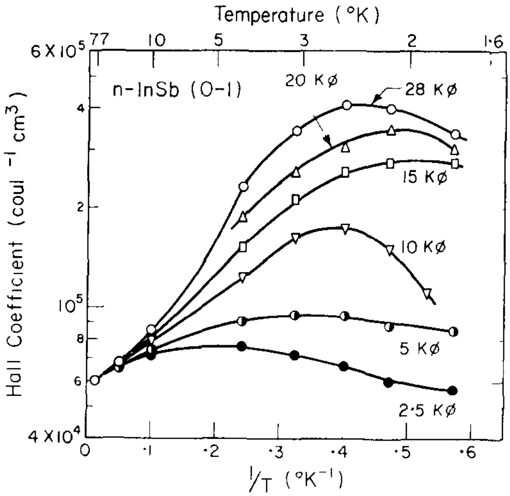

FIG. 1. Hall coefficient at a number of different magnetic field strengths as a function of reciprocal absolute temperature for *n*-type InSb (sample O-1).

If we first turn our attention to Hall data for a given temperature in the liquid-helium range, we see that the Hall coefficient increases markedly with magnetic field strength at high fields. Such large increases in R_H with increases in field strength can be explained only by decreases in the number of electrons in the conduction band.[1] The electrons which have left this band presumably fall into donor impurity-band levels which have been split off from the conduction band as a result of the strong magnetic field. The depth of these levels, i.e. the donor ionization energy, increases with field strength.

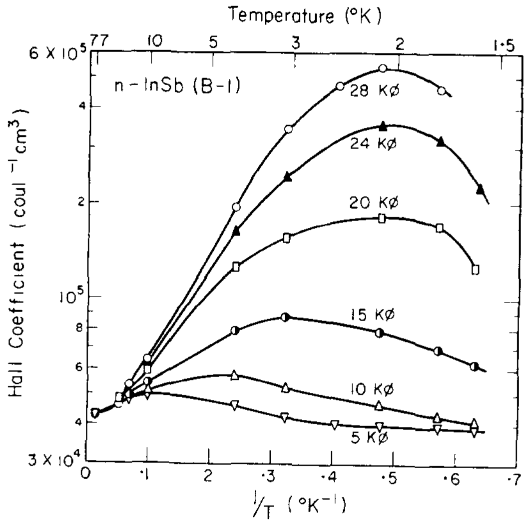

FIG. 2. Hall coefficient at a number of different magnetic field strengths as a function of reciprocal absolute temperature for *n*-type InSb (sample B-1).

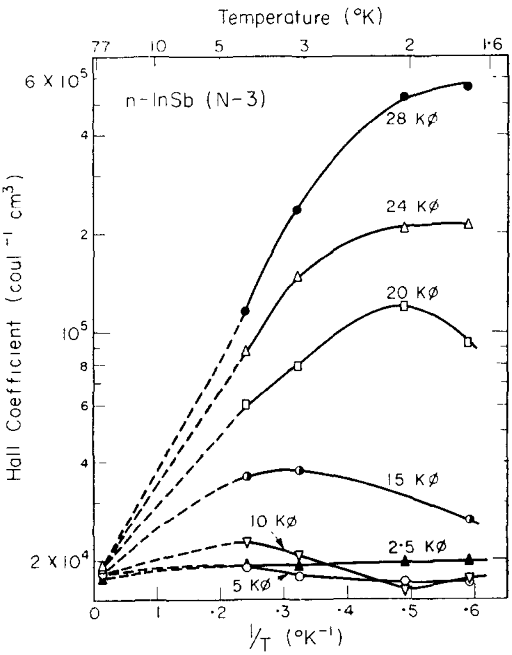

FIG. 3. Hall coefficient at a number of different magnetic field strengths as a function of reciprocal absolute temperature for *n*-type InSb (sample N-3).

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* Exploratory measurements on additional samples (from Dr. S. KURNICK of Chicago Midway Laboratories) having excess donor content of somewhat greater than 10^{15} cm^{-3} and total impurity content of about 10^{16} cm^{-3} indicated no freeze-out.

If electrons in the donor levels had zero mobility, the Hall coefficient for a given field would keep increasing as the temperature decreased. Such is not the case, however. Instead, R_H passes through a maximum at some temperature, if the magnetic field is strong enough (see Figs. 1–3), indicating that electrons in localized donor levels have non-zero mobility. The Hall maximum phenomenon has also been observed in studies of impurity banding in germanium.[10] Unlike the case of germanium, however, a sufficiently strong magnetic field is necessary before any maximum appears, and the height of the maximum increases as the field is increased. This latter can be interpreted as due to an increase in the mobility ratio ($b = \mu_c/\mu_i$) with increasing field strength. Now in view of the large magnetoresistance exhibited by n-InSb even when no freeze-out occurs,[1] we expect the conduction electron mobility (μ_c) to decrease as the magnetic field is increased. Hence, for b to increase with field strength, the mobility of electrons in donor levels (μ_i) must decrease more sharply than μ_c does as the field is increased. Such behavior for μ_i has indeed been observed and will be discussed below.

It may be noted in Figs. 1–3 that at the lowest temperatures the Hall coefficient at low magnetic field strengths sometimes falls somewhat below the exhaustion values. This may indicate that a more sophisticated treatment of the Hall coefficient for electrons in the impurity band is needed than is involved in equation (1).

Comparison of Figs. 1–3 shows that sample N-3, which has the largest concentration of excess donors requires a larger magnetic field to initiate freeze-out of electrons than samples O-1 and B-1. In view of the approximate nature of our knowledge of the total donor concentrations in these samples (see Section 4(d)), this observation is not amenable to a clear-cut interpretation. It may indicate that the spacing between excess donors determines how much the electronic wave functions must be shrunk to produce bound levels.

(b) *Donor Ionization Energy and Number of Impurity-Band (Bound) Levels*

The donor ionization energies for three samples were calculated by the method described in Section 2(a). The results are plotted as a function of magnetic field strength in Fig. 4. The limits of error indicated are taken as the spread in the values obtained for the ionization energy at a particular magnetic field strength, using data for different pairs of temperatures.

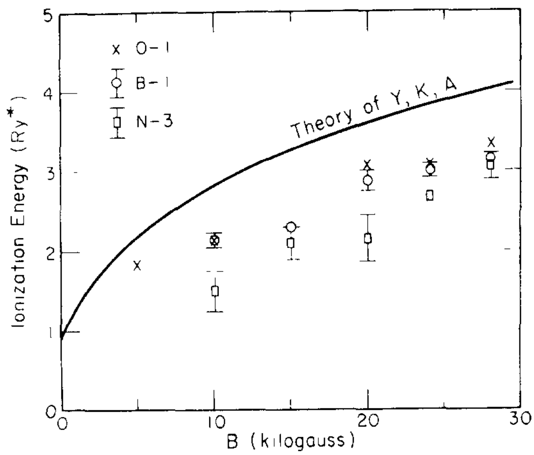

FIG. 4. Donor ionization energy as a function of magnetic field strength. The points were calculated from Hall-effect data on n-InSb samples O-1, B-1, and N-3. The curve is that given by the theory of YAFET *et al.*[5] for an isolated donor.

The ionization energies for samples B-1 and O-1 are equal within the uncertainty of our determinations. The ionization energies for sample N-3 seem to be smaller than for the other two samples, at least at the lower field strengths. However, the difference is not much greater than the uncertainty

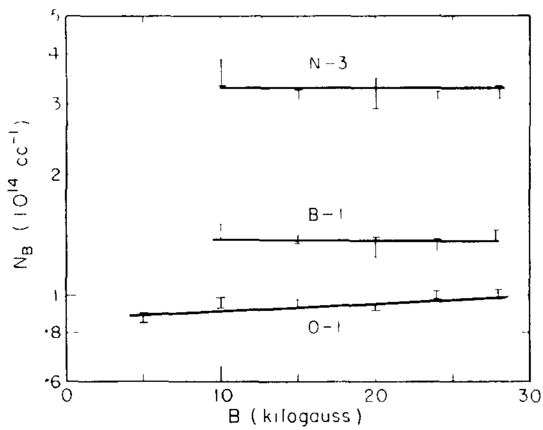

FIG. 5. Number of impurity-band (bound) levels in n-InSb samples O-1, B-1, and N-3 as a function of magnetic field strength. The points were calculated from our Hall data.

in the energies. If real, it is in accord with the fact that the concentration of excess donors is largest in sample N-3, since this implies the greatest overlap between wave functions centered on adjacent excess donors and hence the smallest ionization energy.

For comparison, the theoretical curve for donor ionization energy given by YAFET *et al.*[5] is also included in Fig. 4. Since their theory is for widely separated donors, it may not be quite applicable to our results. It is interesting to note, however, that the field-dependence we find for the binding energy is similar to that predicted by the theory. Our ionization energies are all about 1 Ry less than the theoretical ones at the same magnetic fields.*

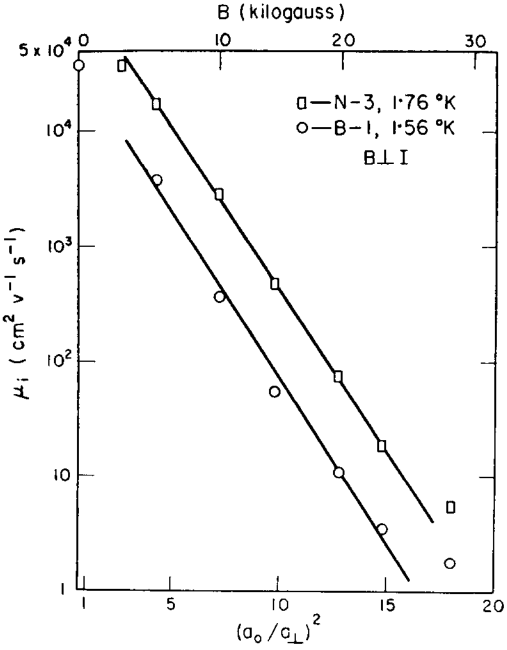

FIG. 6. Effect of a magnetic field transverse to the current direction upon the mobility of electrons in the impurity band in *n*-type InSb samples B-1 and N-3. The logarithm of the mobility is plotted against the square of the reciprocal of the reduced radius of a donor wave function in a direction perpendicular to the magnetic field.

* *Note added in proof:* Our experimental values for the donor binding energy agree with the separation between the theoretical ground state donor level and an excited *p*-like donor level recently obtained theoretically by E. N. ADAMS [private communication] and by H. WALLIS and H. J. BOWLDEN [*J. Phys. Chem. Solids*, to be published].

Fig. 5 shows the number of donor energy levels per cm^3 in the impurity band which we obtain for samples B-1, O-1, and N-3 as a function of magnetic field strength. Note that the number of levels per cm^3 is independent of field strength, at fields where we can determine this number, at any rate, and is about equal to the concentration of excess donors. This latter fact is *a posteriori* justification of the validity of the method we have used to analyze our Hall data to obtain donor ionization energy. In addition, it may indicate that since the total concentration of ionized impurities always seems to be quite large compared to the concentration of excess donors in all of these samples (see Table 1), only the more widely spaced uncompensated (excess) donor ions provide sites for localization of electrons.

(b) *Impurity-Band Mobility*

The effect of magnetic field strength on the mobility of electrons in the impurity band in

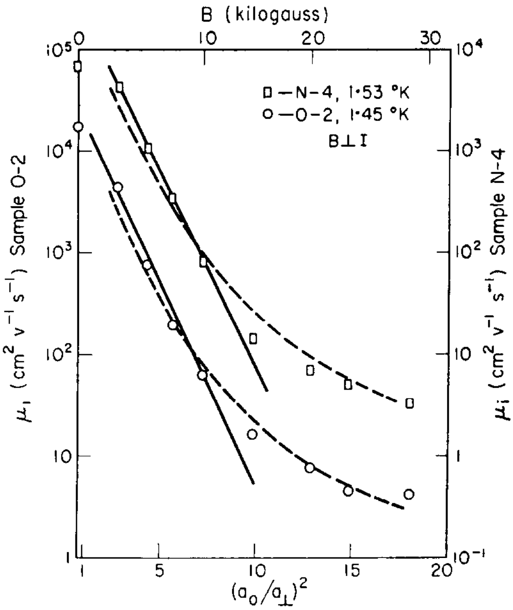

FIG. 7. Effect of a magnetic field transverse to the current direction upon the mobility of electrons in the impurity band in *n*-type InSb, samples N-4 and O-2. The logarithm of the mobility is plotted against the square of the reciprocal of the reduced radius of a donor wave function in a direction perpendicular to the magnetic field. Note that different ordinate scales are used for the two samples. The dashed curves are calculated using equation (9) and made to fit the data near 10,000 G.

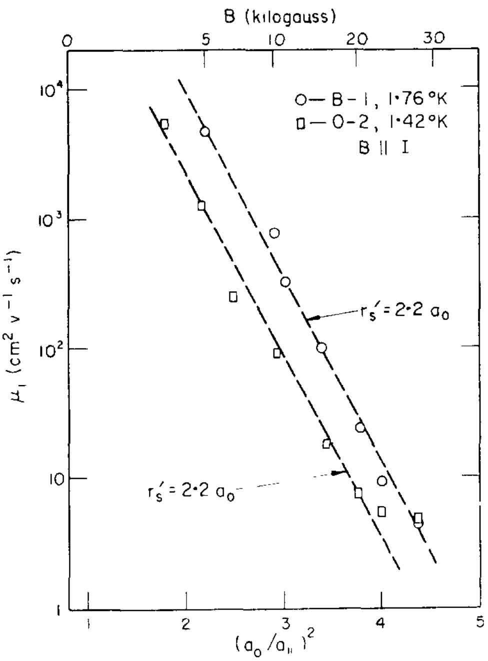

FIG. 8. Effect of a magnetic field parallel to the current direction upon the mobility of electrons in the impurity band in *n*-type InSb, samples B-1 and O-2. The logarithm of the mobility is plotted against the square of the reciprocal of the reduced radius of the donor wave function in a direction parallel to the magnetic field. The dashed curves are calculated using equation (10) and made to fit the data at one field. Under certain conditions r_s' would equal one-half the spacing between donors important for exchange jumping of electrons (see text).

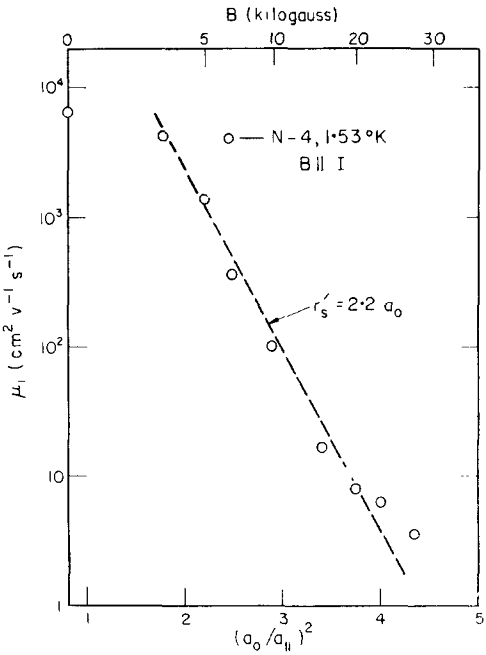

FIG. 9. Effect of a magnetic field parallel to the current direction upon the mobility of electrons in the impurity band in *n*-type InSb, sample N-4. The logarithm of the mobility is plotted against the square of the reciprocal of the reduced radius of the donor wave function in a direction parallel to the magnetic field. The dashed curves are calculated using equation (10) and made to fit the data at one field. Under certain conditions r_s' would equal one-half the spacing between donors important for exchange jumping of electrons (see text).

various *n*-type indium antimonide samples is presented in Figs. 6–9. The magnetic fields used are either perpendicular to, or parallel to, the long sample dimension (and current direction), as indicated in the figures. In each case the logarithm of the mobility, obtained from resistivity data by using equation (6), is plotted against the inverse square of the appropriate radius of the donor wave function taken from the theory of YAFET *et al.*[5] Theory suggests the use of such a plot. The actual magnetic field strengths used are also indicated in these figures.

We were not able to determine the temperature-dependence of μ_i because of the limited temperature range experimentally available, in which most conduction seems to be due to electrons in the impurity band. However our results indicate that μ_i is not strongly temperature-dependent.

To interpret the impurity-band mobility, we consider a conduction mechanism which consists essentially of quantum-mechanical resonance jumps by electrons between donor ions, with the net jumping being non-zero along the direction of the applied electric field due to the presence of this field. The mobility is taken to be proportional to a diffusion constant which is given by the product of the jump frequency times the square of the component of the jump distance along the electric field direction integrated over all possible angles between the jump direction and the electric field direction.

In order to obtain the jump frequency, we calculate the exchange integral, K, since for large enough donor separation $\nu_{\text{jump}} = 2K/h$. According to the theory of YAFET *et al.*,[5] in the presence of a strong magnetic field in the z direction, an electron wave function centered on a donor at x_D, y_D, z_D has the form

$$\psi(x_D, y_D, z_D) = [(2\pi)^{\frac{3}{2}} a_\perp^2 a_\parallel]^{-\frac{1}{2}} \times \times \exp[-\{(x-x_D)^2+(y-y_D)^2\}/4a_\perp^2 - -(z-z_D)^2/4a_\parallel^2]. \quad (7)$$

Using this wave function for donors at x_0, y_0, z_0 and $-x_0$, $-y_0$, $-z_0$, the exchange integral, K, is given by

$$K = \exp\left[-\frac{1}{8}R^2\left\{\frac{\sin^2\theta}{a_\perp^2}+\frac{\cos^2\theta}{a_\parallel^2}\right\}\int_{\substack{\text{electron}\\\text{coordinates}}} \psi^2(0,0,0)\frac{e^2}{\sqrt{(x+x_0)^2+(y+y_0)^2+(z+z_0)^2}}\,dxdydz\right] \quad (8)$$

where outside the integral x_0, y_0, z_0 have been expressed in polar coördinates with z_0 being the polar axis, e.g. $x_0 = \frac{1}{2}R \sin\theta \cos\phi$, and R is the separation between the donors. If, inside the integral, we let $a_\perp = a = a_\parallel$ and $y_0 = 0 = z_0$, then for $R > 2a$, the integral in equation (8) is almost independent of a and in fact is within 30 per cent of the asymptotic value, $2e^2/R$, which it approaches at very large R. Hence the field-dependence of the exchange integral in this case is contained in the exponential factor before the integral in equation (8).

Using the exchange integral obtained from equation (8) in the limit of large R, we obtain for the diffusion constants

$$D_{\text{trans}} = \frac{4\pi e^2}{h}R\left\{\left(1+\frac{1}{2A}\right)\exp(-R^2/8a_\perp^2)\times \times \int_0^1 \exp(A\gamma^2)\,d\gamma - \frac{1}{2A}\exp(-R^2/8a_\parallel^2)\right\} \quad (9)$$

when the magnetic field is perpendicular to the applied electric field, and

$$D_{\text{long}} = \frac{32\pi e^2}{hR}\frac{1}{1/a_\perp^2-1/a_\parallel^2}\times \times\left\{\exp(-R^2/8a_\parallel^2) - -\exp(-R^2/8a_\perp^2)\int_0^1 \exp(A\gamma^2)\,d\gamma\right\} \quad (10)$$

when the magnetic field is parallel to the applied electric field. In both equations (9) and (10), the quantity $A = \frac{1}{8}R^2(1/a_\perp^2 - 1/a_\parallel^2)$.

In order to determine the field-dependence predicted by equation (9) for the mobility in a transverse magnetic field, we insert trial values of R into equation (9). Upon doing so we find that for values of R important for explaining our measured mobilities, equation (9) predicts that at low enough field strengths μ_i^{trans} varies essentially like $\exp(-R^2/8a_\perp^2)$. However, at higher fields it predicts that μ_i^{trans} decreases less strongly than $\exp(-R^2/8a_\perp^2)$, the departure from this exponential being greatest for large R and highest fields.

Looking now at Figs. 6 and 7, we see that the transverse field mobility data do follow these predictions, at least qualitatively, the slopes of the straight lines drawn through the data, and hence the values of R, being greater in samples O-2 and N-4 than in samples B-1 and N-3.

To test just how well the observed field-dependence of the mobility can be fitted by using equation (9) when the data depart from a simple exponential behavior, we have included the dashed curves in Fig. 7. These dashed curves are calculated from equation (9), using the values of R obtained from the slopes of the straight lines drawn through the data points as discussed in the next paragraph, and are made to agree with the observed mobilities near 10,000 G.

When the observed impurity-band mobility, in the presence of a transverse magnetic field, is simply proportional to $\exp(-\text{constant}/a_\perp^2)$, we identify the constant with $\frac{1}{2}r_s^2$, where r_s is equal to one-half the donor spacing, R. The value of this constant for each sample is obtained from the slope of the pertinent straight line in Figs. 6 and 7, and the values of r_s are tabulated in Table 1. By assuming that donors important for the electron-jump process are uniformly distributed, each occupying a spherical volume equal to $(4\pi/3)r_s^3$, we can calculate a donor concentration, N_D, for each sample. These values of N_D are given in Table 1.

From Table 1 it can be seen that the value of N_D for a given sample is always greater than the exhaustion electron concentration, n_0 (and thus the concentration of excess donors) and is less than the total concentration of ionized impurities, N_I. For all samples, N_D is between about one-sixth and one-third the total concentration of ionized impurities. Thus the values we obtain for N_D from the dependence of the impurity-band mobility upon transverse magnetic field strength are quite reasonable. (See Section 4(d) for a further discussion of the various impurity concentrations.)

Leaving now the field-dependence of the impurity-band mobility, we should like to consider the magnitude of this mobility in the presence of a transverse magnetic field. In order to calculate its magnitude, we need to relate the mobility to the diffusion constant given by equation (9). Employing the Einstein relation

$$\mu = \frac{eD}{kT}, \tag{11}$$

where the symbols have their usual meanings, we obtain values for μ_i which are about a factor of ten higher than the corresponding measured mobilities. This may be a consequence of the fact that equation (11) is for non-interacting particles in a Boltzmann distribution. In our samples the occupation of the bound donor levels due to excess donors is almost complete, thus electrons occupy these levels in accordance with equation (4). If we use the distribution given by equation (4), thus permitting electron jumps only to empty bound donor levels, in the exact form of the Einstein relation for a system of interacting particles, i.e. $\mu = eD/(d\zeta/d\log n)$,[11] we obtain

$$\mu_i = \frac{eD}{kT}\cdot\frac{1}{2}\exp\{(\epsilon_B-\zeta)/kT\}. \tag{12}$$

This expression yields values for μ_i which agree in order of magnitude with measured mobilities at low magnetic field strengths. However, at high field strengths it yields values of μ_i which are as much as an order of magnitude smaller than the mobilities measured at these fields. These discrepancies may indicate that some of the compensated donors provide sites for electrons to jump into. These extra sites are of importance only at the highest fields, where the occupation of bound levels is most complete.

Since the magnitudes predicted for the mobility by equations (11) and (12) seem to bracket the observed values, we believe that a satisfactory explanation of the magnitudes of the measured mobilities requires taking into account both the unavailability of neutral donors as jump sites and the presence of compensated donors, the role played by the latter being complicated by the presence of compensated acceptors.

Turning now to the case of a longitudinal magnetic field, we find that, upon inserting various values of R in equation (10), both the factor in braces and the $1/a_\perp^2 - 1/a_\parallel^2$ factor contribute to the field-dependence of D_{long}, the relative contributions depending on the particular value of R. For $R \approx 2a_0$ most of the field-dependence of D_{long} is contained in the $1/a_\perp^2 - 1/a_\parallel^2$ factor, but for somewhat larger values of R the field-dependence of D_{long} is sensitive to the value of R. If R is large enough,

$$D_{\text{long}} \sim [1/a_\perp^2 - 1/a_\parallel^2]^{-1} \exp(-R^2/8a_\parallel^2).$$

Now for each of our samples the value of R seems to be intermediate between the extremes mentioned above (as indicated by the values of R which fit the dependence of the mobility upon transverse magnetic field strength). Thus, in each of our samples, the variation of the mobility with longitudinal magnetic field strength should depend on the value of R characteristic of the sample and in fact at high field strengths should become proportional to $[1/a_\perp^2 - 1/a_\parallel^2]^{-1}\exp(-R^2/8a_\parallel^2)$ according to equation (10).

Looking now at the data for a longitudinal magnetic field (see Figs. 8 and 9), we see that the mobility is almost as strongly dependent on field strength as when a transverse field is present. The presence of the $[1/a_\perp^2 - 1/a_\parallel^2]^{-1}$ factor in equation (10) does account for part of this strong dependence on field strength when we use the same value for R as in the transverse-field case.

However, if we try to fit the shape of the (longitudinal field) mobility data accurately by calculating curves, using equation (10) and making them agree in magnitude with the data at one field (see Figs. 8 and 9), we find that equation (10) cannot reproduce the tailing off of the observed mobilities at high fields. In addition, a larger value of $R(=2r_s')$ is needed to fit the overall field-dependence of the mobility than was needed in the case of a transverse field. The quantity r_s' seems to be the same for all samples, i.e. $r_s' = 2{\cdot}2\ a_0$. Since such a large value of r_s' would yield a value for N_D less than the concentration of excess donors, we reject it as unrealistic. Furthermore the observed field-dependence of the mobility in a longitudinal magnetic field also fits the function $\exp(-r_s^2/2a_\perp^2)$ with r_s very close to that obtained from the mobility when a transverse magnetic field is present (see Fig. 10).

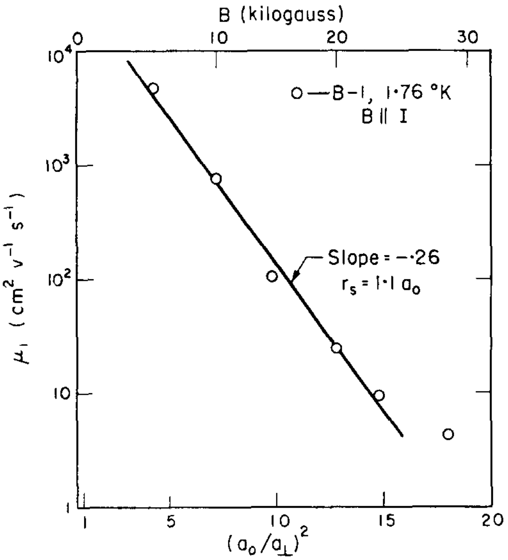

FIG. 10. Logarithm of the mobility of electrons in the impurity band in *n*-type InSb, sample B-1, in the presence of a *longitudinal* magnetic field plotted against the reciprocal squared of the reduced radius of a donor wave function in a direction *perpendicular* to the magnetic field. Under certain conditions r_S is equal to one-half the spacing between donors important for exchange jumping of electrons (see text).

The above difficulties in interpreting the impurity-band mobility in the case of a longitudinal magnetic field may arise because representing the wave function of a bound state around a donor as a gaussian in the direction parallel to the magnetic field is not adequate for treating the longitudinal-field-mobility results,[12] although such a form is adequate for a variational calculation of donor ionization energy.[5]

We have used a gaussian form for the donor wave function in the direction parallel to the magnetic field because it allows the exchange integral to be calculated in a straightforward manner. In addition, it yields a more reasonable interpretation of the mobility than another tractable wave function, namely an exponential of the first power of $R/a_\parallel$.

In ending the discussion of the impurity-band mobility, we should like to point out that thus far we have neglected another factor which should influence the mobility (when either a transverse or a longitudinal magnetic field is present). This factor is the non-uniform spacing between donors due to the latter not being distributed uniformly in the crystal. Rough estimates we have made indicate that the effect of such non-uniformity would be to reduce the overall field-dependence of μ_i. These estimates also indicate that μ_i should be most field-dependent at highest fields. Thus we do not believe non-uniform donor spacing is the reason for the deviations in observed mobilities from that expected on the basis of the simple conduction model we have used.

(d) *Impurity Concentrations*

We should like to review in this section just what information we have about the concentrations and types of impurities present in our samples and how this information was obtained. Consideration of the latter allows assessment of the degree of accuracy of this information.

First of all, the sign of the Hall coefficient and high mobilities of charge carriers at 77°K indicate that our samples are *n*-type, i.e. there is an excess in the concentration of donors over that of acceptors. This excess-donor concentration is directly determinable from the Hall coefficient when all electrons from excess donors are in the conduction band (exhaustion) and there are no intrinsic carriers. We have used Hall measurements at 77°K at fields sufficiently large so that the Hall coefficient is independent of field strength in order to determine the exhaustion electron concentration, n_0, and thus the excess-donor concentration. Values which we obtain for n_0 (listed in Table 1) are about as accurate as our Hall measurements (provided, of course, that the conduction band is spherical, as has been reported).[2]

Since ionized impurity scattering seems to be the important scattering mechanism in *n*-InSb at low temperatures,[13] we have attempted to estimate total impurity concentration, N_I, from the conduction-electron mobility at such temperatures. Unfortunately, the relationship between N_I and the mobility given by theory is simple and unambiguous only when the Born approximation is valid.* In addition, the theory applies only if the impurities scatter independently, and this may not be the case in our samples because the wavelengths of conduction electrons are comparable to the spacing between impurities. Nevertheless, we shall use a theoretical Born approximation formula for the mobility because such a formula does yield correct values for N_I† in uncompensated samples of n-type InAs in which the electron wavelengths are comparable to impurity spacing.[15]

Since the range of the scattering potential of an individual ion is also involved in the theoretical formulae for the mobility, we need to know it in order to be able to determine N_I. Fortunately, the mobility is much less sensitive to this range than it is to N_I. This range has been taken by some authors[16] to be just half the mean distance between impurity ions. However, theoretical calculations have also been made in which the screening effect of conduction-band electrons determines the range of the potential.[17,19] When only donor impurities are present, and they are all ionized, both the above methods of estimating the range of potential yield the same results. However, this may not be the case when there are appreciably more scattering ions than electrons, for example, when the acceptor concentration is not small compared to the donor concentration. SCLAR[18] points out that the range of the potential should be taken as the smaller of the two values obtained from the mean impurity spacing and the screening length due to the electron cloud.

When we apply this prescription to our samples, we obtain values for N_I from the mobility of conduction-band electrons at 77 and 1·5°K which agree very well (within about 30 per cent for all but one sample). This is satisfying, since at 77°K the screening length is much larger than one-half the mean impurity spacing, whereas at 1·5°K the two are comparable. Thus we were encouraged to obtain estimates of N_I from the conduction-band mobility.

The values of N_I listed in Table 1 were obtained by using as the mobility the product of the conductivity measured at 1·5°K in zero applied magnetic field and the exhaustion Hall coefficient (determined at 77°K). The particular theoretical formula for the mobility which we used was that appropriate to a degenerate electron distribution (in the conduction band) as given by DINGLE.[19] In view of the above discussion, the values thus obtained for N_I should not be in error by as much as an order of magnitude. It is interesting to note that the value we obtain for N_I in a given sample is always greater than our estimates of N_D and n_0 in that sample, as we would expect if all these quantities were known accurately.

The other impurity concentration involved in this work is the quantity N_D, estimated from the dependence of the impurity-band mobility on transverse magnetic field strength. According to our interpretation of this impurity-band mobility, N_D should be the concentration of donors involved in the electron-jump process. In the case of no compensation, N_D would presumably be the total ionized donor concentration and be equal to both n_0 and N_I. Since we find N_I always appreciably greater than n_0 (or the excess donor concentration), the question arises as to just what effect acceptor ions have on the jump process. It is reassuring that (in a transverse magnetic field at any rate) N_D turns out to be between the excess-donor concentration ($= n_0$) and the total concentration of donors given by $\frac{1}{2}(N_I+n_0)$; see Table 1.

(e) *Non-ohmic Behavior*

Experimental confirmation of the model used in interpreting our Hall data above is provided by measurements of the Hall coefficient at a fixed low temperature and high magnetic field as a function of electric field strength. Representative Hall-coefficient data are presented in Figs. 11 and 12. At fields $\gtrsim 0{\cdot}5$ V/cm, R_H decreases as the electric field is increased until it reaches its exhaustion value. Further increases in electric field strength cause no further change in R_H. Such behavior is readily interpreted in terms of the above model as follows.

* Recent discussions of ionized impurity scattering, which also review previous work, have been given by BLATT[14] and SCLAR.[18]

† By correct N_I values we mean that for a given sample the value of N_I determined from the mobility agreed with the concentration of extrinsic electrons, and thus the concentration of donor ions, obtained from the Hall coefficient.

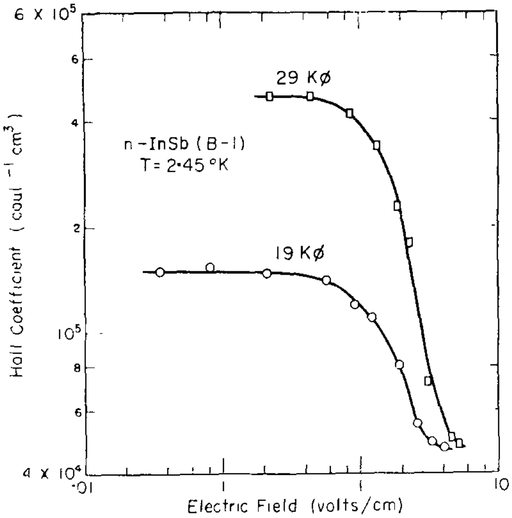

FIG. 11. The Hall coefficient as a function of electric field strength for n-type InSb, sample B-1, at two different magnetic field strengths.

Electrons which had fallen into localized donor levels in the presence of the magnetic field are freed from the donor ions by conduction-band electrons which have been accelerated sufficiently by the electric field to cause impact ionization of the donors similar to that occurring in germanium.[20] When all electrons have been raised from the impurity-band levels into the conduction band,

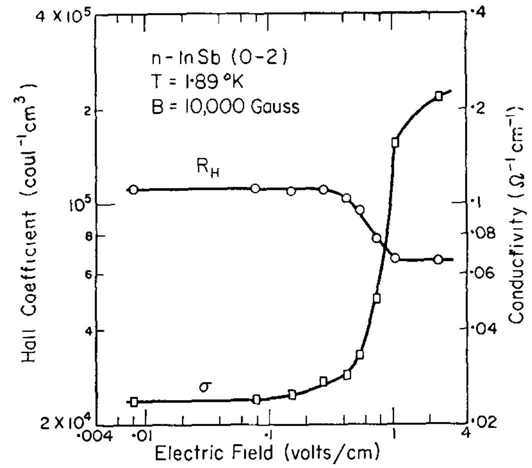

FIG. 12. The Hall coefficient and electrical conductivity as a function of electric field strength for n-type InSb, sample O-2.

the Hall coefficient becomes saturated. That energetic or "hot" electrons are available is indicated by the fact that the electrical conductivity starts to increase with electric field before R_H begins to decrease (see Fig. 12), and field-enhanced mobility for "hot" conduction-band electrons is to be expected when scattering by ionized impurities is dominant.[21]

There is no theory available for deducing the donor ionization energy from the electric field strengths required to cause the above effects. However, our results are in qualitative agreement with what is known about the dependence of donor ionization energy and electron mobilities upon magnetic field strength. For example, it can be seen in Figs. 11 and 12 that the Hall coefficient saturates sooner, the smaller the magnetic field. This is to be expected, since donor ionization energy decreases and conduction-electron mobility increases with a decrease in magnetic field strength.

One might expect R_H to begin decreasing at lower electric field strengths for successively weaker magnetic fields. Our measurements indicate no appreciable dependence of the onset of the decrease in R_H upon magnetic field strength, at least between 10,000 and 29,000 G. However, this onset is so gradual that it is difficult to define.

The complication introduced by the non-zero mobility of electrons in donor levels prevents quantitative analysis of the Hall-coefficient and conductivity versus electric-field data over most of the range of electric fields in the non-ohmic range on the basis of present data.

5. CONCLUSIONS

A unique opportunity for investigating impurity banding occurs in n-type indium antimonide. Both the position of the impurity-band levels and the mobility of electrons in these levels can be studied for varying amounts of overlap between donor wave functions in one and the same sample, since this overlap depends on the strength and direction of the applied magnetic field.

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