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# Frequencies of ferromagnetic resonance of ferromagnet–antiferromagnet–ferromagnet (FM/AFM/FM) trilayers

G.M. Genkin, M.V. Sapozhnikov \*, I.D. Tokman

*Institute of Applied Physics, Nizhny Novgorod, Russia*

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## Abstract

The properties of the ferromagnetic resonance of FM/AFM/FM trilayer, namely, the dependence of the resonance frequencies on the external magnetic field and on the constant of the exchange interaction between layers were examined in our work. The principal dependence of the macroscopical property (FMR frequency) on the microscopical parameter (on the parity of the number of the atomic monolayers in AFM layer) was shown. The corrections to the FMR frequencies of FM layers and AFMR frequencies of magnetic sublattices of the AFM layer were defined in the approximation of the weak (in the comparison with other parameters) exchange interaction at the boundary between the layers. Besides, the numerical calculations were performed to obtain the dependence of the frequency curves on the value of the exchange interaction at the boundary between the layers.

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## 1. Introduction

The study of superlattices consisting of alternating superthin layers of ferromagnet (FM) and antiferromagnet (AFM) is of current concern. The effective exchange interaction between FM layers through AFM layers can exist in the some conditions and magnetic moments of FM layers separated by AFM layer aspires to have opposite directions. One of the possible explanation of mechanism of such interaction is the existence of short-order exchange interaction between neighboring atoms of FM and AFM layers on the boundaries between them (Fig. 1) [1]. The character of the interaction in this case depends strongly on the crystal structure.

One of the most developed methods to study magnetic materials is the method of ferromagnetic resonance (FMR). It is widely used to examine complex systems consisting of alternating layers of different magnetic and nonmagnetic materials [2–4]. The dependence of resonance frequencies on the

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\* Corresponding author.

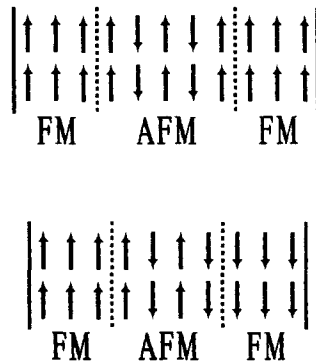


Fig. 1. Stable spin configurations of FM/AFM/FM trilayer if external field is absent. The effective interaction between FM layers has ferromagnetic character when the number of atomic monolayers in AFM layer is odd (upper figure). The interaction is antiferromagnetic when the number of atomic monolayers in AFM layer is even (lower figure).

value of the external magnetic field was studied in [5] for the case of an FM/AFM/FM trilayer, but the mechanism of interaction between layers was not taken into account. As for our work, the frequencies of uniform FMR and AFMR (antiferromagnetic resonance) for an FM/AFM/FM trilayer, which are caused by the existence of exchange interaction between layers, were determined. The both cases, with FM and AFM effective exchange between FM layers through the AFM layer was taken into account, and the dependence of the resonance frequencies on the external magnetic field and constant of the exchange interaction on the boundaries was studied. As a result, we obtained algebraic expressions for resonance frequencies in the case of weak exchange on the boundaries, and the dependence of the resonance curves on different parameters was defined by numerical methods in the wider range of the coupling between the layers.

The paper has the following plan. The description of the used model and possible configurations of the magnetic moments in its frameworks are put in Section 2. The dependence of stability of these configurations on different system parameters are discussed there also.

As for Section 3, it includes Eqs. (1)–(4), describing the behavior of the magnetic moments of the system and boundary conditions (5)–(8). The derivation of the system of equations for natural frequencies of uniform oscillations of the magnetic moments (Eqs. (9)–(11)) is done there too. The method of derivation is shown on an example of the solution in the case of effective ferromagnetic interaction between FM layers through the AFM layer. Averaging of the magnetization over the layers is done to simplify the solution for the case of uniform oscillations. Those who are interested only in final results may omit this section.

There are results for the frequencies of uniform oscillations in the case of effective ferromagnetic interaction between FM layers through the AFM one, obtained in approximation of the weak coupling on the boundaries (Eq. (12)–(16)), in the first part of Section 4. The second part of the section is devoted to a discussion of the results of numerical calculations of frequencies in the case of the same configuration (Figs. 3 and 4).

The situation when the effective interaction between FM layers through the AFM one has antiferromagnetic character is considered in Section 5. The results which are obtained in the approximation of the weak interaction on the boundaries (Eqs. (17)–(22), (24)–(28)) and by numerical methods are cited there.

Section 6 is devoted to short final conclusions and the possibility of some possible generalizations is discussed there.

## 2. The model

The model is represented in Fig. 2. The system consists of three layers: two FM layers with thickness  $d_1$  are separated by an AFM layer with thickness  $d_2$ . The layer planes are perpendicular to the  $z$ -axis. The external magnetic field  $H$  lies in the layers plane and it is directed along the  $x$ -axis. It is considered that the AFM layer has axis anisotropy and its axis coincides with the direction of the external field. As for FM layers, they have cubic crystal anisotropy. Magnetic moments of the FM layers are marked  $\mathbf{M}_1$ ,  $\mathbf{M}_4$  ( $M_1 = M_4$ ), and magnetic moments of the AFM sublattices are marked  $\mathbf{M}_2$  and  $\mathbf{M}_3$ , accordingly ( $M_2 = M_3$  – antiferromagnet is compensated). In the stable state  $\mathbf{M}_1$  and  $\mathbf{M}_2$  are directed along the magnetic field and  $\mathbf{M}_3$  is directed against it. The direction of the  $\mathbf{M}_4$  depends on the situation and possible cases will be examined below. There is ferromagnetic interaction between the layers on the boundary between them. The magnetic moments movement on the boundaries between the layers and the external boundaries is ordered by Rado boundary conditions. If the AF-magnet is assumed to consist of atomic monolayers and besides spins are coupled ferromagnetically inside every monolayer, but spins from neighboring monolayers are coupled antiferromagnetically, and if such monolayers lie in the plane of the AFM layer, then it is evident that the situation greatly depends on the number of the monolayers in the AFM layer. If the number is odd, then there are atoms of one and the same sublattice and it is coupled with both of the FM-layers. The second sublattice of the AF-magnetic isn't coupled directly with FM layers and effective interaction between FM layers through the AFM layer has ferromagnetic character in that case.

If the number of monolayers in the AFM layer is even, then there are atoms of different sublattices on the opposite boundaries of the AFM layer. The effective coupling between FM layers has AFM character in that case, and their magnetic moments aspire to have opposite directions. Two different situations are possible in that case (Figs. 1 and 2(1)):

(1) If the external field is greater than the effective interaction, then the magnetic moments of both FM layers lie along the external magnetic field. Besides, if the exchange interaction inside the layers are greater than that on the boundaries between the layers, then the AFM layer will have strong AFM order.

(2) If the external field is weaker than effective coupling, then magnetic moments of the FM layer are directed oppositely to each other: one lies along the magnetic field while the other is against it.

As a matter of fact, the form of internal spin configuration of FM/AFM structures with effective antiferromagnetic interaction depends strongly on different parameters of the system: constants of exchange interaction, anisotropy constants, external magnetic field. A detailed study of the possible stable configurations of spins and the stability of this configurations in the case when the coupling

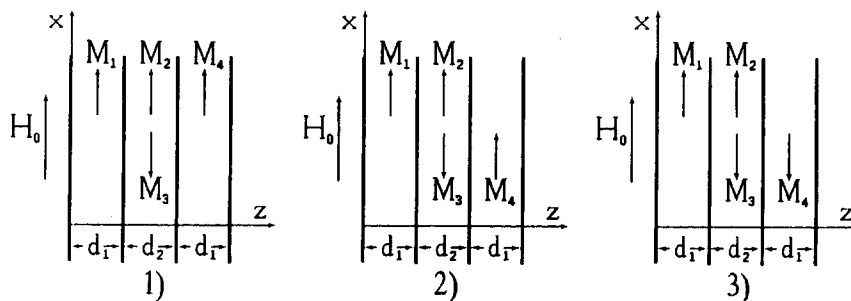


Fig. 2. The possible system configurations in the framework of the used model.  $\mathbf{M}_1$ ,  $\mathbf{M}_4$  are the magnetizations of the FM layers,  $\mathbf{M}_2$ ,  $\mathbf{M}_3$  are the magnetizations of the AFM layer sublattices,  $\mathbf{H}_0$  is the external magnetic field. (1) The effective interaction is ferromagnetic; (2) the effective interaction is antiferromagnetic but it is weaker than the external field; (3) the effective ferromagnetic interaction is stronger than the external field.

constant on the boundary between layers and inter layer exchange constant have approximately equal values was done in [1,6,7]. A more general case, when the constants may be different, has been studied in detail by us and the digest was prepared [8] and the separate paper is also prepared for future publication. Here we can only say that uniform oscillations of the magnetic moment of layers are possible only in the case when interaction on the boundaries is weaker than that inside the layers. As for the model used in this work (we consider average magnetic moments of the layers), the conversion of the frequency to the imaginary value is the sign of the configuration instability. Evidently, if one of the frequencies of the system is imaginary the amplitude of oscillations rises infinitely, i.e. the system becomes unstable. The influence of the AFM layer on the FM layer is similar to well known unidirectional anisotropy [9] if the interaction between layers is weaker than interaction inside them and the reorientation of the magnetic moment of the second FM layer along the magnetic field doesn't change the spin structure of the AFM layer.

### 3. The initial equations and the method of solution

We represent here the method of solution on the example of the case of ferromagnetic effective interaction between FM layers through AFM one. To find the resonance frequencies we use the method which was suggested in [10,11] for the resonance frequency calculations of a FM/FM bilayer.

The fundamental frequencies of precession are found by solving a system of the Landau–Lifshitz equations:

$$-\frac{1}{\gamma_i} \frac{\partial \mathbf{M}_i}{\partial t} = [\mathbf{M}_i \times \mathbf{H}_{\text{ef}i}], \quad (1)$$

$i = 1, 2, 3, 4$ . These numbers correspond to: 1 – first FM layer, 2, 3 – sublattices of the AFM layer, 4 – second FM layer. The solution is found in the usual form:

$$\mathbf{M}_i(z, t) = M_{0i} \mathbf{x} + \mathbf{m}_i \exp(i\omega t), \quad (2)$$

where  $M_1 = M_4 = M_{01}$ ,  $M_2 = M_{02}$ ,  $M_3 = -M_{02}$  and  $M_{0i}^2 \gg m_i^2$ .

The effective magnetic fields  $\mathbf{H}_{\text{ef}i}$  have the following form:

$$\mathbf{H}_{\text{ef}1} = H_0 \mathbf{x} + \mathbf{H}_{\text{kf}} - 4\pi m_{1z}(z, t) \mathbf{z} + \frac{2A_1}{M_{01}^2} \frac{\partial^2 \mathbf{M}_1}{\partial z^2} + \mathbf{T}_{12} \delta(z - d_1), \quad (3a)$$

$$\mathbf{H}_{\text{ef}2} = H_0 \mathbf{x} + \mathbf{H}_{\text{k}} - 4\pi(m_{2z} + m_{3z}) \mathbf{z} - \kappa \mathbf{M}_3 + \frac{2A_2}{M_{02}^2} \frac{\partial^2 \mathbf{M}_3}{\partial z^2} + \mathbf{T}_{21} \delta(z - d_1) + \mathbf{T}_{24} \delta(z - d_1 - d_2), \quad (3b)$$

$$\mathbf{H}_{\text{ef}3} = H_0 \mathbf{x} - \mathbf{H}_{\text{k}} - 4\pi(m_{2z} + m_{3z}) \mathbf{z} - \kappa \mathbf{M}_2 + \frac{2A_2}{M_{02}^2} \frac{\partial^2 \mathbf{M}_2}{\partial z^2}, \quad (3c)$$

$$\mathbf{H}_{\text{ef}4} = H_0 \mathbf{x} + \mathbf{H}_{\text{kf}} - 4\pi m_{4z}(z, t) \mathbf{z} + \frac{2A_1}{M_{01}^2} \frac{\partial^2 \mathbf{M}_4}{\partial z^2} + \mathbf{T}_{42} \delta(z - d_1 - d_2), \quad (3d)$$

$\mathbf{x}, \mathbf{y}, \mathbf{z}$  are the unit vectors of the corresponding axes here and below. The components of  $\mathbf{H}_{\text{ef}}$  have the following meaning:

(1)  $H_0 \mathbf{x}$  is the external constant magnetic field.

(2)  $\mathbf{H}_{\text{kf}}$  is the field of the cubic anisotropy in FM layers. If the plane of the layer coincides with the [001] side of the cubic lattice and the  $x$ -axis coincides with the [100] axis, then  $\mathbf{H}_{\text{kf}}$  has the components  $-(2K_1/m_{01}^2)m_y \mathbf{y}$  and  $-(2K_1/m_{01}^2)m_z \mathbf{z}$  in the limit of small deviations of the magnetic moments.

(3)  $H_k$  is field of axis anisotropy in the AFM layer.

(4)  $-4\pi m_{1,4z}z$  and  $-4\pi(m_{2z} + m_{3z})z$  are the demagnetization field of FM and AFM films respectively.

(5)  $(2A_i/M_{0i}^2)(\partial^2 M_i/\partial z^2)$  corresponds to nonuniform exchange interaction.

(6)  $\kappa M_i$  is the uniform interaction between sublattices of AF-magnetic.

(7)  $T_{12}, T_{21}, T_{24}$  and  $T_{42}$  are the effective surface fields of coupling between FM and AFM layers (they have the dimension of Oe  $\times$  cm) and surface energy of such interaction has the form of  $E = -JM_i M_j$ . For small deviations of the magnetic moment from the  $x$ -axis it takes the form:

$$T_{ij} = -JM_{0j} \left[ \left( \frac{m_{iy}}{M_{0i}} - \frac{m_{jy}}{M_{0j}} \right) y + \left( \frac{m_{iz}}{M_{0i}} - \frac{m_{jz}}{M_{0j}} \right) z \right] \Big|_{d_k}, \quad (4)$$

where  $T_{ij}$  is the effective surface field which acts on the  $i$ -th layer from the  $j$ -th layer,  $d_k = d_1$  if it is the boundary between the first FM layer and AFM layer, and  $d_k = d_1 + d_2$  if it is the boundary between the AFM layer and another FM layer. Evidently, this effective fields depends on the directions of the magnetic moments in the neighboring layers only.

The used boundary conditions (the conditions of Rado and Weertman [12]) are the result of preservation of the normal components of the energy fluxes through boundaries. On the external surfaces they have the form

$$\frac{2A_1}{M_{01}^2} \frac{\partial M_i}{\partial z} \Big|_0 = \frac{2A_1}{M_{01}^2} \frac{\partial M_i}{\partial z} \Big|_{2d_1+d_2} = 0, \quad (5)$$

The conditions on the internal boundaries have a similar form:

$$\frac{2A_1}{M_{01}^2} \frac{\partial M_1}{\partial z} \Big|_{d_1} = T_{12}, \quad (6a)$$

$$\frac{2A_2}{M_{02}^2} \frac{\partial M_2}{\partial z} \Big|_{d_1} = \frac{2A}{M_{02}^2} \frac{\partial M_2}{\partial z} \Big|_{d_1+d_2} = 0, \quad (6b)$$

$$\frac{2A_2}{M_{02}^2} \frac{\partial M_3}{\partial z} \Big|_{d_1} = -T_{21}, \quad \frac{2A_2}{M_{02}^2} \frac{\partial M_3}{\partial z} \Big|_{d_1+d_2} = T_{24}, \quad (6c)$$

$$\frac{2A_1}{M_{01}^2} \frac{\partial M_4}{\partial z} \Big|_{d_1+d_2} = -T_{42}, \quad (6d)$$

After a concrete form of the  $T_{01}, T_{02}, T_{12}, T_{21}, T_{24}$  and  $T_{42}$  was taken into account, the boundary conditions take the scalar form

$$A_1 \frac{\partial m_{1y,z}}{\partial z} \Big|_0 = 0, \quad (7)$$

on the external boundaries of the first FM layer. The conditions on the external boundary of the second FM layer are similar. We consider the surface anisotropy on the external and inter-layer surfaces to be absent. Evidently, if this anisotropy has great value, i.e. pinning of spins on the surfaces exists, the uniform oscillations become impossible, because the condition (10) isn't fulfilled. Besides, if the pinning on the boundaries is strong, it leads to independent behavior of the magnetic moments of the neighboring layers, due to stationarity of spins on the surfaces, which are transmitters of the interaction between the layers.

For the inter layer boundaries we have symmetrical expressions:

$$2 \frac{A_1}{M_{01}} \frac{\partial m_{1y,z}}{\partial z} \Big|_{d_1} = JM_{01}M_{02} \left( \frac{m_{2y,z}(d_1)}{M_{02}} - \frac{m_{1y,z}(d_1)}{M_{01}} \right) = 2 \frac{A_2}{M_{02}} \frac{\partial m_{2y,z}}{\partial z} \Big|_{d_1}, \quad (8a)$$

$$2 \frac{A_2}{M_{02}} \frac{\partial m_{3y,z}}{\partial z} \Big|_{d_1+d_2} = JM_{01}M_{02} \left( \frac{m_{4y,z}(d_1+d_2)}{M_{01}} - \frac{m_{2y,z}(d_1+d_2)}{M_{02}} \right) = 2 \frac{A_1}{M_{01}} \frac{\partial m_{4y,z}}{\partial z} \Big|_{d_1+d_2}. \quad (8b)$$

As we are interested in frequencies of uniform oscillations, we may integrate the system (1) over the layers between the limits  $0 + \epsilon$  and  $d_1 - \epsilon$ ;  $d_1 + \epsilon$  and  $d_1 + d_2 - \epsilon$ ;  $d_1 + d_2 + \epsilon$  and  $2d_1 + d_2 - \epsilon$  for each layer correspondingly. Here  $\epsilon$  is an infinitely small value ( $\epsilon \rightarrow 0$ ). So we can eliminate the terms containing the  $\delta$ -function. On the other hand, this integrating reduces the degree of derivatives included in the equations to the first degree. After that, we can eliminate the terms containing derivatives using the correlations (5)–(8) on the boundaries. As the result we obtain the system of eight scalar equations for magnetic moments values integrated over the layers and described as, for example:

$$M_{1y,z} = \frac{1}{d_1} \int_{0+\epsilon}^{d_1-\epsilon} m_{1y,z}(z) dz. \quad (9)$$

Besides, if the value of the exchange interaction on the boundary is less than inside the layers, we can substitute the values of the magnetization on the boundaries by their average (9) values, as for example

$$m_{1i}(0) \approx m_{1i}(d_1) \approx M_{1i}, \quad (10)$$

Thus for the integral values of the magnetic moments one can obtain the system of eight algebraic equations of the following type:

$$\frac{i\omega}{\gamma_1} M_{1y} + \left( B + \frac{JM_{02}}{d_1} \right) M_{1z} - \frac{JM_{01}}{d_1} M_{2z} = 0, \quad (11a)$$

$$\left( H + \frac{JM_{02}}{d_1} \right) M_{1y} - \frac{i\omega}{\gamma_1} M_{1z} - \frac{JM_{01}}{d_1} M_{2y} = 0, \quad (11b)$$

$$-\frac{JM_{02}}{d_2} M_{1z} + \frac{i\omega}{\gamma_2} M_{2y} + \left( H_0 + H_k + \kappa M_{02} + 4\pi M_{02} + 2 \frac{JM_{01}}{d_2} \right) M_{2z} + (\kappa + 4\pi) M_{02} M_{3z} - \frac{JM_{02}}{d_2} M_{4z} = 0, \quad (11c)$$

$$-\frac{JM_{02}}{d_2} M_{1y} + \left( H_0 + H_k + \kappa M_{02} + 2 \frac{JM_{01}}{d_2} \right) M_{2y} - \frac{i\omega}{\gamma_2} M_{2z} + (\kappa + 4\pi) M_{02} M_{3y} - \frac{JM_{02}}{d_2} M_{4y} = 0, \quad (11d)$$

$$-(\kappa + 4\pi) M_{02} M_{2z} + \frac{i\omega}{\gamma_2} M_{3y} + (H_0 - H_k - \kappa M_{02} - 4\pi M_{02}) M_{3z} = 0, \quad (11e)$$

$$-\kappa M_{02} M_{2z} + (H_0 - H_k - \kappa M_{02}) M_{3z} - \frac{i\omega}{\gamma_2} M_{3y} = 0, \quad (11f)$$

$$-\frac{JM_{01}}{d_1}M_{2z} + \frac{i\omega}{\gamma_1}M_{4y} + \left(B + \frac{JM_{02}}{d_1}\right)M_{4z} = 0, \tag{11g}$$

$$-\frac{JM_{01}}{d_1}M_{2y} + \left(H + \frac{JM_{02}}{d_1}\right)M_{4y} - \frac{i\omega}{\gamma_i}M_{4z}, \tag{11h}$$

$H = H_0 + 2K_1M_{01}$ ; and  $B = H_0 + 2K_1M_{01} + 4\pi M_{01}$  here and below, as the magnetic field coincides with the [100] axis.

Fundamental frequencies are found from the condition that the determinant of this system is equal to zero. The equation of the eight degree for  $\omega$  is obtained from this condition. Due to the symmetry of the system it is an equation of the fourth degree for  $\omega^2$ . Principally, such equations has a precise mathematical solution, but due to the cumbersome coefficients the solution is to be cumbersome too (it must consist of about 1000 terms) and therefore it isn't clear and suitable for interpretation. Therefore we have undertaken two things. First, an algebraic solution was obtained in the approximation of the weak exchange interaction on the boundary between the layers (in that case our equation of the fourth degree disintegrates into two equations of the first degree and one equation of the second degree), and second, graphs of the dependents of the resonance frequencies on the magnetic field value are obtained numerically in a more general case.

#### 4. Odd number of atomic monolayers in the AFM layer (effective ferromagnetic interaction between FM layers)

##### 4.1. The solution in the limit of the weak exchange interaction between the layers

Effective exchange interaction is characterized by  $J/d_1$  and  $J/d_2$  parameters. Notice, that coupling parameters have the dimension of length. A dimensionless parameter  $\alpha$  is the ratio of surface exchange energy and the Zeeman energy of interaction between the magnetic moment of the layer (sublattice) and external magnetic field  $\alpha \sim JM/H_0d$ . If the exchange interaction on the surfaces is weaker than the internal one, then the characteristic equation of the fourth degree of  $\omega^2$  is divided into two equations of the first degree and one of the second degree. It corresponds to the case, when the coupling between the layers has the 'static' character, and the resonance frequency of the layer is influenced only by the existence of the great constant component of the magnetic moment of the neighboring layer directed along the  $x$ -axis. The influence of a small alternative part of the magnetic moment of another layer, which has 'dynamic' character, is proportional to smaller values and therefore it hasn't been taken into account. Evidently, that in the case of the 'static' interaction every expression for resonance frequency corresponds to precession of a magnetic moment of the defined layer. The expression for the precession of the magnetic moments in both FM layers is equal in the case of ferromagnetic character of effective interaction between FM layers:

$$\left(\frac{\omega_{1,4}}{\gamma_1}\right)^2 = \left(B + \frac{JM_{02}}{d_1}\right)\left(H + \frac{JM_{02}}{d_1}\right). \tag{12}$$

Notice that the resonance frequency corrections, caused by the coupling, are proportional to  $\alpha H_0$ , which is natural for the case of weak interaction. As shown below, this frequency degeneration in the case of 'static' interaction caused by the equivalence of the FM layers which interact with the same sublattice of the AFM layer, disappears when the interaction is greater.

Having solved the equation of the second degree of  $\omega^2$  corresponds to the resonance in the AFM layer, one can obtain the following expressions for the resonance frequencies:

$$\left(\frac{\omega_{2,3}}{\gamma_2}\right)^2 = \left(\frac{\omega_{2,3}^0}{\gamma_2}\right)^2 + \left(\frac{\omega_{2,3}^1}{\gamma_2}\right)^2, \tag{13}$$

where

$$\left(\frac{\omega_{2,3}^0}{\gamma_2}\right)^2 = H_0^2 + H_k^2 + H_k(2\kappa + 4\pi)M_{02} \pm \sqrt{16\pi^2 H_k^2 M_{02}^2 + 4H_0^2 H_k (H_k + (2\kappa + 4\pi)M_{02})}, \quad (14)$$

corresponds to the frequency for the separate AFM layer. In the case of infinite AF-magnetic the demagnetization fields are absent and an ordinary expression is derived:

$$\left(\frac{\omega_{2,3}^0}{\gamma_2}\right)^2 = \left(H \pm \sqrt{H_k(H_k + (2\kappa + 4\pi)M_{02})}\right)^2. \quad (15)$$

The second term  $-(\omega_{2,3}^1/\gamma_2)^2$  defines the corrections to the square of the frequencies stipulated by the existence of exchange interaction between FM and AFM layers. It has the first order of  $J/d_2$ . Usually the exchange interaction between sublattices of AF-magnets is greater than other parameters (external magnetic field, anisotropy field etc).

$$\left(\frac{\omega_{2,3}^1}{\gamma_2}\right)^2 = \frac{JM_{01}}{d_2} \left[ 2\kappa M_{02} \pm \sqrt{2\kappa M_{02} H_k} \left( \frac{H_0}{H_k} + 2 \right) \right]. \quad (16)$$

Evidently, in this case the existence of exchange interaction between the layers leads to the increasing of both resonance frequencies in the AFM layer as the first term is greater than that with the alternating sign.

#### 4.2. The solution of the more general case

The calculation of the frequency branches  $\omega_{\tau_i} = \omega_{\tau_i}(H)$  in a wider range of exchange interaction values was provided by numerical methods on the basis of the obtained characteristic equation. The following concrete values of the parameters were used in our calculations:  $M_{10} = M_{02} = 1700$  Oe (the magnetic moments of the layers),  $\gamma_f = \gamma_a$  – gyromagnetic constants of layers,  $\kappa = 60$ ,  $H_k = 500$  Oe (the field of anisotropy in the AFM layer),  $2K/M_{01} = 200$  Oe (the effective field of the anisotropy in the FM layer). The layer thicknesses  $d_1 = d_2$  are considered to be equal to nine atomic monolayers. The results of numerical calculations are shown in Figs. 3 and 4 for  $J_{\text{int}} = 1, 10$  respectively. Here  $J_{\text{int}} = J/a$  ( $a$  the inter atomic distance) is the dimensionless constant of the interaction between the layers. The following main conclusions can be made: evidently, the lower branch  $\omega_{\tau_1} = \omega_{\tau_1}(H)$  rises with the increasing of the external field, and then is lowered to the axis  $\omega = 0$ . The point of the intersection corresponds to the boundary of the stability of that system. If the external field is greater than this critical value, spin-phase transformation takes place in the AFM layer, and this instability displaces in the imaginary component of one of the fundamental frequencies. Thus frequency branches have their sense only when  $H < H_c$ , i.e. when the configuration is stable. Evidently,  $H_c$  raises with the growth of  $J_{\text{int}}$  and a stable region of this configuration widens. An interesting peculiarity is the intersection of the second and third frequency branches. For the given model (when integral magnetic moments are considered) it is caused by the existence of such type of oscillations, when the interaction between magnetic moments has ‘static’ character with every value of exchange interaction. This type of oscillations is associated with the existence of definite symmetry in the system. Its frequency can be easily calculated for every value of exchange interaction between the layers; its expression coincides with (12).

As shown in Fig. 3 this kind of oscillation is degenerated with another one at small values of  $J_{\text{int}}$ , but this degeneration disappears with the increase of inter layers exchange. This kind of oscillation is showed in Fig. 5 in detail.

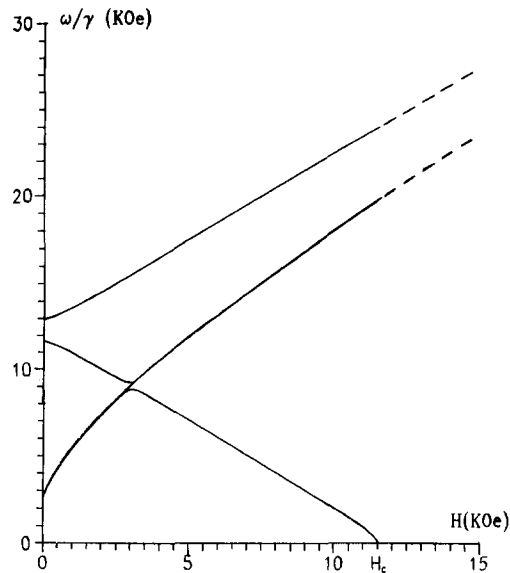


Fig. 3. The frequency curves in the case of the effective ferromagnetic interaction when  $J_{\text{int}} = 1$ . Two frequency branches are practically degenerated.  $H_c \approx 11.5$  kOe.

The existence of the interaction leads to the displacement of all branches in the direction of the of frequency increase. In this case the rising parts of the first and third braces (the numbering of branches goes from lower frequencies of higher) is weakly influenced by the value of the interaction. As a result, the value of the external field, when they (1st and 3rd branches) draw together more strongly, shifts (with the increasing of the interaction) to the region of the higher values of the external field.

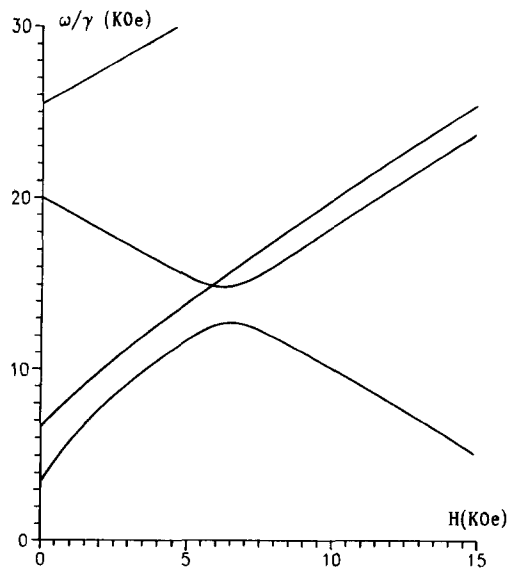


Fig. 4. The frequency curves in the case of the effective ferromagnetic interaction when  $J_{\text{int}} = 10$ .

## 5. Even number of the atomic monolayers in the AFM layer (effective antiferromagnetic interaction between FM layers)

As was noticed above, the effective interaction between FM layers has the AFM character and uniform oscillations of the magnetization are possible for two kinds of configuration:

(1) magnetic moment of the second FM layer lies along the magnetic field (the effective interaction is weaker in that case than the external field), and

(2) this magnetic moment lies oppositely to the magnetic field (the external field is weaker than the effective interaction).

The solution method for that case is analogical to the one for the case of the ferromagnetic effective exchange interaction. It is necessary to take into account the changing in the form of the effective magnetic fields and boundary conditions conditioned by the fact that the magnetic moments of the different sublattices of the AFM layers interacts with the magnetic moments of the different FM layers. Therefore we cite final results for this two last cases here only.

### 5.1. The magnetic moment of the second FM layer directed along the external field

This situation is schematically shown in Fig. 2(2). The characteristic equation of the fourth degree desintegrates into three equations (two equations of the first degree and one of the second degree) in the limit of weak inter-layers coupling when a great role is played by the ‘static’ exchange but the ‘dynamic’ exchange is too weak. Under these conditions the equations for resonance frequencies may be attributed to each FM or AFM layer separately. Evidently, in such limits the expression for the frequency of the magnetic moment precession of the first FM layer (which magnetic moment lies along the external field) has a form similar to that in the first case:

$$\left(\frac{\omega_1}{\gamma_1}\right)^2 = \left(B + \frac{JM_{02}}{d_1}\right)\left(H + \frac{JM_{02}}{d_1}\right). \quad (17)$$

The effective field of the exchange interaction of the first FM layer is directed along the external field, and therefore the resonance value of frequency increases. As for the magnetic moment of the second FM layer, the effective field of the exchange interaction is directed counter to the external field, and in that case the resonance value of frequency decreases as compared to the individual FM layer:

$$\left(\frac{\omega_4}{\gamma_1}\right)^2 = \left(B - \frac{JM_{02}}{d_1}\right)\left(H - \frac{JM_{02}}{d_1}\right), \quad (18)$$

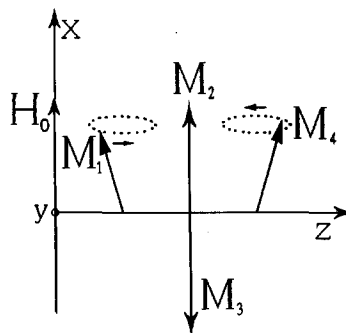


Fig. 5. The ‘symmetrical’ kind of oscillation in the case of the effective ferromagnetic interaction. The magnetic moments of the FM layers oscillate in antiphase. In this case their ‘dynamic’ actions on the magnetic moment of the first AFM sublattice compensate each other and there is no oscillation. Therefore it influences the magnetic moments of the FM layers only ‘statically’.

As for the AFM layer, the Néel sublattices are coupled with different FM layers, and from the mathematical point of view, the ‘static’ interaction plays a role of the addition to the external magnetic field. So the expression for resonance frequencies in the AFM layer is more simple than in the first problem:

$$\left(\frac{\omega_{2,3}}{\gamma_2}\right)^2 = \left(\frac{\omega_{2,3}^0}{\gamma_2}\right)^2 + \left(\frac{\omega_{2,3}^1}{\gamma_2}\right)^2, \quad (19)$$

where

$$\left(\frac{\omega_{2,3}^0}{\gamma_2}\right)^2 = H_0^2 + H_k^2 + H_k(2\kappa + 4\pi)M_{02} \pm \sqrt{16\pi^2 H_k^2 M_{02}^2 + 4H_0^2 H_k (H_k + (2\kappa + 4\pi)M_{02})}, \quad (20)$$

corresponds to the frequency for the separate AFM layer. Evidently, it is similar to that in the previous problem, which is natural. As for the second term,  $(\omega_{2,3}^1/\gamma_1)^2$ , the correction to the frequency associated with the exchange interaction has another form:

$$\left(\frac{\omega_{2,3}^1}{\gamma_2}\right)^2 = 2\frac{JM_{01}}{d_2}H_0 \pm 2\frac{JM_{01}}{d_2} \frac{4H_0(H_k^2 + H_k(2\kappa + 4\pi)M_{02})}{\sqrt{16\pi^2 H_k^2 M_{02}^2 + 4H_0^2 H_k (H_k + (2\lambda + 4\pi)M_{02})}}. \quad (21)$$

In reality, the exchange interaction between sublattices of the AFM layer is greater than the influence of other factors. In such conditions the expression (21) may be simplified and we obtain the following expression for the correction:

$$\left(\frac{\omega_{2,3}^1}{\gamma_2}\right)^2 = 2\frac{JM_{01}}{d_2} \left(H_0 \pm \sqrt{2\kappa M_{02} H_k}\right). \quad (22)$$

Evidently, the resonance frequencies move in different directions if the inter-layer exchange exists. The main reason of such an effect is that, from the formal point of view, the existence of the ‘static’ interaction has an influence similar to the increase of the external magnetic field, as noticed above.

The numerical calculations were carried out with the same values of the parameters as in the previous case, but the thickness of the AFM layer is considered to be equal to 10 interatomic layers. The frequency curves  $\omega_{\tau_i} = \omega_{\tau_i}(H)$  are represented on the graph for the values of the dimensionless constant of the interaction  $J_{\text{int}} = 1$  and 5 ( $J_{\text{int}} = J/a$ ) (Figs. 6 and 7). Evidently, two critical values of the external field, which limit the stable region of that configuration, exist.  $H_{c1}$  limits it from below and  $H_{c2}$  limits from above. If the value of the external field increases or decreases over its critical values, one of the frequencies becomes imaginary, which points to instability of such configuration in that case. If the external field is less than  $H_{c1}$ , the overturning of the magnetic moment of the second FM layer takes place, as its interaction with the AFM layer begins to play a greater role. If the external magnetic field is greater than  $H_{c2}$ , the spin-flop phase transition takes place in the AFM layer and the system becomes unstable again.  $H_{c1}$  increases and  $H_{c2}$  decreases with the increasing of the interaction between the layers, i.e. the stable region narrows. The frequency curves behave themselves in the following way with the increase of  $J_{\text{int}}$  (the numeration has the order from the lowest frequency to the highest one): the first frequency branch presses itself against the axis  $\omega = 0$ , while the fourth, on the contrary, gives displacements in the region of higher frequencies. As for the second and third ones, they draw together in the region of weak fields and go away from each other in the region of the fields values, where this frequency dependencies have positive derivatives. If  $J_{\text{int}} = 0$ , the degeneration of two kinds of oscillations exists, but it disappears much quicker with the increase of the interaction constant, than it does in the first case.

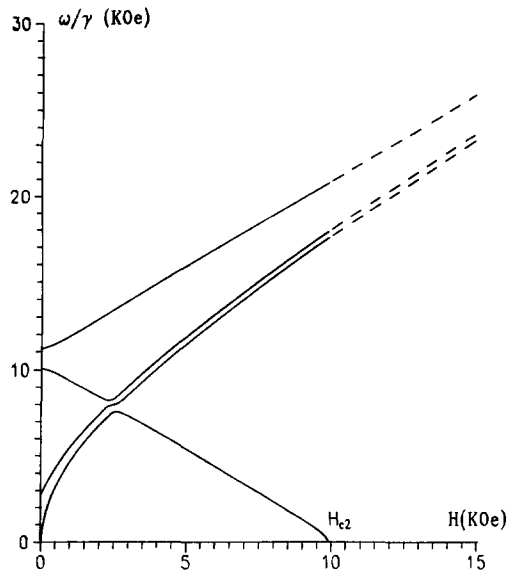


Fig. 6. The frequency curves in the case of effective antiferromagnetic interaction weaker than the external field when  $J_{\text{int}} = 1$ . Two frequency branches lie very closely to each other.  $H_{c1} < 0$  due to the anisotropy,  $H_c \approx 9.9$  kOe.

5.2. The magnetic moment of the second FM layer directed against the external field

This situation is schematically shown in Fig. 2(3). The magnetic moments of the FM layers lie along the magnetic moments of the corresponding sublattices of the AFM layer. The whole scheme of the solution is analogical. The formal changes are minimal and associated with the fact that now

$$M_4(z, t) = -M_{01}\mathbf{x} + \mathbf{m}_4 \exp(i\omega t), \tag{23}$$

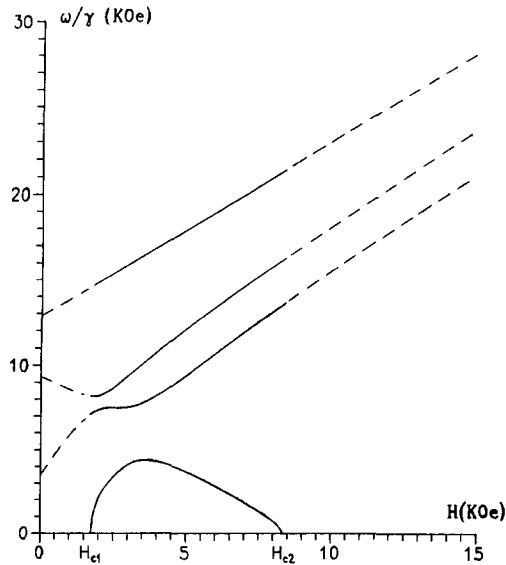


Fig. 7. The frequency curves in the case of effective antiferromagnetic interaction weaker than the external field when  $J_{\text{int}} = 5$ .  $H_{c1} \approx 1.75$  kOe,  $H_c \approx 8.4$  kOe.

as the magnetic moment of the second FM layer is turned by 180°. In the limit of the ‘static’ character of the exchange interaction the equation of the fourth degree desintegrates into three equations again. The following expressions for resonance frequencies for the FM layers are obtained:

$$\left(\frac{\omega_1}{\gamma_1}\right)^2 = \left(B_1 + \frac{JM_{02}}{d_1}\right)\left(H_1 + \frac{JM_{02}}{d_1}\right), \quad (24)$$

where  $B_1 = H_0 + 2K/M_{01} + 4\pi M_{01}$  and  $H_1 = H_0 + 2K/M_{01}$ ;

$$\left(\frac{\omega_4}{\gamma_1}\right)^2 = \left(B_2 + \frac{JM_{02}}{d_1}\right)\left(H_2 + \frac{JM_{02}}{d_1}\right), \quad (25)$$

where  $B_2 = -H_0 + 2K/M_{01} + 4\pi M_{01}$  and  $H_2 = -H_0 + 2K/M_{01}$ .

It is necessary to remember that in this case  $JM_{01}/d_1$ , and consequently,  $H_0$  are considered to be small values, because such configuration is stable only when the influence of the external magnetic field on the magnetic moment of the second FM layer is less than the influence of the exchange interaction between layers. Evidently, the resonance frequencies of the FM layers  $\omega_1$  and  $\omega_4$  behave themselves as resonance frequencies in a pure AF-magnet. If the external field is absent,  $\omega_1 = \omega_4 = \omega_0$  and degeneration occurs (if the demagnetization field isn't taken into account). The frequencies separate in the presence of the external magnetic field. As for the AFM layer, due to the type of the configuration, ‘static’ interaction plays the role of the correction to the field of the anisotropy and the frequency has the following type:

$$\begin{aligned} \left(\frac{\omega_{2,3}}{\gamma_2}\right)^2 &= H_0^2 + \left(H_k + \frac{JM_{01}}{d_2}\right)^2 + \left(H_k + \frac{JM_{01}}{d_2}\right)(2\kappa + 4\pi)M_{02} \\ &\pm \sqrt{16\pi^2 M_{02}^2 \left(H_k + \frac{JM_{01}}{d_2}\right) + 4H_0^2 \left(H_k + \frac{JM_{01}}{d_2}\right)\left(H_k + \frac{JM_{01}}{d_2} + (2\kappa + 4\pi)M_{02}\right)}. \end{aligned} \quad (26)$$

Picking out the terms of zero and first order of smallness we obtain respectively:

$$\left(\frac{\omega_{2,3}^0}{\gamma_2}\right)^2 = H_k^2 + H_k(2\kappa + 4\pi)M_{02} \pm 4\pi H_k M_{02}, \quad (27)$$

$$\left(\frac{\omega_{2,3}^1}{\gamma_2}\right)^2 = [2H_k + (2\kappa + 4\pi)M_{02} \pm 4\pi M_{02}] \frac{JM_{01}}{d_2}. \quad (28)$$

It is of great interest that frequencies are independent of the value of the external magnetic field, which has the same order of smallness as the inter-layer exchange, in the first approximation. The existence of the exchange interaction leads to the increase of both frequencies.

Numerical calculations were carried out with the same values of the parameters as in the second problem. The results are given in Figs. 8 and 9 for the values of the dimensionless exchange constant  $J_{\text{int}} = 1$  and 10. The critical value of the external field exists. If the field is greater than its critical value, one of the frequencies becomes imaginary, the system comes to an unstable state and overturning of the magnetic moment of the second FM layer into the position along external field occurs. Evidently, the stable region widens when the increase of the exchange interaction takes place. The interaction has such a character, that all frequency branches shift to the region of higher frequencies. Two lower branches corresponding to primary oscillations of the magnetic moments of the FM layers are specially interesting.

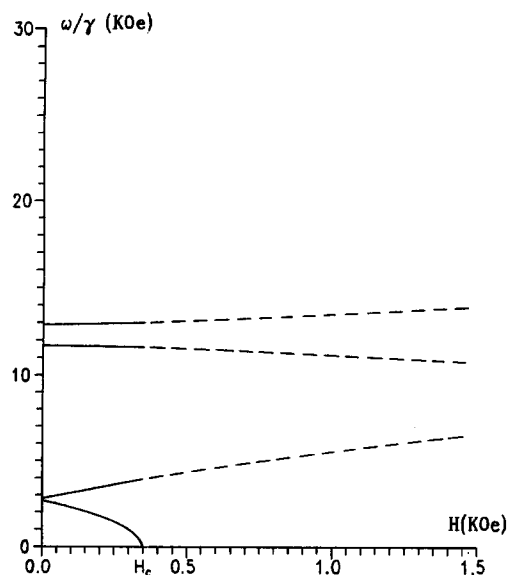


Fig. 8. The frequency curves if the effective antiferromagnetic interaction is stronger than the external field when  $J_{\text{int}} = 1$ .  $H_c \approx 0.3$  kOe.

Their form confirms the existence of the exchange interaction between FM layers through AFM layer. Evidently, these branches look like those of a pure AF-magnet: they go away from each other with the increase of the external magnetic field. Degeneration with  $H_0 = 0$  is absent due to the influence of demagnetization fields of thin film.

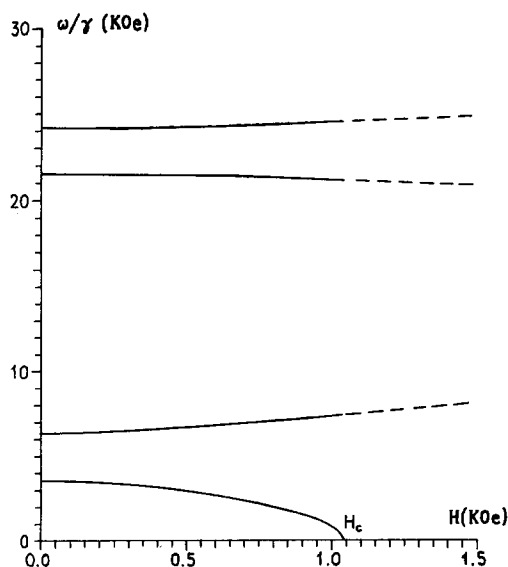


Fig. 9. The frequency curves if the effective antiferromagnetic interaction is stronger than the external field when  $J_{\text{int}} = 10$ .  $H_c \approx 1.04$  kOe.

## 6. Conclusions

The resonance properties (the uniform ferromagnetic resonance frequencies behavior) of complex magnetic systems consisting of alternative superthin layers of ferromagnetic and antiferromagnetic materials (FM/AFM/FM trilayer) are studied in our work. The possibility of the existence of one of the mechanisms providing the effective exchange interaction between FM layers through AFM layers, which has alternating sign depending on the AFM layer thickness, is shown. Besides, it is shown that the macroscopic properties of such systems, namely, the resonance frequencies behavior, depends dramatically on the microscopical parameter – the number of the atomic monolayers in the AFM layer. Besides, if effective interaction between FM layers has AFM character, the behavior of the system depends strongly on the ratio between the energy of the interaction between layers and the Zeeman energy of interaction between magnetic moments and external the magnetic field.

These configurations of the system are represented in Fig. 2. The frequencies of the uniform resonance are obtained in all these cases. The frequencies of the system with effective ferromagnetic interaction between FM layers through AFM one are represented in Eqs. (12)–(16). There are four frequencies, two of them correspond to the frequencies of uniform resonance in the first and the second FM layers and the next two are the frequencies of antiferromagnetic resonance in the AFM layer. The corrections, caused by the exchange interaction on the boundary between the layers, are calculated for them. Analogical solutions are obtained for the case of effective antiferromagnetic interaction between the layers, if the system is in a strong magnetic field (Fig. 2(2)), which are represented by Eqs. (17)–(22); and for the same system in a weak magnetic field (Fig. 2(3), Eqs. (24)–(28)). Moreover, numerical calculations were performed to obtain the solution in a more general case, i.e. when the interaction on the boundaries between the layers isn't weak in comparison with other parameters of the system. The obtained results are represented in Figs. 3 and 4 for a system with effective ferromagnetic interaction and in Figs. 6, 7, 8 and 9 for two possible configurations of a system with effective antiferromagnetic interaction.

It is interesting to compare the frequency branches for the cases of effective ferromagnetic and antiferromagnetic interaction between FM layers through the AFM layer (odd and even number of the atomic monolayers in the AFM layer). The main difference is the behavior of the two lower frequency branches. If the coupling is weak ('static interaction'), they correspond to the resonance in the FM layers. Both of the frequencies increase with the increase of the external field in the case of the FM character of the effective interaction. If the effective interaction has AFM character, then they go away from each other before the overturning of the magnetic moment, which is directed counter to the external field, takes place. The resonance frequencies of the usual antiferromagnetic are similar. As for other frequency branches, the difference has a quantitative character first of all in cases of different characters of the effective exchange. The frequency branches corresponding to the primary oscillations of the magnetic moment of the FM layer, directed along the external field (the second from below with weak external fields and third with greater values of the field), coincided practically in both cases. As for other branches, the frequencies are higher in the case of the effective ferromagnetic exchange interaction, if the other parameters of the system are similar. Besides, the degeneration of resonance frequencies with the increase of the exchange constant disappears quicker in the case of the AFM character of the effective exchange. It is due to the absence of the symmetry about the rearrangement of the FM layers in that case.

The results of our work can be generalized easily for the case of an FM/AFM superlattice and its typical frequencies can easily be obtained. Besides, we have examined the geometry of such a system, when the external magnetic field lies in the plane of the structure. The results, which can be obtained for the case of perpendicular orientation of the external magnetic field, will have no principal differences.

Possible differences will be associated with another influence of the demagnetization fields. Probably, our results can be generalized to the case of the arbitrary position of the external field also.

## 7. References

- [1] L.L. Hinchey and D.L. Mills, *Phys. Rev. B* 33 (1986) 3329.
- [2] K.Yu. Guslienko, N.A. Lesnik, A.I. Mitsek and B.P. Vozniuk, *J. Appl. Phys.* 69 (1991) 5316.
- [3] S.A. Oliver, C. Vittoria, J.J. Krebs, P. Lubitz, A. Chaiken and G.A. Prinz, *J. Appl. Phys.* 69 (1991) 4550.
- [4] J.J. Krebs, D.M. Lind and S.D. Berry, *J. Appl. Phys.* 73 (1993) 6457.
- [5] J.J. Krebs, P. Lubitz, A. Chaiken and G.A. Prinz, *Phys. Rev. Lett.* 63 (1989) 1645.
- [6] L.L. Hinchey and D.L. Mills, *Phys. Rev. B* 34 (1986) 1689.
- [7] H. Hasegawa, *Phys. Rev. B* 42 (1990) 2368.
- [8] G.M. Genkin, M.V. Sapozhnikov and I.D. Tokman, *Digests of International Magnetism Conference, Stockholm, Sweden (1993)* p. AS-04.
- [9] W.H. Meeiklejohn and C.P. Bean, *Phys. Rev.* 102 (1956) 1413.
- [10] J.F. Cochran, B. Heinrich and A.S. Arrott, *Phys. Rev. B* 34 (1986) 7788.
- [11] B. Heinrich, S.T. Purcell, J.R. Dutcher, K.B. Urquhart, J.F. Cochran and A.S. Arrott, *Phys. Rev. B* 38 (1988) 12879.
- [12] G.T. Rado and J.R. Weertman, *J. Phys. Chem. Solids* 11 (1959) 315.