Journal of
Nanoscience and Nanotechnology
Vol. 9, 1011–1014, 2009

Thermoelastic Dissipation in MEMS/NEMS Flexural Mode Resonators

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Understanding the energy dissipation mechanisms in single-crystal silicon MEMS/NEMS resonators are particularly important to maximizing an important figure of merit relevant for miniature sensor and signal processing applications: the Quality factor (Q) of resonance. This paper discusses thermoelastic dissipation (TED) as the dominant internal-friction mechanism in flexural mode MEMS/NEMS resonators. Criteria for optimizing the geometrical design of flexural mode MEMS/NEMS resonators are theoretically established with a view towards minimizing the TED for single-crystal silicon MEMS/NEMS flexural mode resonators.

Keywords: Thermoelastic Dissipation (TED), MEMS/NEMS Resonators, Flexural Vibration.

1. INTRODUCTION

Microfabricated silicon resonators have been viewed as attractive replacements for quartz crystal resonators as electronic filters and reference oscillators for wireless communication applications due to their small size and fabrication compatibility with CMOS processing.[1] Recent work[2] has demonstrated the potential of silicon MEM resonators in applications as timing references for reference oscillators meeting the GSM phase noise specification at an output frequency of approximately 13 MHz. Understanding the dominant energy dissipation mechanisms in MEMS/NEMS resonators is relevant for both sensor and signal processing applications. In signal processing applications such as reference oscillators, enhancing the resonator Quality factor in vacuum can lead to reduced power consumption, enhanced stability of oscillator output frequency and in lowered phase noise.[3,4] In a mass sensor application, enhancement of Quality factor for resonant sensors can lead to improvements in resolution and/or sensitivity.[5]

In MEMS/NEMS resonators, the net Quality factor (Q) of resonance is determined by a number of loss mechanisms:

$$Q^{-1} = Q_{\mathrm{INT}}^{-1} + Q_{\mathrm{EXT}}^{-1} + Q_{\mathrm{OTHER}}^{-1} \tag{1}$$

where the Q_{INT} is the internal friction of the material, which consists of thermoelasic dissipation, phonon scattering dissipation, and losses due to impurities and micro-cracking.[6] The Q_{EXT} refers to the external dissipation, which consists of surface effects, gas damping, anchor-loss mechanisms, and energy losses due to environment and coupling of different vibration modes. Q_{OTHER} represents all other dissipation mechanisms, which are either not well understood or hard to model analytically.[7]

The thermoelastic dissipation (TED) is a fundamental energy dissipation mechanism in MEMS/NEMS flexural mode resonators,[8] which arises from the local heat current generated by the stress inhomogeneities in a vibrating body. That is, the local temperature of the body increases under volume compression and decreases under volume tension with a positive thermal expansion coefficient. Vibration in pure flexural or bending modes results in a linear stress gradient being created within the resonator volume which in turn sets up a temperature gradient. This results in irreversible heat flow from the resonator and an increase in system entropy. Under high vacuum conditions, TED can play a dominant role. The period of vibration relative to the thermal relaxation time for the resonator establishes the importance of TED relative to other energy dissipation mechanisms. If the period of vibration is much longer than the thermal relaxation time, a thermodynamic equilibrium is established with the environment. If the period of vibration is much shorter than the thermal relaxation time, thermal relaxation of the system is no longer possible and an adiabatic regime is reached. TED disappears at very low temperatures.[4,9,10]

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1533-4880/2009/9/1011/004

doi:10.1166/jnn.2009.C075

2. DESIGN OPTIMIZATION FOR TED IN FLEXURAL MODE RESONATORS

The quality factor of TED in a flexural mode vibrating beam is given by[11]

$$Q_{\text{TED}} = \frac{Q_T}{2}\left(\frac{\zeta}{f_0} + \frac{f_0}{\zeta}\right) \quad \text{with } Q_T = \frac{2C_P\rho}{\alpha^2 TE} \tag{2}$$

where C_P is the heat capacity under constant pressure, ρ is the density, E is the Young's modulus, α is the thermal expansion coefficient, f_0 and ζ are the resonant frequency and the TED characteristic frequency of the flexural mode vibrating beam.

When $f_0 = \zeta$, Q_{TED} is minimized to be Q_T, that represents the maximum TED in a flexural mode vibrating beam. The TED characteristic frequency ζ is determined by

$$\zeta = \frac{\pi}{2} D t^{-2} \quad \text{with } D = \frac{\kappa}{\rho C_P} \tag{3}$$

where D is the thermal diffusion (the spontaneous spreading of the heat of certain material) and t is the thickness of the beam in the vibration plane. Typical values of thermal and mechanical properties of silicon oxide and silicon are listed in Table I.[12, 13]

The theoretical model of Q_{TED} as a function of normalized frequency for transverse vibration is shown in Figure 1. Q_{TED} is a function of the flexural mode frequency (Eq. (2)), while the frequency is related to the resonator dimension. For the flexural vibration beam, the frequency can be expressed as[14]

$$f_0 = \frac{\alpha_n}{4\sqrt{3}\pi}\sqrt{\frac{E}{\rho}}\frac{t}{L^2} \tag{4}$$

where E is the Young's Modulus, ρ is the material density, α_n is the boundary condition parameters, L and t are the length and thickness in the vibration plane of the resonant beam, respectively. Values of α_n for various boundary conditions are listed in Table II.

Substituting Eqs. (3) and (4) into $f_0 = \zeta$, we obtain the maximum TED arises when

Table I. Thermal and mechanical properties of SiO_2 and Si at a temperature of 300 K.

Symbol	Characteristic	SiO_2	Si
E [GPa]	Young's modulus	70	165
ρ [kg m^{-3}]	Density	2200	2330
C_P [J kg^{-1} K^{-1}]	Heat capacity under constant pressure	700	900
κ [W m^{-1} K^{-1}]	Thermal conductivity	1.4	148
D [m^2 s^{-1}]	Thermal diffusion of the material	9.1×10^{-7}	7.1×10^{-5}
α [K^{-1}]	Thermal expansion coefficient	4×10^{-7}	2.6×10^{-6}
Q_T	Minimum Q_{TED}	926000	12500

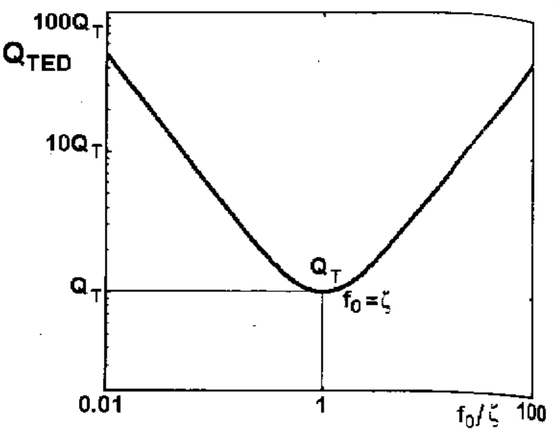


Fig. 1. Theoretical model of Q_{TED} as a function of the normalized frequency (f_0/ζ) for a flexural mode vibrating beam. Q_T is the minimum Q_{TED} at a given temperature in the case of $f_0 = \zeta$.

$$\frac{\alpha_n}{4\sqrt{3}\pi}\sqrt{\frac{E}{\rho}}\frac{t}{L^2} = \frac{\pi}{2}\frac{\kappa}{\rho C_P}t^{-2} \tag{5}$$

The ratio of the resonant frequency f_0 to the TED characteristic frequency ζ of a flexural mode vibrating beam is given by

$$\frac{f_0}{\zeta} = \frac{\alpha_n C_P\sqrt{E\rho}}{2\sqrt{3}\pi^2\kappa}\frac{t^3}{L^2} \tag{6}$$

The maximum TED arises when the thickness t and the length L of the resonator satisfies

$$\frac{t^3}{L^2} = \frac{2\sqrt{3}\pi^2\kappa}{\alpha_n C_P\sqrt{E\rho}} \tag{7}$$

For instance, for a flexural mode vibrating cantilever, the maximum TED arises when the cantilever length are 3.5 μm, 110 μm and 3500 μm with the thickness of 1 μm, 10 μm and 100 μm, respectively.

A geometrical design guide to escape high TED for single-crystal silicon MEMS/NEMS flexural mode resonators is shown in Figure 2. The figure describes the comparison of the geometrical dimension, Q_{TED} and frequencies of resonators with various beam lengths and thicknesses in the vibration plane.

The area enclosed by the red dashed lines is the normal dimension of MEMS/NEMS flexural mode resonators. The resonant frequencies of the resonators are indicated by the green lines. The maximum TED line, on which the Q_{TED} is equal to Q_T is indicated in black bold line. The shaded blue area is the TED forbidden region in which Q_{TED} is less than $10 \times Q_T$ and the TED is expected

Table II. Numerical values α_n for flexural mode resonators.

Boundary conditions	1st mode	2nd mode	3rd mode	4th mode
Clamped–clamped	22.373	61.673	120.90	199.86
Clamped-free	3.5160	22.034	61.697	120.90
Free–free	22.373	61.673	120.90	199.86

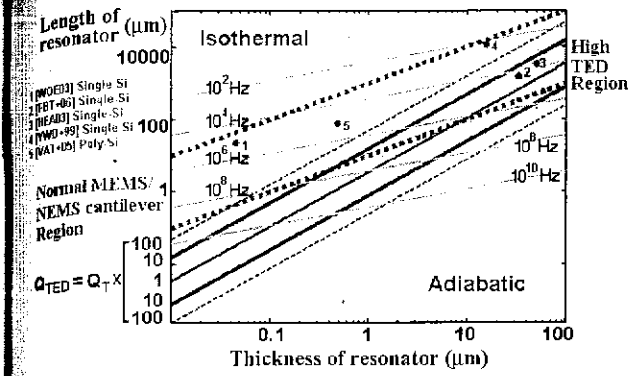


Fig. 2. Comparison of the geometrical dimension, Q_{TED} and frequencies of the single-crystal silicon MEMS/NEMS flexural mode resonators.

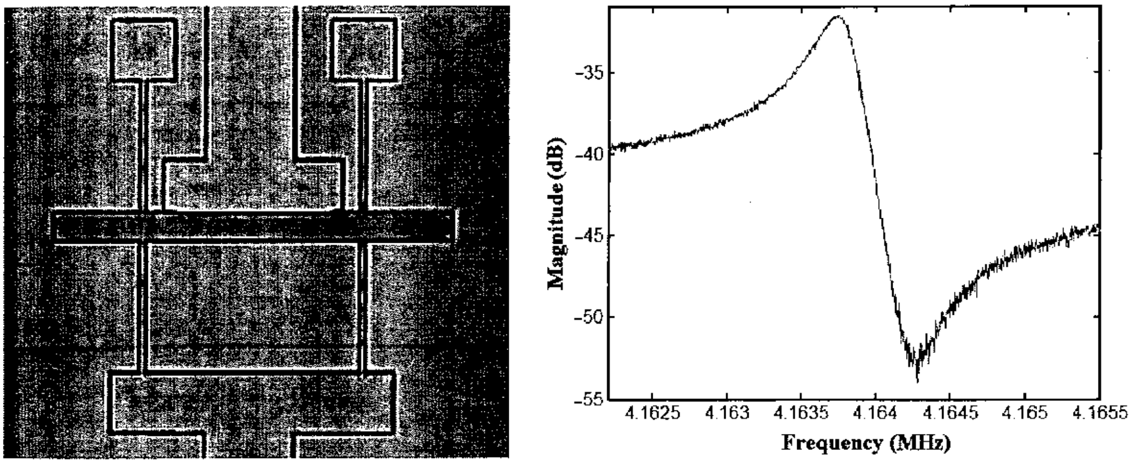
to be the dominant energy loss mechanism. The area above the upper blue dashed line is the isothermal regime, and the area below the lower blue dashed line is the adiabatic regime. In both regimes, Q_{TED} is greater than $100 \times Q_T$ and the TED can be neglected. The points 1–5 list typical MEMS/NEMS flexural mode resonators found in Refs. [3, 13, 15, 16, 17]. The particular values indicated in this plot are plotted for a cantilever (a beam with clamped-free boundary conditions), and this general geometrical design guide can be extended for other boundary conditions as well.

Table III. Comparison between measured quality factor and calculated quality factors of the fundamental mode CC and FF beam flexural mode resonators.

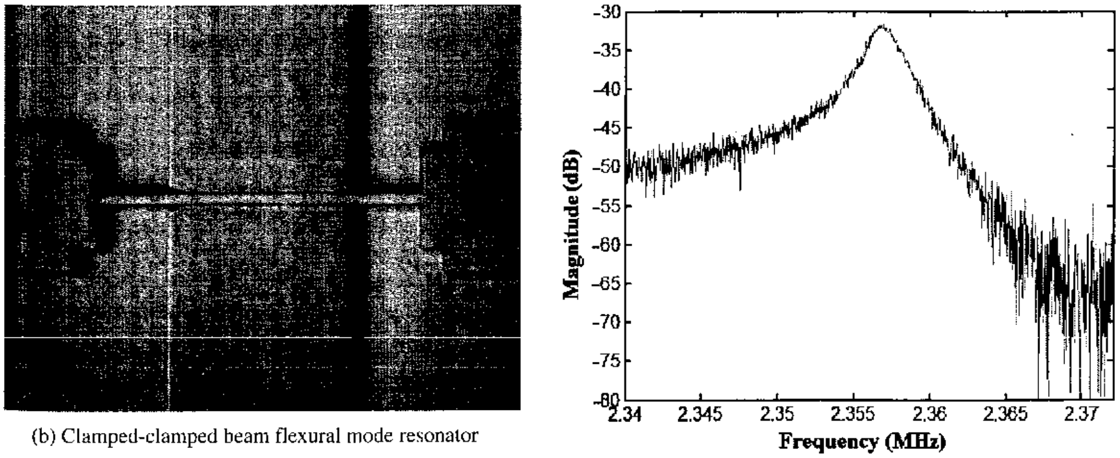
	Q_{TED}	Q_{EXTERNAL}	Q_{TOTAL}	Q_{MEASURED}
CC	20645	3419	2933	2765
FF	20966	33082	12833	10332

3. EXPERIMENTAL VERIFICATION

Clamped–clamped (CC) and free–free (FF) MEMS resonators were designed and fabricated as shown in Figure 3 (left) in a silicon-on-insulator MEMS technology. The resonators have been electrically characterized in high vacuum and a network analyzer based setup is used to extract S-parameters. Figure 3 (right) shows the S-parameters for



(a) Free-free beam flexural mode resonator



(b) Clamped-clamped beam flexural mode resonator

3. Optical micrographs and S-parameter measurement results of in-plane flexural mode free–free beam resonator and clamped–clamped beam nator.

the electro-mechanical device under an operating pressure of 1×10^{-3} mbar in a vacuum chamber with an applied DC bias of 30 V. Table III compares the measured quality factor with that obtained from calculations, which consists of the calculated internal friction quality factor (TED quality factor) and the calculated external quality factor and for the fundamental mode of the CC and FF beam resonators. The external quality factor can be calculated by employing analytical models for anchor loss dissipation[3] and surface loss dissipation[17] in the MEMS/NEMS resonators.

4. CONCLUSIONS

The mechanism of the thermo-elastic dissipation in the MEMS/NEMS resonators is described in this paper. A criteria for geometrical design for flexural mode beam resonators with clamped–clamped, clamped–free or free–free boundary conditions is theoretically established with a view to minimizing the internal friction loss, i.e., the TED in MEMS/NEMS resonators. Current research is focused on discovering the optimal geometrical designs involving external dominant dissipation mechanisms including surface and anchor losses.

Acknowledgments: Funding from the European Community Framework 6 Project entitled 'NanoTIMER' is acknowledged.

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Received: 7 June 2007. Accepted: 30 November 2007.