The Analysis on Improving Linearity of Microring-assisted Mach-Zehnder Modulator

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Abstract

The microring-assisted (MRA) Mach-Zehnder (MZ) modulator has very high linearity. In this paper, the parameter expressions, which can remove the 2nd-and 3rd-order harmonic terms of the modulation curve simultaneously at the bias point, is investigated. From the analysis, the analytic formulas of the design parameters in the most usual case, the modulator with one or two microrings assisted, can be attained. These expressions provide a very efficacious and convenient approach for designing and analyzing the MRA-MZ modulators. And by the formulas, the specific characteristic of the MRA-MZ modulators is presented in this paper.

Keywords: Integrated Optics, Linearity Modulation, Optical Microring Resonator, All-pass Filter

1 Introduction

With the development of optical communication, the demands for high linearity modulators are rapidly increasing in recent years. The Mach-Zehnder (MZ) interferometer as one of the simplest configurations is widely used in the design of optical modulators. But the sublinear characteristic of the modulation curve severely affects its performance and limits its applications. Consequently, the superlinear [1] characteristic of the phase response of the microring-based all-pass filter (APF) [2] is investigated to improve the linearity. Xie et al. proposed that the microring-assisted (MRA) configuration could eliminate the 3rd-order harmonic term of the modulation curve [1, 3]. Tazawa et al. discussed the bandwidth of MRA-MZ modulators [4]. Since loss can't be avoided and it makes heavy impact on the response of microring resonators, Yang et al. analyzed the influence of loss on the linearity characteristic of one-ring-assisted MZ modulators [5]. The analysis was based on numerical methods, so the influence of the structure parameters can't be demonstrated directly and clearly. In this paper, a more common configuration MRA-MZ modulator with N rings is analyzed. The basic conditions to achieve 3rd-order linearity are derived with analytical formulas. Moreover, the concise formulas of the most usual cases that the MRA-MZ modulator owns one or two microrings are presented. Utilizing these formulas, the characteristics of the MRA-MZ modulator are investigated in this paper. These formulas offer an efficacious and convenient way for designing and analyzing MRA-MZ modulators.

2 General Formulas

The microring as an important functional structure has been applied in optical filters and optical dispersion compensation [6-9]. Fig. 1 schematically shows the MRA-MZ intensity modulator in which N rings are employed. In Fig.1, each microring resonator is coupled with one of the arms of the MZ interferometer, forming an all-pass filter (APF). The modulation signal is applied to the microrings, modulating the output phase of the all-pass filter and then resulting in the intensity-modulated output of the MZ interferometer.

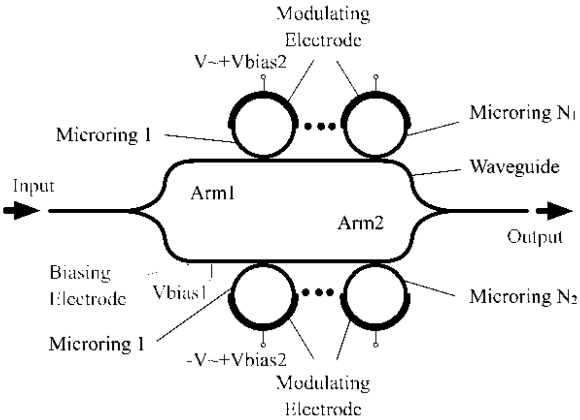

Fig. 1. Schematic diagram of the linearity-enhanced MZ modulator ith *N* microring resonators assisted. The modulation signal is applied to the microrings.

Let each microring work at the same bias point, and assume that the structure parameters of each ring are the same. Then the output intensity of the N-ring-assisted MZ modulator can be written as:

$$I_{out} = \frac{I_0}{4}[\sigma_{01}^2 a_r^{2N_1}(\theta_1) + \sigma_{02}^2 a_r^{2N_2}(\theta_2) + 2\sigma_{01}\sigma_{02} a_r^{N_1}(\theta_1) a_r^{N_2}(\theta_2)\cos(\Delta\varphi + N_1\varphi_r(\theta_1) - N_2\varphi_r(\theta_2))] \qquad (1)$$

where I_0 is the intensity of the input light, N_i is the number of the rings coupled with the arm i (=1, 2) of the MZ interferometer. σ_{0i}=exp ($-\alpha_{0i}L_{0i}$) is the amplitude attenuation factor of arm i (=1, 2), and $\Delta\varphi = \varphi_1 - \varphi_2$ is the phase delay difference of the two arms, $\varphi_i = \beta_{0i}L_{0i}$ is the phase delays introduced by arm i (=1, 2), α_{0i}, β_{0i} and L_{0i} are the amplitude attenuation coefficient, the propagation constant, and the waveguide length of arm i (=1, 2), respectively. The intensity and phase response, $a_r^2(\theta_i)$ and $\varphi_r(\theta_i)$, of the microring-based APF can be written as follows, respectively：

$$a_r^2(\theta_i) = \frac{\rho^2 + \sigma^2 - 2\sigma\rho\cos(\theta_0 + \theta_i)}{1 + \sigma^2\rho^2 - 2\sigma\rho\cos(\theta_0 + \theta_i)} \qquad (2)$$

$$\varphi_r(\theta_i) = \arctan\left[\frac{(1-\rho^2)\sigma\sin(\theta_0 + \theta_i)}{(1+\sigma^2)\rho - \sigma(1+\rho^2)\cos(\theta_0 + \theta_i)}\right] \qquad (3)$$

Here $\rho=(1-\kappa^2)^{1/2}$ is the transmission coefficient and κ is the ring-waveguide amplitude coupling coefficient, σ = exp ($-\alpha L_r$) is the round-trip amplitude attenuation factor, α is the amplitude attenuation coefficient of the microring, and $\theta_0=\pi$ is the initial modulating point. It can be found that the linearity of the MZ modulator can be significantly improved when $\theta_0=\pi$ [5]. And we can get the parameter expression conveniently at this bias point. $\theta_1=\Delta\theta$, $\theta_2=-\Delta\theta$, $\Delta\theta$ is the phase delay difference of the modulation.

For the MRA-MZ modulator, its transfer function given by Eq. (1) can be written in the Fourier series:

$$I_{out}(\Delta\theta) = \sum_n \frac{1}{n!} I^{(n)}(0)(\Delta\theta)^n \qquad (4)$$

where $I^{(n)}(\Delta\theta) = \frac{dI_{out}^n(\Delta\theta)}{d^n(\Delta\theta)}$ is the coefficient of the nth-order harmonic term.

By some calculation, the parameter equations, which can remove the 3rd-order harmonic terms of the modulation curve, can be attained as follows:

$$A_1+A_2+B+C=0 \tag{5}$$

where

$$A_1 = N_1^2\sigma^2\left(\rho^2-1\right)^2+3N_1\sigma\rho\left(\rho^2-1\right)\left(\sigma^2-1\right),\ A_2 = N_2^2\sigma^2\left(\rho^2-1\right)^2+3N_2\sigma\rho\left(\rho^2-1\right)\left(\sigma^2-1\right),$$

$$B = 2N_1N_2\sigma^2\left(\rho^2-1\right)^2,\ C=\rho\left[\rho\left(\sigma^2-1\right)^2-\sigma\left(\sigma^2+1\right)\left(\rho^2+1\right)-4\sigma^2\rho\right]$$

These letters A_1, A_2, B, C are used only for simplifying the expression.

The condition for removing the 3rd-order harmonic term is independent of phase delay difference and the loss coefficient of the two interference arms. The solution of the transmission coefficient only lies on the amplitude attenuation factor of the rings.

Further more, utilizing the solution of Eq. (5), the 2nd-order harmonic term can be removed by adjusting the phase difference of the arms $\Delta\varphi$. Following these steps, the 2nd and 3rd order harmonic terms of MRA-MZ modulator are eliminated simultaneity at the bias point.

The phase delay difference of the two arms for making the 2nd-order term equal to zero is given by:

$$\cos\left(\Delta\varphi\right)=\frac{-\rho\left(\sigma^2-1\right)\left[N_1\sigma_{01}^2\left(\frac{\sigma+\rho}{1+\sigma\rho}\right)^{2N_1}+N_2\sigma_{02}^2\left(\frac{\sigma+\rho}{1+\sigma\rho}\right)^{2N_2}\right]}{\sigma_{01}\sigma_{02}\left(N_1+N_2\right)\left[\sigma N_1\left(\rho^2-1\right)+\sigma N_2\left(\rho^2-1\right)+\sigma^2\rho-\rho\right]\left(\frac{\sigma+\rho}{1+\sigma\rho}\right)^{N_1+N_2}} \tag{6}$$

Note that, if the waveguide loss can be ignored, the even-order distortion of the modulation curve given by Eq. (4) can be eliminated by setting $\Delta\varphi=\pi/2$. And the Eq. (5) can be rewritten as:

$$(N_1+N_2)^2(\rho-1)^2=0 \tag{7}$$

The solution is:

$$\rho=\frac{1+\left(N_1+N_2\right)^2-\left[2\left(N_1+N_2\right)^2+1\right]^{\frac{1}{2}}}{\left(N_1+N_2\right)^2} \tag{8}$$

From Eq. (8), if the sum of N_1 and N_2 increases, the transmission coefficient should be raised, as shown in Fig. 2. Consequently, more microrings can be applied in order to simplify the fabrication.

With the number N_i (i =1, 2) of microrings increasing, the solution of Eq. (5) becomes too complex to attain. It is better to choose the numerical method to calculate Eq. (5) under this condition.

3 Characteristics of MRA-MZ with one or two rings

3.1 The formulas

MRA-MZ modulators with one or two rings are the most usually used structures for application. From the equations above, the formulas of the design parameters of the structures can be attained.

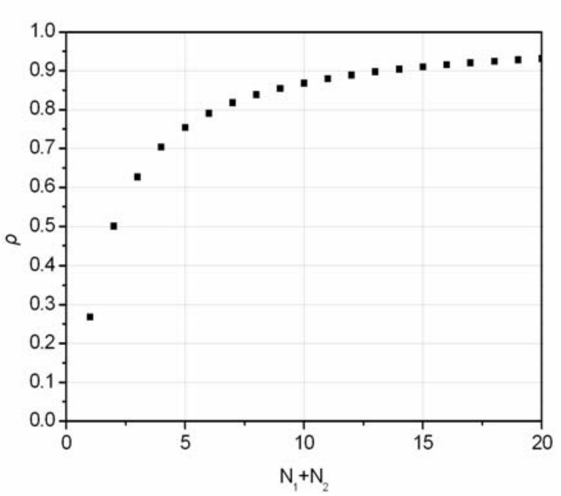

Fig. 2. At the loss-free case, the power transmission ρ vs. the sum of N_1 and N_2

3.1.1 MRA-MZ with one ring (N_1=1, N_2=0)

When MZ modulators with one ring assisted, the Eq. (5) can be rewritten as:

$$\sigma^2\rho^2-4\sigma\rho+1=0 \tag{9}$$

Due to $0<\rho<1$, the solution is:

$$\rho=\frac{2-\sqrt{3}}{\sigma} \tag{10}$$

$$\cos\Delta\varphi=\frac{\sqrt{3}\sigma_{01}}{6\sigma_{02}}\frac{(\sigma^2-1)}{\sigma} \tag{11}$$

3.1.2 MRA-MZ with two microrings coupled to the same arm (N_1=2, N_2=0)

When MZ modulators with two microrings coupled to the same arm, the Eq. (5) can be rewritten as:

$$4\rho^3\sigma^2+\rho^2\sigma^3\text{-}7\rho^2\sigma\text{-}7\rho\sigma^2+\rho+4\sigma=0 \tag{10}$$

Using the solution of the root of 3rd-order equation, the answer can be attained:

$$\rho=-ZX-ZY/X-\frac{1}{12\sigma}\left(\sigma^2-7\right)-\sqrt{3}iZ\left(X-Y/X\right) \tag{11}$$

where

$$X=\left[-\sigma^6-105\sigma^4-111\sigma^2+217+18\left(-11\sigma^8-1036\sigma^6-1794\sigma^4-1036\sigma^2-11\right)^{\frac{1}{2}}\right]^{\frac{1}{3}}\quad Y=\sigma^4+70\sigma^2+37,$$

$$Z=1/\left(24\sigma\right)$$

In fact, the solution of the transmission coefficient in Eq. (11) is the real root of the Eq. (10). These letters X, Y, Z are used only for simplifying the expression.

Applying Eq. (11) to the Eq. (6), the phase delay difference of the two arms is given by:

$$\Delta\varphi=\cos^{-1}\left[-\frac{\sigma_{01}}{\sigma_{02}}\frac{\rho\left(\sigma+1\right)\left(\sigma-1\right)\left(\rho+\sigma\right)^2}{\left(1+\rho\sigma\right)^2\left(2\rho^2\sigma+\rho\sigma^2-\rho-2\sigma\right)}\right] \tag{12}$$

Note that the design parameters have been determined, and at the bias point the 2nd-order and 3rd-order harmonic terms are eliminated at the same time.

3.1.3 MRA-MZ with two microrings forming a push-pull configuration (N_1=1, N_2=1)

When the two-ring-assisted MZ works in a push-pull operation, Eq. (5) can be also simplified as Eq. (10). So the solution of the transmission coefficient is Eq. (11), too.

Applying Eq. (11) to Eq. (6), the phase delay difference of the two arms is attained as follows:

$$\Delta\varphi = \cos^{-1}\left(-\frac{\sigma_{01}^2+\sigma_{02}^2}{2\sigma_{01}\sigma_{02}}\frac{\rho(\sigma+1)(\sigma-1)}{\left(2\rho^2\sigma+\rho\sigma^2-\rho-2\sigma\right)}\right) \tag{13}$$

3.2 Property Analysis

Taking advantage of the formulas in 3.1, the property of these designs can be analyzed conveniently. With the various round-trip amplitude attenuation factor of the rings, the transmission coefficients ρ, and the phase delay difference of the arms $\Delta\varphi$ that makes $I'''(0)$ and $I''(0)$ equal zero respectively are shown in Fig.3, if the loss of the two arms is ignored. With the loss of the microrings increasing, the power transmission coefficient ρ becomes larger to keep 3rd-order term been eliminated, and the bias of the two arms will shift from π/2 to π. If the waveguide loss is ignored and σ<0.268, the 2nd-order and 3rd-order can't be removed, and the linearity of the modulator will be deteriorated significantly.

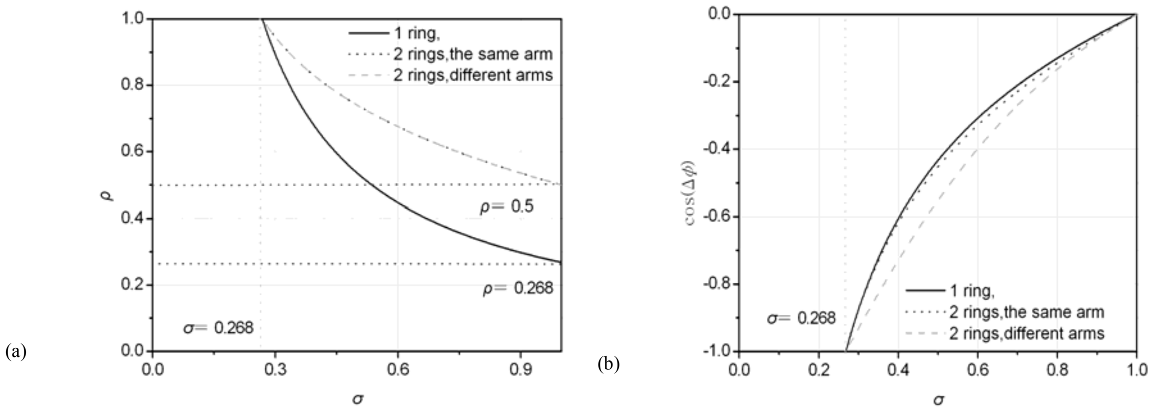

Fig. 3. Applying 1 ring, 2 rings coupled to the same arm or 2 rings to the different arms, regardless of loss of the two arms (σ_{01}=1, σ_{02}=1) , (a) ρ vs. σ (b) cos ($\Delta\varphi$) vs. σ

Regardless of the loss of the two arms (σ_{0i}=1 (i=1, 2)), the loss of the microrings will result in output power dropping. Fig. 4 shows the influence of the loss of the microrings on the maximum of the output power when the loss of the arms is neglected.

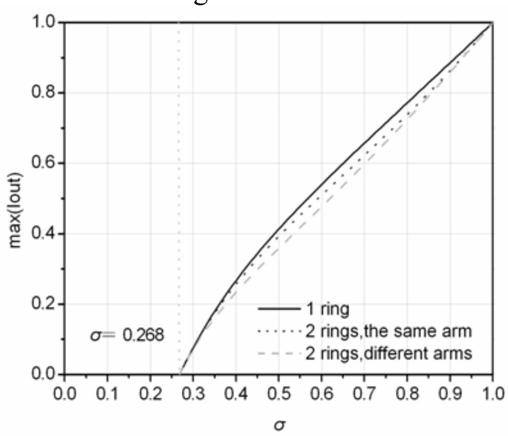

Fig. 4. The round-trip amplitude attenuation factor σ vs. the maximum output power

The maximum modulation depth of a modulator is an important criterion. Fig.5 shows the relationship between the maximum modulation depth and the loss of the microrings without the consideration of the loss of the arms. When setting $N_1=N_2$, the device forms a pull-push configuration. The form will not introduce the power difference between the two arms. So when the loss of microrings is small, the configuration can keep the maximum modulation depth has magnitude 1. But when the loss of the microrings increases over the limit ($\sigma<0.55$), the maximum modulation depth will descend sharply. The reason for this phenomenon is that σ gets so small that the range for the change of the phase becomes too small to fill the need of the MZ interferometer. When $N_2=0$ (or $N_1=0$), the microrings are applied to couple to the same arm. Using the structure, the modulation ways and the configuration of the electrodes can be simplified. The same modulation form can be applied to each microring. We don't need to use the two different modulation voltages. But the uneven distribution of the microrings will introduce the imbalance of the power of the arms. So if the loss of the microring can't be ignored, the imbalance of the arms will result in the maximum modulation depth decreasing. For the consideration, additional loss will be introduced to the other arm to compensate the imbalance. Fig.6 illustrates that the loss of arm introduced compensates the imbalance. Certainly the insertion loss of the device will increase accordingly.

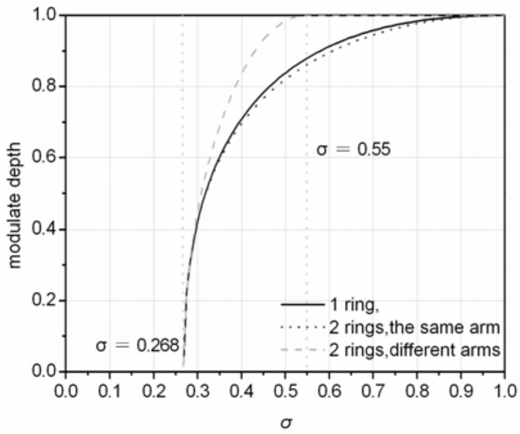

Fig. 5. The corresponding maximum modulation depth

(a) (b)

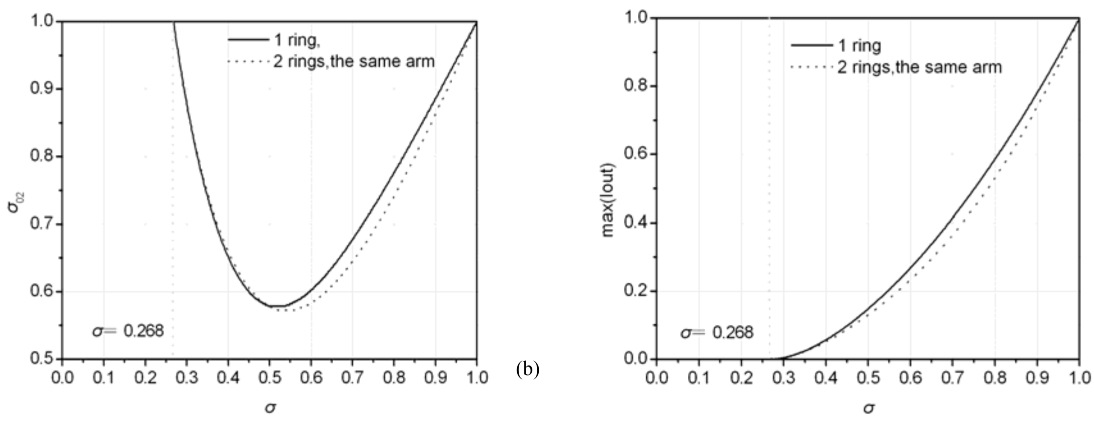

Fig. 6. Setting σ_{01}=1, we increase the loss of arm 2 to introduce the imbalance between the two arms to increase the modulation depth (a) the amplitude attenuation factor of arm σ_{02} vs. the round-trip amplitude attenuation factor σ, (b) the maximum of output power vs. the round-trip amplitude attenuation factor σ

Moreover, we can set intentionally the power imbalanced between the two arms of the MZ interferometer, which can be realized by properly setting the splitting ration of the two Y-branch couplers [5]. With the imbalance between the two arms increasing, the loss of the microrings must be reduced to ensure to make the 2nd-order term zero. Fig.7 illustrates the affect of the imbalance we introduce.

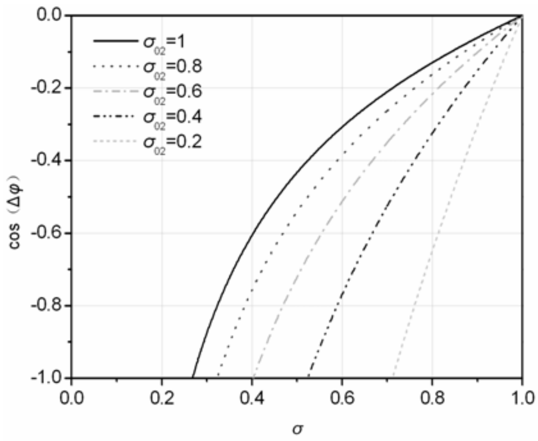

Fig. 7. Setting σ_{01}=1, applying 1 ring coupled to arm 1, the diagram shows cos($\Delta\varphi$) vs. σ with the varied loss of arm 2 (σ_{02})

4 Conclusion

In this paper, the design way for improving the linearity and the property of the MRA-MZ modulator is investigated. We attain the formulas, which can tell us how to determine the design parameters to eliminate the 2nd-order and 3rd-order terms simultaneously at the bias point when the modulator owns 1 or 2 microrings. Utilizing the analytic formulas, it becomes more conveniently and accurately to design and analyze this type of high linearity modulator. The pull-push configuration will not deteriorate the maximum modulation depth when the loss of microrings is small, but the setting of modulation is more complex. Applying microrings coupled to the same arm, the configuration of modulation, such as the structure of electrodes and the electrode voltages, is easier to implement, but the maximum modulation depth will decrease. By setting properly imbalance of the power between the two arms, the influence of the loss of microrings can be compensated, so the maximum modulation depth will be improved.

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