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Stability Analysis of Embankments and Slopes

By Sarada K. Sarma,[1] M. ASCE

Introduction

In the stability analysis of embankments and slopes there are at present two basic lines of approach. The first is the limit equilibrium approach and the second is the stress-strain analysis. With the advent of the finite element technique, given the material properties and the cross section of the embankment or the slope, it is not difficult to analyze the section for deformation and safety by computing the stresses and strains in the structure. However, the use of such a sophisticated method requires very accurate input data. Otherwise, the results obtained from such analysis become as doubtful as the input data itself. Therefore, for a designer who has yet to satisfy himself about the soil properties obtained from various laboratory tests, and to finalize a cross section of a dam or an embankment, the use of the finite element method becomes uneconomical.

On the other hand, simple methods such as those based on the limit equilibrium principle, even though these cannot give the deformation picture of the structure under stress, are able to produce comparable results as regards the safety of the structure. Furthermore, the strength parameters computed from analysis of slope failures agree reasonably well with laboratory tests results, which gives more confidence in the analytical procedure of the limit equilibrium approach. Once a final design section is arrived at, it can always be analyzed with a finite element or a similar sophisticated method to check the results obtained from the simple analysis.

The basic idea behind the limit equilibrium approach of analysis is to assume a surface which is likely to fail and try to find a state of stress along the surface so that the free body, contained within the slip surface and the free ground surface, is in static equilibrium. This state of stress, which is known as the mobilized stress is not necessarily the true state along this surface. However, this state of stress is then compared with the available strength, i.e., the stress

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[1] Lect., Dept. of Civ. Engrg., Imperial College of Sci. and Tech., London, England.

necessary to cause failure along the surface. The factor of safety, F, is then defined as that factor by which the available shear strength should be reduced so as to bring it into equilibrium with the mobilized shear stress. Within the same principle, the critical acceleration factor, K_c, gives the horizontal load as a fraction of the total weight of the free body, which, when applied to the free body, brings the stresses along the slip surface into equilibrium with the available strength. The factor K_c is called the critical acceleration factor because of the association of the horizontal load with the earthquake problem. In seismic stability analysis, the quantity $K_c g$ gives the critical horizontal acceleration which produces a factor of safety equal to one for the slip surface. In the absence of a real horizontal load, K_c may serve as a measure of the factor of safety [Sarma and Bhave (1974)].

There are at present several methods of stability analysis in existence which apply the limit equilibrium principle. Most of these methods apply the technique of slices, Fellenius (1936), Bishop (1955), Kenney (1956), Janbu (1957), Chugaev (1964), Morgenstern and Price (1965), Spencer (1967), Skempton and Hutchinson (1969), and Sarma (1973). In these methods, the available strength is computed on the basis of the Mohr-Coulomb failure criterion. These methods mainly differ in the shape of the assumed slip surfaces and in the handling of the indeterminacy of the problem as shown later. There are other methods based on the limit equilibrium principle which do not apply the slices technique. These are generally of limited use and are not considered here.

In the method of slices, there are two kinds of solution. The first is a "simplified" solution where the conditions of static equilibrium are not rigorously satisfied. Assumptions are made to obtain the solution in a simple form. The second is a "rigorous" solution where the equilibrium conditions are completely satisfied. Furthermore, these solutions have to satisfy the conditions of acceptability, such that the forces obtained from the solution do not violate the Mohr-Coulomb failure criterion anywhere within the sliding body, no tension is implied, and the direction of forces are kinematically admissible. It is not always possible to obtain a completely acceptable solution without going through a lot of iterations. Therefore, experience is needed to seek out a seemingly unacceptable solution that can be treated as acceptable. However, an acceptable rigorous solution does not necessarily provide the actual stresses in the slope and it is rigorous only within the context of the method of solution.

In the method of slices, the body of mass contained within the assumed slip surface and free ground surface is divided into n slices. It is not necessary for the slices to be vertical or even the edges to be parallel. But almost all methods use vertical slices as shown in Fig. 1. Then for n slices, we have the following $6n - 2$ unknowns: n numbers of the effective normal force N' or total normal force N; n numbers of the shear force T; $n - 1$ numbers of the body force E' in terms of effective stresses or E in terms of total stresses; $n - 1$ numbers of the body forces X; $n - 1$ numbers of the points of application of the E' or E forces given by z; n numbers of the points of application of the N' or N forces given by l; and 1 number of the factor of safety F or critical acceleration factor K_c.

The X, E forces and the point of application of E are defined at both ends of the slip surface. From the static equilibrium conditions, we have the following three equations for each slice: Σ moment $= 0$; Σ vertical forces $= 0$; and

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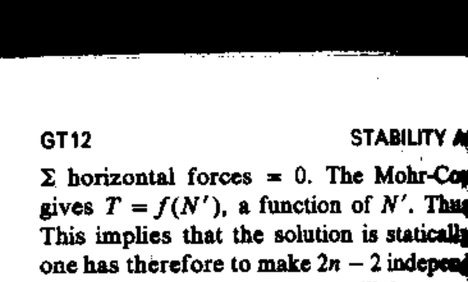

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Σ horizontal forces = 0. The Mohr-Coulomb failure criterion for each slice gives $T = f(N')$, a function of N'. Thus, for n slices, we have $4n$ equations. This implies that the solution is statically indeterminate. To obtain a solution, one has therefore to make $2n - 2$ independent assumptions. If more assumptions are made, more unknowns will have to be introduced or iterations must be applied to change the assumed values. Since there is an infinite number of ways in which these assumptions can be made, there is an equally large number of solutions. Now, let us look into the various available solutions and see how these assumptions are made.

Bishop's Simplified Method.—In this method, the assumptions are of n numbers of point for the application of the N forces and $n - 1$ numbers for the magnitude of the X forces ($X = 0$). Since one more assumption is made than required, one of the known conditions cannot be satisfied. In this case the horizontal equilibrium of one slice cannot be satisfied with the computed safety factor. For circular slip surfaces this error is small and the result is generally acceptable.

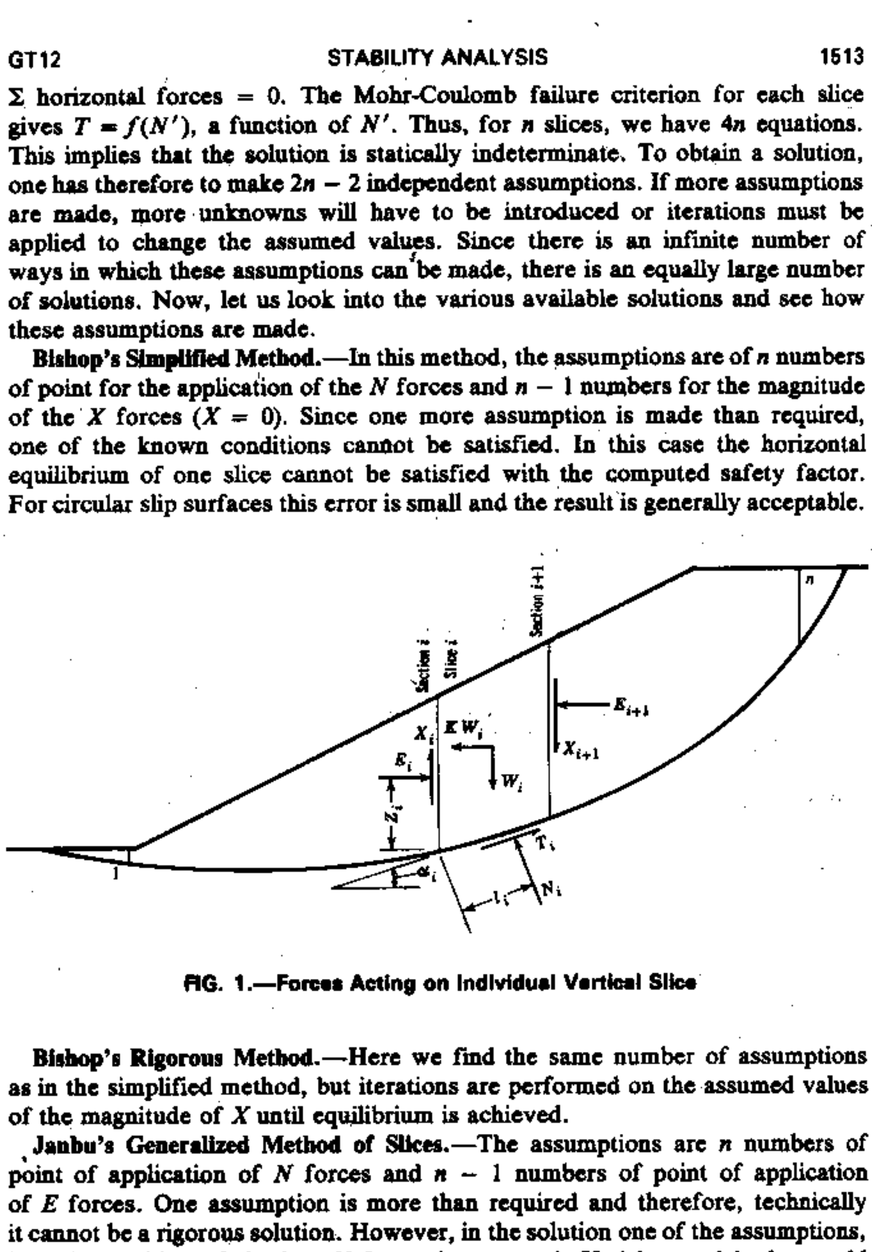

FIG. 1.—Forces Acting on Individual Vertical Slice

Bishop's Rigorous Method.—Here we find the same number of assumptions as in the simplified method, but iterations are performed on the assumed values of the magnitude of X until equilibrium is achieved.

Janbu's Generalized Method of Slices.—The assumptions are n numbers of point of application of N forces and $n - 1$ numbers of point of application of E forces. One assumption is more than required and therefore, technically it cannot be a rigorous solution. However, in the solution one of the assumptions, i.e., the position of the last N force, is not used. Had he used it, he would have found that the moment equilibrium of the last slice is not satisfied, i.e., $M_n \neq 0$, which he does not test. Moreover, as Madej (1971) points out, Janbu's moment equilibrium equation is incomplete. However, this error is small and it only affects the position of the line of thrust and not the factor of safety.

Kenney's Method.—The same assumptions as in Janbu's method are made. In his solution he uses all the assumed values and therefore is unable to satisfy one of the known conditions which in this case is that E_{n+1} is not equal to the defined value.

Morgenstern and Price Method.—The assumptions are n numbers of point of application of N forces and $n - 1$ numbers of relative relationship between X and E forces. Of course, in this solution, since infinitesimal slices are used, n tends to infinity but the principle remains the same. Since one more assumption is made than required, they introduce an extra unknown. This method therefore satisfies static equilibrium conditions rigorously.

Spencer's Method.—He makes the same assumptions as Morgenstern and Price, but instead of introducing an unknown, he iterates on the relationship between X and E to obtain the solution for circular arc slip surfaces only.

The Writer's Method (1973).—The assumptions are n numbers of point of application of N forces and $n - 1$ numbers of relative magnitude of X forces and an extra unknown is introduced to obtain a solution.

There are other methods both simplified and rigorous which are not mentioned here. As the necessary and sufficient number of assumptions that are required are $2n - 1$, if one can make these assumptions satisfactorily, a solution can be obtained. Given herein are some possible combinations of assumptions that can be made:

1. The $n - 1$ numbers of relationship between X and E forces and $n - 1$ numbers of point of application of E or N forces. A method of solution based on these assumptions is provided here. The solution is in fact an extended wedge method of solution. See also Chugaev (1964).
2. The $n - 1$ numbers of point of application of E forces and $n - 1$ numbers of point of application of N forces. This is the Janbu method of solution but the position of the last N force should be found as part of the solution.
3. The $n - 1$ numbers of absolute magnitude of X forces and $n - 1$ numbers of points of application of E or N forces. This method of solution, although simple, is not recommended as the solution depends on the absolute magnitudes of X forces which may be very difficult to define.
4. The n numbers of relative magnitude of N' forces and $n - 1$ numbers of points of application of E or N' forces and introduce an extra unknown. This comes out to be a very quick solution and the basics of the analysis will be provided here.

It may be possible to find other sets of assumptions and devise solutions. But the usefulness of such methods will depend on being able to arrive at an acceptable solution.

The writer (1973) has shown that the stability analysis solution obtained in the form of the critical acceleration factor K_c is straightforward and easy. Factor K_c itself can be used as a measure of the stability or alternatively, as he has shown, the solution obtained in the form of K_c can be used to determine the factor of safety. In order to obtain the static factor of safety or the factor of safety for any given earthquake acceleration K, one has only to reduce the strength parameters on the slip surface by a known factor F representing the factor of safety and determine the value of K_c corresponding to the reduced strength. A trial of three or four values of F will produce a graph as shown in Fig. 2 which can be used to determine the factor of safety for any K. Thus, even though an iterative process cannot be avoided for determining the factor of safety, the intermediate results in the process of iteration become useful,

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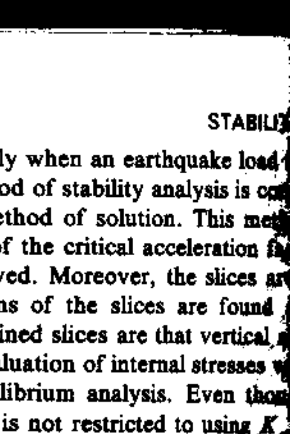

FIG. 2.—Relationship between Facto

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Analysis

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A method of stability analysis is considered in the following. It is an extended wedge method of solution. This method is new to the extent that it gives the solution of the critical acceleration factor K_c in a closed form and no graphics are involved. Moreover, the slices are not necessarily vertical and the critical inclinations of the slices are found as part of the solution. The reason for using inclined slices are that vertical slice interfaces are sometimes not suitable for an evaluation of internal stresses which is the real purpose of a sophisticated limit equilibrium analysis. Even though the solution is obtained in the form of K_c, it is not restricted to using K_c as a measure of stability. As mentioned earlier, a factor of safety can always be computed by iterations. Graphical solutions to determine the factor of safety with vertical slices are available which involve repetitive drawing of force polygons with assumed values of F until complete equilibrium is achieved [Seed (1966); see also Taylor (1948)].

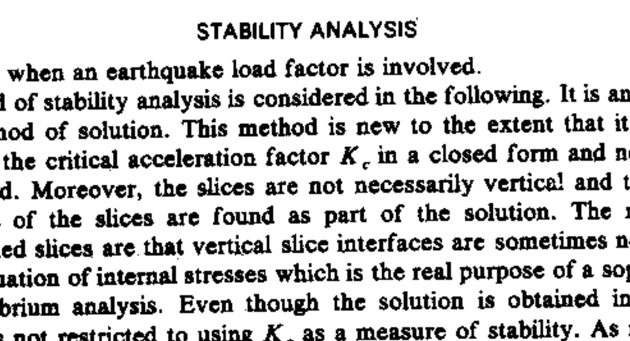

FIG. 2.—Relationship between Factor of Safety F and Acceleration Factor K

The method presented here invokes the possibility of shear failures inside the sliding mass and therefore is a suitable method for back analysis of strength parameters from actual slips. In order to use the method for computing factors of safety greater than one, it will be necessary to make assumptions regarding the degree of mobilization of shear strength inside the soil mass. If we assume that the factor of safety is the same on the internal shear planes as the factor of safety on the slip surface, the method is directly applicable, by simply reducing the strength parameters everywhere by the same factor of safety.

Analysis

Consider a failure surface as shown in Fig. 3. The failure surface comprises a series of straight lines. Since any curved surface can be formed of a series of straight lines, the method is applicable to any surface.

In this method the mass contained within the slip surface and the free ground surface is divided into n slices where the slices need not be vertical or even parallel, as shown in Fig. 3. The inclination of the slices are so chosen that a kinematic slip mechanism can develop. Since these inclinations are not known in advance, we start with assumed inclined planes where sliding can take place inside the mass and later iterate on these inclinations to find a critical set. The idea of the inclined failure plane inside the failed mass was used by Seed and Sultan (1967) for the wedge solution with two slices.

The forces acting on the ith slice are as shown in Fig. 3. We assume that under the influence of the force KW_i, the factor of safety on the slip surface

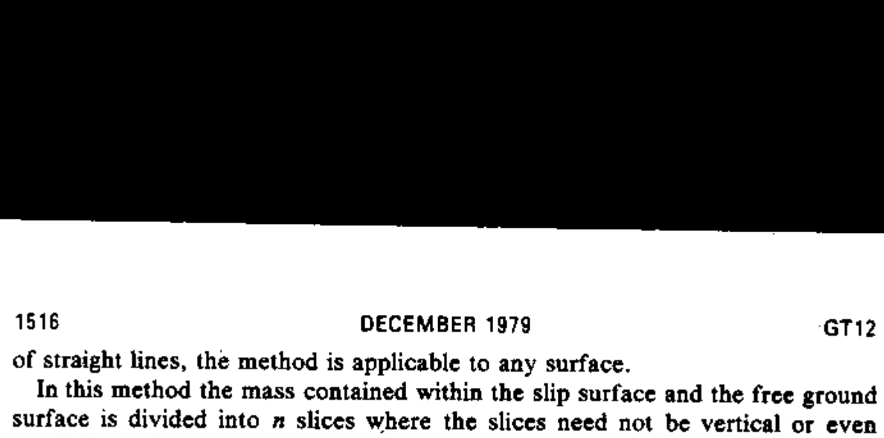

FIG. 3.—Forces Acting on Individual Inclined Slice

is equal to one in which case K is the critical acceleration factor K_c.

From the vertical and horizontal equilibrium of the slice, we obtain

$$N_i \cos\alpha_i + T_i \sin\alpha_i = W_i + X_{i+1}\cos\delta_{i+1} - X_i\cos\delta_i - E_{i+1}\sin\delta_{i+1} + E_i\sin\delta_i \quad (1)$$

$$T_i\cos\alpha_i - N_i\sin\alpha_i = K_c W_i + X_{i+1}\sin\delta_{i+1} - X_i\sin\delta_i + E_{i+1}\cos\delta_{i+1} - E_i\cos\delta_i \quad (2)$$

Mohr Coulomb failure criterion gives

$$T_i = (N_i - U_i)\tan\phi_i' + c_i' b_i \sec\alpha_i \quad (3)$$

Even if the mass contained within the slip surface is in a state of limiting equilibrium, the mass will not be able to move unless shear surfaces are formed within the body. Let us assume that the body forces X and E on the slice boundaries are such that they are also in a state of limiting equilibrium producing a local factor of safety equal to one, so that

$$X_i = (E_i - PW_i)\tan\bar{\phi}_i' + \bar{c}_i' d_i \quad (4a)$$

$$X_{i+1} = (E_{i+1} - PW_{i+1})\tan\bar{\phi}_{i+1}' + \bar{c}_{i+1}' d_{i+1} \quad (4b)$$

in which $\bar{\phi}'$ = the average friction angle
cohesion on the same plane; d = the
= the force due to water pressure on
in Eqs. 1 and 2 thereby eliminating T_i
from the resulting two equations, we ob

$$E_{i+1} = a_i - p_i K_c + E_i e_i$$

Eq. 5a is a recurrence relation and from

$$E_{n+1} = a_n - p_n K_c + E_n e_n;$$

$$E_{n+1} = (a_n + a_{n-1} e_n) - (p_n + p_{n-1} e_n$$

and proceeding further

$$E_{n+1} = (a_n + a_{n-1} e_n + a_{n-2} e_n e_{n-1} + $$

$$- K_c (p_n + p_{n-1} e_n + p_{n-2} e_n e_{n-1} + \ldots$$

In the absence of all external forces E_n

$$K_c = \frac{a_n + a_{n-1} e_n + a_{n-2} e_n e_{n-1} + \ldots}{p_n + p_{n-1} e_n + p_{n-2} e_n e_{n-1} + \ldots}$$

in which

$$a_i = \frac{W_i \sin(\phi_i' - \alpha_i) + R_i \cos\phi_i' + S_{i+1} \sin}{\cos(\phi_i' - \alpha_i + \bar{\phi}_{i+}}$$

$$p_i = \frac{W_i \cos(\phi_i' - \alpha_i)}{\cos(\phi_i' - \alpha_i + \bar{\phi}_{i+1}' - \delta_{i+1}) \sec\bar{\phi}}$$

$$e_i = \frac{\cos(\phi_i' - \alpha_i + \bar{\phi}_i' - \delta_i)\sec\bar{\phi}_i'}{\cos(\phi_i' - \alpha_i + \bar{\phi}_{i+1}' - \delta_{i+1})\sec\bar{\phi}_{i+}}$$

$$R_i = c_i' b_i \sec\alpha_i - U_i \tan\phi_i'$$

$$S_i = \bar{c}_i' d_i - PW_i \tan\bar{\phi}_i'$$

also $\bar{\phi}_1' = \delta_1 = \bar{\phi}_{n+1}' = \delta_{n+1} = 0$

Once the K_c value is determined,
$E_1 = 0$, so that Eq. 5a will give all
the X_i. Using Eqs. 1 and 3, the normal

$$N_i = (W_i + X_{i+1}\cos\delta_{i+1} - X_i\cos\delta_i$$

$$+ E_i \sin\delta_i)\cos\phi_i' \sec(\phi_i' - \alpha_i)$$

Knowing N_i forces, Eq. 3 gives all the

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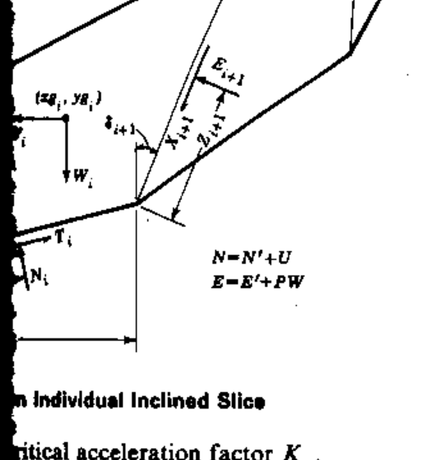

n Individual Inclined Slice

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librium of the slice, we obtain

$$\ldots_{+1} - X_i \cos\delta_i - E_{i+1} \sin\delta_{i+1} \quad (1)$$

$$\ldots \delta_{i+1} - X_i \sin\delta_i + E_{i+1} \cos\delta_{i+1} \quad (2)$$

$$\ldots \quad (3)$$

slip surface is in a state of limiting move unless shear surfaces are formed he body forces X and E on the slice a state of limiting equilibrium producing that

$$\ldots \quad (4a)$$

$$\ldots_{+1} \quad (4b)$$

in which $\bar{\phi}'$ = the average friction angle on the inclined plane; $\bar{c}'$ = the average cohesion on the same plane; d = the length of the inclined plane; and PW = the force due to water pressure on the plane. Putting Eqs. 3, 4*a*, and 4*b* in Eqs. 1 and 2 thereby eliminating T_i, X_i, and X_{i+1} and then eliminating N_i from the resulting two equations, we obtain

$$E_{i+1} = a_i - p_i K_c + E_i e_i \quad (5a)$$

Eq. 5*a* is a recurrence relation and from this we can obtain

$$E_{n+1} = a_n - p_n K_c + E_n e_n;$$

$$E_{n+1} = (a_n + a_{n-1} e_n) - (p_n + p_{n-1} e_n) K_c + E_{n-1} e_n e_{n-1} \quad (5b)$$

and proceeding further

$$E_{n+1} = (a_n + a_{n-1} e_n + a_{n-2} e_n e_{n-1} + \ldots \text{ to } n \text{ terms})$$

$$- K_c (p_n + p_{n-1} e_n + p_{n-2} e_n e_{n-1} + \ldots \text{ to } n \text{ terms}) + E_1 e_n e_{n-1} e_{n-2} \ldots \quad (6)$$

In the absence of all external forces $E_{n+1} = E_1 = 0$. Therefore

$$K_c = \frac{a_n + a_{n-1} e_n + a_{n-2} e_n e_{n-1} + \ldots + a_1 e_n e_{n-1} \ldots e_3 e_2}{p_n + p_{n-1} e_n + p_{n-2} e_n e_{n-1} + \ldots + p_1 e_n e_{n-1} \ldots e_3 e_2} \quad (7)$$

in which

$$a_i = \frac{W_i \sin(\phi_i' - \alpha_i) + R_i \cos\phi_i' + S_{i+1} \sin(\phi_i' - \alpha_i - \delta_{i+1}) - S_i \sin(\phi_i' - \alpha_i - \delta_i)}{\cos(\phi_i' - \alpha_i + \bar{\phi}_{i+1}' - \delta_{i+1}) \sec\bar{\phi}_{i+1}'} \quad (8)$$

$$p_i = \frac{W_i \cos(\phi_i' - \alpha_i)}{\cos(\phi_i' - \alpha_i + \bar{\phi}_{i+1}' - \delta_{i+1}) \sec\bar{\phi}_{i+1}'} \quad (9)$$

$$e_i = \frac{\cos(\phi_i' - \alpha_i + \bar{\phi}_i' - \delta_i) \sec\bar{\phi}_i'}{\cos(\phi_i' - \alpha_i + \bar{\phi}_{i+1}' - \delta_{i+1}) \sec\bar{\phi}_{i+1}'} \quad (10)$$

$$R_i = c_i' b_i \sec\alpha_i - U_i \tan\phi_i' \quad (11)$$

$$S_i = \bar{c}_i' d_i - PW_i \tan\bar{\phi}_i' \quad (12)$$

$$\text{also} \quad \bar{\phi}_1' = \delta_1 = \bar{\phi}_{n+1}' = \delta_{n+1} = 0 \quad (13)$$

Once the K_c value is determined, we may start from the known value of $E_1 = 0$, so that Eq. 5*a* will give all the E_i values. Eq. 4 then will give all the X_i. Using Eqs. 1 and 3, the normal force N_i is given by

$$N_i = (W_i + X_{i+1} \cos\delta_{i+1} - X_i \cos\delta_i - E_{i+1} \sin\delta_{i+1}$$

$$+ E_i \sin\delta_i) \cos\phi_i' \sec(\phi_i' - \alpha_i) \quad (14)$$

Knowing N_i forces, Eq. 3 gives all the T_i forces.

In this analysis we have so far made $n - 1$ assumptions giving the relationship between the X and E forces. In order to determine the line of thrust of E forces, assumptions must be made about the points of applications of all but one N_i forces. Or, alternatively, points of applications of N_i forces can be determined by assuming the line of thrust of E forces. In either case, the number of assumptions required is $n - 1$. The governing equation is obtained by taking moment of all the forces on the ith slice about a corner point A. This gives (with the coordinate system as shown in Fig. 3)

$$N_i l_i - X_{i+1} b_i \sec\alpha_i \cos(\alpha_i + \delta_{i+1}) + E_{i+1}\,[Z_{i+1} + b_i \sec\alpha_i \sin(\alpha_i + \delta_{i+1})]$$
$$- E_i Z_i - W_i(xg_i - x_i) + K_c W_i(yg_i - y_i) = 0 \quad \ldots \ldots \ldots \ldots \ldots \quad (15)$$

in which (xg_i, yg_i) are the coordinates of the center of gravity of the slice.

Again starting from the first slice, where $Z_1 = 0$ assuming l_i, Z_{i+1} can be determined or vice versa. Proceeding to the last slice, last l_n should be determined when $Z_{n+1} = 0$. As in the other methods of slices, the solution thus obtained should satisfy the criterion of acceptability such that all the N_i, T_i values should be positive. The values Z_i and l_i should lie within the slice, preferably in the middle third. Since innumerable variations are possible with l_i and Z_i values, it should always be possible to determine an acceptable line of thrust, but this may also take a considerable number of trials. It is felt, therefore, that

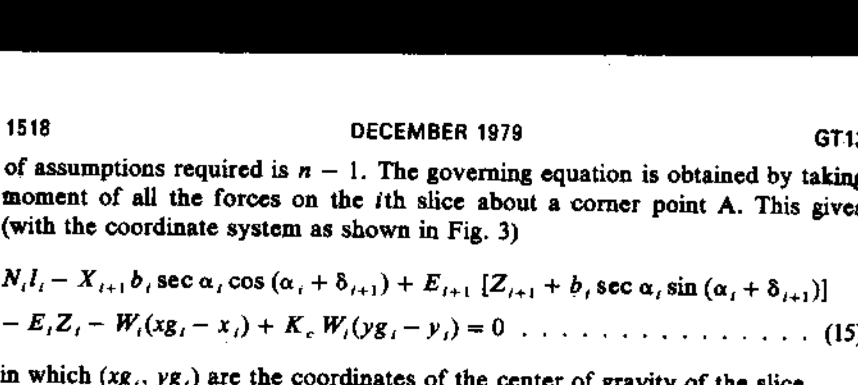

FIG. 4.—Example 1: Application of Method to Bearing Capacity Problem

in this method the line of thrust need not be a criterion of acceptability.

Since the moment equilibrium equation does not play any part in the determination of K_c (it is essential for the complete solution), the slices can be as large as possible and in fact should be controlled by the change of inclination of the slip surface.

The solution K_c depends on the assumed set of values of δ_i, i.e., on the location of the shear surfaces inside the sliding mass. It is possible that the assumed set may not be the one required and therefore it is necessary to find the most critical set. We define a critical set as one which produces the smallest value of K_c. The technique for finding this set is a trial and error procedure. One possible way of obtaining this set is to increase or decrease the δ_i values at the ith point, holding the rest of the δ values constant, and determine the K_c values in each case until a minimum K_c is reached for the ith point.

While increasing δ at the ith point, if the inclined plane crosses the plane at the $i + 1$ point, then δ_{i+1} is also change simultaneously. But while decreasing

δ at the ith point, it was not allowed to
at the $i - 1$ point.

Then, holding the δ_i at this critical
repeat the process until all the slip li
minimum K_c value with the correspond
It is found that proceeding from the
obtained is acceptable. The uniqueness
the results indicate that the procedure

The present method is the only one t
for the solution of the problem. [The
to produce the relative X force distri
because of the parameter λ involved
higher the internal strength, the higher
the internal strength, the lower is the
to produce a given factor of safety.

Because of the number of iteration
δ_i, this method is not really suitable f
numbers of probable slip surfaces have

TABLE 1.—Values of

N_q = 18.3		N_q
β (1)	K_c (2)	β (3)
10°	0.01066	10°
5°	0.00430	5°
2°	0.00268	2°
1°	0.00246	1°

version will be to use $\delta_i = 0$ for all
of safety FL_i for each of $\bar{\phi}'_i$ and $\bar{c}'_i$
1.1 which is obtained from a limited
material. In this case, in Eqs. 8–13, re

$$\tan\bar{\psi}'_i = \frac{\tan\phi'_i}{FL_i} \quad \ldots \ldots \ldots \ldots$$

Example 1.—In the first case we m
problem which was tested by Hansen (
analysis. He has shown that none of
analysis could give the Prandtl's bear
as shown in Fig. 4. The foundation
with $\phi' = 30°$. The surface load q
failure surface for the purpose of an
ABCD. The curved log-spiral section B
BB_1, B_1B_2 etc. so that they make an

A value of N_q is assumed. The a
which produces the starting value of

erning equation is obtained by taking
about a corner point A. This gives
ig. 3)

$[Z_{i+1} + b_i \sec \alpha_i \sin (\alpha_i + \delta_{i+1})]$

$= 0$ . . . . (15)

he center of gravity of the slice.
re $Z_1 = 0$ assuming l_i, Z_{i+1} can be
last slice, last l_n should be determined
of slices, the solution thus obtained
such that all the N_i, T_i values should
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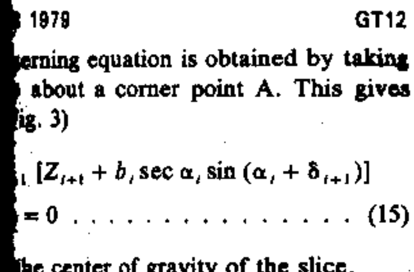

to Bearing Capacity Problem

a criterion of acceptability.
not play any part in the determina-
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o increase or decrease the δ_i values
values constant, and determine the
is reached for the ith point.
the inclined plane crosses the plane
simultaneously. But while decreasing

δ at the ith point, it was not allowed to cross the critical plane already obtained at the $i - 1$ point.

Then, holding the δ_i at this critical angle, proceed to the $i + 1$ point and repeat the process until all the slip line points are exhausted. The ultimate minimum K_c value with the corresponding δ_i values should be the critical set. It is found that proceeding from the toe point to the crest point, the result obtained is acceptable. The uniqueness of this set of δ_i is not tested. However, the results indicate that the procedure should provide the critical set.

The present method is the only one that uses the internal strength of material for the solution of the problem. [The writer (1973) uses the internal strength to produce the relative X force distribution but the exact effect is not felt because of the parameter λ involved in that solution.] It is found that the higher the internal strength, the higher is the K_c. In other words, the higher the internal strength, the lower is the strength required on the slip surface to produce a given factor of safety.

Because of the number of iterations involved in finding the critical set of δ_i, this method is not really suitable for the analysis of sections where large numbers of probable slip surfaces have to be analyzed. In this case, a simplified

TABLE 1.—Values of K_c for Different Values of N_q

N_q = 18.3		N_q = 18.5		N_q = 18.4	
β (1)	K_c (2)	β (3)	K_c (4)	β (5)	K_c (6)
10°	0.01066	10°	+0.00600		
5°	0.00430	5°	−0.00040		
2°	0.00268	2°	−0.00202		
1°	0.00246	1°	−0.00225	1°	0.00010

version will be to use $\delta_i = 0$ for all slices and then introduce a local factor of safety FL_i for each of $\bar{\phi}_i'$ and $\bar{c}_i'$. A reasonable value of FL_i is about 1.1 which is obtained from a limited number of test cases with homogeneous material. In this case, in Eqs. 8–13, replace $\bar{\phi}_i'$ by $\bar{\psi}_i'$ and $\bar{c}_i'$ by $\bar{c}_i'/FL_i$ where

$$\tan \bar{\psi}_i' = \frac{\tan \phi_i'}{FL_i} \quad . . . . (16)$$

Example 1.—In the first case we make use of the classical bearing capacity problem which was tested by Hansen (1966) to check various methods of stability analysis. He has shown that none of the commonly used methods of stability analysis could give the Prandtl's bearing capacity factor N_q. The problem is as shown in Fig. 4. The foundation material is weightless and cohesionless with $\phi' = 30°$. The surface load q is 1.0. We use the Prandtl's theoretical failure surface for the purpose of analysis, as shown in the figure by the line ABCD. The curved log-spiral section BC is broken up into several linear segments BB_1, B_1B_2 etc. so that they make an equal angle β at 0.

A value of N_q is assumed. The analysis proceeds with $\delta_i = 0$ for all slices which produces the starting value of K_c. Then first change δ at point B as shown by the line BE while keeping the rest of the angles constant and determine K_c again. This K_c is found to be smaller than the previous K_c. Then we move

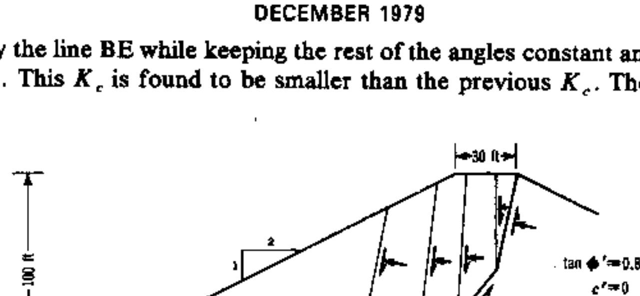

FIG. 5.—Example 2: Comparison of Solutions by Different Methods

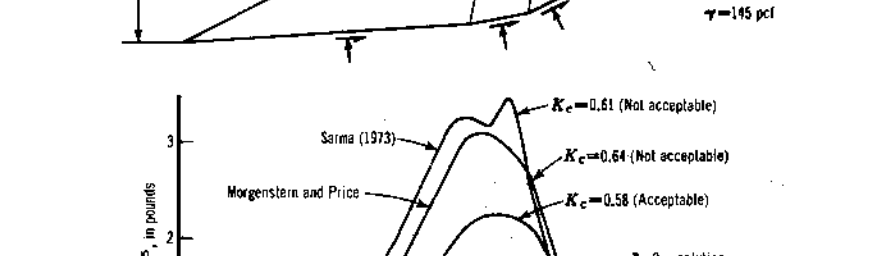

FIG. 6.—Example 3: Wellingborough Slip of May 1961 [Chandler (1974)]

the point E further away and repeat. As the point E passes over the points B_1, B_2 etc. the angles at B_1, B_2 etc. are also changed simultaneously, until

the minimum K_c is reached correspo
angle obtained is $E_o BO$. Then procee
at B_1. It is found that δ value at B_1
K_c. The angle at B_1 is therefore $E_1 E_1$
Proceeding further on, it is found t
can be changed and all the lines pas
C, it is found that the angles become

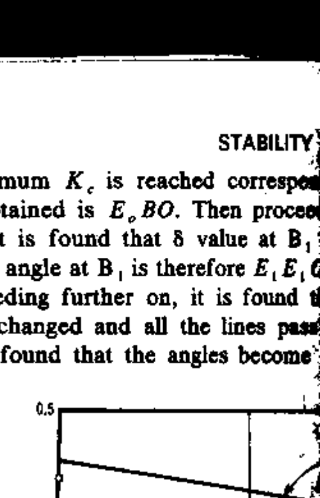

FIG. 7.—Effect of Shear Strength on
Strength on Slip Surface

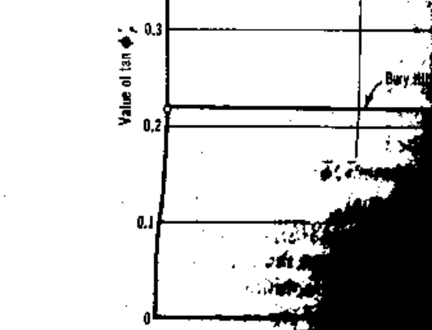

FIG. 8.—Example 4: Bury

all the lines pass through O. Thus, a
producing the same kinematic mechanis
value of K_c depends on the assumed
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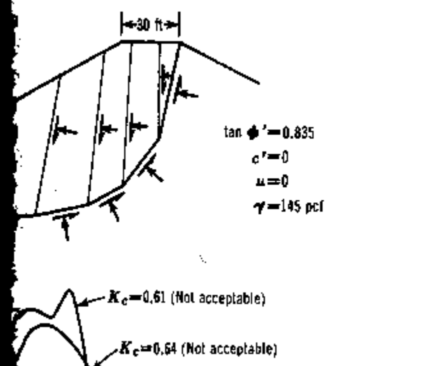

f Solutions by Different Methods

lip of May 1961 [Chandler (1974)]

s the point E passes over the points
re also changed simultaneously, until

the minimum K_c is reached corresponding to the point B. In this case the angle obtained is $E_o BO$. Then proceed to point B_1 and change the angle δ at B_1. It is found that δ value at B_1 cannot be changed without increasing K_c. The angle at B_1 is therefore $E_1 E_1 O$.

Proceeding further on, it is found that none of the angles up to point F can be changed and all the lines pass through O. After point F until point C, it is found that the angles become negative to produce minimum K_c and

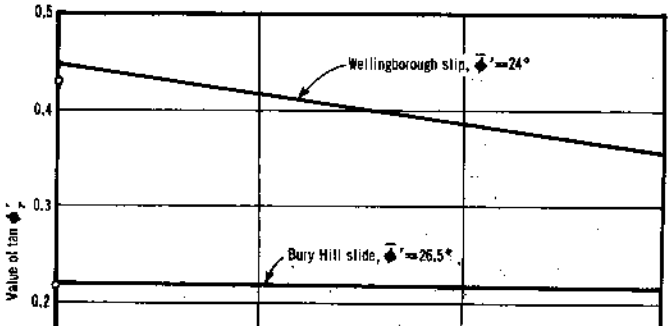

FIG. 7.—Effect of Shear Strength on Internal Shear Surfaces on Calculated Residual Strength on Slip Surface

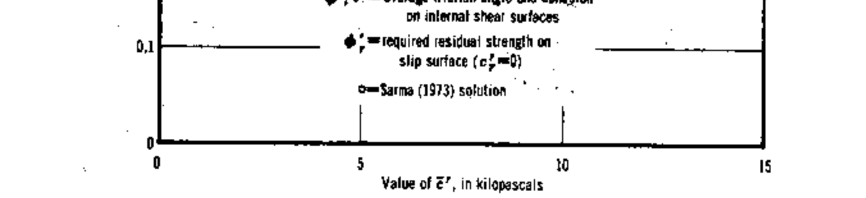

FIG. 8.—Example 4: Bury Hill Slide [Hutchinson, et al. (1973)]

all the lines pass through O. Thus, all local shear surfaces pass through O, producing the same kinematic mechanism as given by Prandtl. The final minimum value of K_c depends on the assumed value of N_q and on the angle β (see Table 1).

The following shows the N_q values required for $K_c = 0$ or static factor of safety = 1. As angle β is made smaller and smaller, N_q approaches the Prandtl's value of 18.401.

β	N_q
10°	18.750
5°	18.483
2°	18.414
1°	18.405

This example clearly shows that the present method is capable of finding the kinematic mechanism as well as the K_c value for any given slip surface. From this, to find the critical surface, it is only a process of trial and error.

Example 2.—In this problem, the slip surface is an arbitrary one. This is a simple problem but chosen because in this case both the writer (1973), and Morgenstern and Price (1965) methods were unable to produce an acceptable result. The present method does give an acceptable solution. The final results are as shown in Fig. 5. It is possible that both the Morgenstern and Price method and the writer's method will produce an acceptable result if one tries with the different variations of their assumptions for a long time.

Example 3.—These two cases are chosen as examples of actual slips. The first one is a rotational slide and is described by Chandler (1974) and the second one is a translational slide described by Hutchinson, et al. (1973). In studying these cases, the nonhomogeneity of the internal material is disregarded.

Case 3, the Wellingborough slip [Chandler (1974)], is shown in Fig. 6 for the pre-1961 section. Pore pressures are assumed on the slip surface. The tension crack is assumed to be filled with water. (The solution is modified easily to take this external force into account.) The results are as shown in Fig. 7. This example shows clearly the effect of the shear strength of material on the internal shear surfaces on the residual strength required for $F_s = 1$ or $K_c = 0$.

Case 4, the Bury Hill slide [Hutchinson, et al. (1973)], is shown in Fig. 8. The tension crack is assumed to be dry. The results are shown in Fig. 7. This example shows that for translational slides, the shear strength on the internal shear surfaces does not have a dramatic effect on the required residual strength, but still can account for about a degree in friction angle.

It is often noticed that the residual shear strength measured from torsional shear tests in the laboratory is lower than that computed from analysis of actual slips, Hutchinson, et al. (1973). It is now seen that the method of analysis itself can account for some of this difference, depending on the shape of the slip surface and the peak strength.

From the test cases, it is found that the method is suitable for analyzing actual slips that has occurred. Because large slices can be used, the time required for each analysis is comparatively small.

A computer program was written for the analysis with homogeneous materials only and the program was used in the Imperial College Computer CDC-6400.

Conclusions

In conclusion, it appears that there is still scope for the development of limit equilibrium methods of stability analysis where different methods can serve different purposes. The method presented in this paper is suitable for analysis of slope failures taking into account the kinematics of sliding blocks. It shows

that the shear strength on the
computed factor of safety.

Acknowledgments

The main body of the work wa
Engineers under grant agreement N
thank J. Hutchinson and R. Chand
of the example cases. Acknowledg
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Acknowledgments

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Appendix II.—Notation

The following symbols are used in this paper:

- b = width of slice measured horizontally;
- c' = cohesion in terms of effective stresses on slip surface;
- $\bar{c}'$ = average cohesion in terms of effective stresses on side of slice;
- DE = difference between normal body forces, E, on two sides of slice;
- DX = difference between shear forces, X, on two sides of slice;
- d = length of inclined internal shear plane measured from slip surface to ground surface;
- E = normal body force on side of slice in terms of total stresses;
- E' = normal body force on side of slice in terms of effective stresses;
- F = factor of safety for given seismic coefficient;
- F_s = static factor of safety;
- h = depth of slip surface measured vertically from ground surface;
- K = seismic coefficient;
- K_c = critical acceleration factor;
- l = distance of point of application of normal force, N, from corner of slice measured along slip surface;
- N = normal force on base of slice in terms of total stresses;
- N' = normal force on base of slice in terms of effective stresses;
- n = number of slices;
- P = normal force function on base of slice in terms of effective stresses giving distribution of normal stresses on slip surface;
- PW = force due to pore water pressure on side of slice;
- Ru = pore pressure ratio;
- T = shear force on slip surface at base of slice;
- U = force due to pore water pressure on base of slice;
- u = pore water pressure at point;
- W = total weight of sliding mass;
- W_i = weight of slice;
- X = shear force on side of slice;
- z = distance of point of application of E force measured from corresponding point on slip surface;
- α = angle made by slip surface with horizontal;
- δ = angle made by internal shear surface with vertical;
- σ' = normal stress function on slip surface;
- ϕ' = friction angle on slip surface in terms of effective stresses; and
- $\bar{\phi}'$ = average friction angle on side of slice in terms of effective stresses.

ENG

Equivalent Linear
Settlements

By Issa S. O

Introduction

Several methods have been propo
on sands. Table 1 summarizes so
Penetration Test (SPT) N values.
The purpose of this paper is fo
way considers important parameters
methods, with the intent of improvin
The key to the proposed method is e
is dependent on the mean-effective
compactness of sand.
The first five methods in Table 1 are
and Peck's (23) well-known settleme
allowable foundation pressure such t
Meyerhof's (11,12) method is an analy
chart. Meyerhof (12) and D'Appoloni
and Peck chart (and thus Meyerhof's m
grossly overestimates) settlements as
of this, they concluded the Terzaghi
increased by 50%. Meyerhof (12) fur
the water table be ignored, since its
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when the depth to water table below
than one-half foundation width. The

Note.—Discussion open until May 1, 1
a written request must be filed with the E
paper is part of the copyrighted Journal
Proceedings of the American Society of Civ
1979. Manuscript was submitted for revie
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[1]Vice Pres., Converse Ward Davis Dixon