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THE OPTIMAL DESIGN OF SOLAR CELL GRID LINES

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Abstract—The shape of grid lines or fingers, used to reduce conductive losses in photovoltaic cells, is shown to be optimized when the current flux in the line remains constant. This result is derived for cells of arbitrary geometry assuming the fraction of the cell area shaded is small. The shapes of grid lines for three special cases are provided. Optimal shapes for grid lines are also derived for cases when the area of the lines is a significant fraction of the cell area.

INTRODUCTION

The achievement of high efficiency in solar cells is dependent upon minimizing parasitic losses. A potential source of an efficiency degradation is the surface sheet resistance. Conductive grid lines or fingers are utilized to reduce the effective sheet resistance but at some loss of current generating capacity since the grids are normally opaque to incident radiation. Lehovec and Fedotowsky[1] have derived a relationship for grid spacing to minimize shading and sheet resistance losses, and Serreze[2] provides a technique for minimizing shading and ohmic losses in grid lines of predetermined shape. But, the actual shape of the grid lines themselves has either been neglected or assumed to be rectangular or triangular (linear taper).

In this paper an expression for the optimal grid line shape is derived. It shows that the shading and ohmic losses are minimized when the current flux in the grids remains constant. The maintenance of constant current flux can be attained in grids of constant sheet resistance by either increasing the width of the lines, increasing the number of lines or both.

ANALYSIS

A solar cell of length L and of arbitrary shape is shown in Fig. 1, where the function $g(x)$ describes the shape of the cell boundary and $f(x)$ describes the shape of the grid line under consideration. Note that the role of this grid line is unspecified so it could be a busbar, a finger leading to a busbar or a feeder line to a finger. For simplicity, all current from the cell is assumed to flow normal to the grid line. The current in the grid line at a point x is given by $i(x) = \int_0^x j[g(\xi) - f(\xi)]\,d\xi$ where j is the current generated per unit area of cell. If $f(x) \ll g(x)$, then the fraction of the cell area shaded is small (low shading factor) and current in the grid line can also be approximated by $i(x) = j\int_0^x j(\xi)\,d\xi$ or $i(x) = jh(x)$, where $h(x)$ is the integral of the exposed cell area from 0 to x. The ohmic power loss in the finger PL is $PL = \int_0^L \rho(i^2(x)/f(x))\,dx$, where ρ is the sheet resistance of the finger in $\Omega/\square$. This can be rewritten as $PL = \rho j^2 \int_0^L (h^2(x)/f(x))\,dx$.

The shape of the grid line can be optimized by allowing its contour to vary while its area remains constant and when the power loss is minimized the shape will be an optimum. This can be implemented by utilizing the calculus of variations. An error function F is first formed with the power loss PL and is augmented with a Lagrange multiplier P and the constraint that the grid line area remain constant.

$$F = \rho j^2 \int_0^L \frac{h^2(x)}{f(x)}\,dx + P\left[\int_0^L f(x)\,dx - A\right].$$

The constraint of constant grid line area, $\int_0^L f(x)\,dx = A$, is included in the error function in a form such that it is identically zero and therefore regardless of the value of the Lagrange multiplier P, the value of the error function remains unchanged. A minimum in the error function is found by differentiating the integrand I with respect to $f(x)$ and setting the resultant expression equal to zero (since derivatives of $f(x)$ are not present).

$$\frac{dI}{dX} = -\rho j^2 \frac{h^2(x)}{f^2(x)} + P = 0$$

or

$$f(x) = jh(x)\sqrt{\left(\frac{\rho}{P}\right)}.$$

But we know from the constraint that

$$\int_0^L f(x)\,dx = A$$

so

$$A = j\sqrt{\left(\frac{\rho}{P}\right)}\int_0^L h(x)\,dx.$$

The cell area and the area of the grid line are constant so $\int_0^L h(x)\,dx$ is also a constant. Therefore $P = \rho[(j/A)\int_0^L h(x)\,dx]^2$ and is a constant. Now an expression for the contour can be obtained.

$$f(x) = \frac{Ah(x)}{\int_0^L h(x)\,dx}$$

Since both A and $\int_0^L h(x)\,dx$ are constants $f(x)$ can be

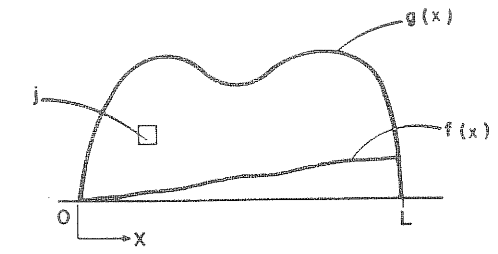

Fig. 1. Cell of length L and arbitrary shape $g(x)$ with grid line of shape $f(x)$.

expressed by

$$f(x) = \alpha h(x) \tag{1}$$

or

$$f(x) = \alpha \int_0^x g(\xi)\, d\xi, \tag{2}$$

where α is a constant dependent upon cell and light intensity characteristics. This results in the width of the grid line increasing as the integral of cell area thereby maintaining a constant current flux in the grid line. The shapes for three special cases are given in Fig. 2.

$$f_1(x) = c_1 x \tag{3}$$

$$f_2(x) = c_2 r^2 \tag{4}$$

$$f_3(x) = c_3\left[\left(\frac{x}{R}-1\right)\sqrt{\left[\frac{x}{R}\left(2-\frac{x}{R}\right)\right]} + \sin^{-1}\left(\frac{x}{R}-1\right) + \frac{\pi}{2}\right]. \tag{5}$$

The solution form for the more general case when $f(x) \cong g(x)$ is derived and results for Cases 1 and 2 given below.

When $f(x) \cong g(x)$, the complete form for $h(x)$ must be used

$$h(x) = \int_0^X [g(\xi) - f(\xi)]\, d\xi$$

from eqn (1),

$$f(x) = \alpha \int_0^X [g(\xi) - f(\xi)]\, d\xi$$

or

$$\frac{df(x)}{dx} = \alpha[g(x) - f(x)]$$

and rearranging terms and utilizing D for the differential operator (d/dx) yields

$$(D + \alpha) f(x) = \alpha g(x). \tag{6}$$

A solution to the homogenous equation is in the form

$$f(x)_H = a_1 e^{-\alpha x}.$$

For Case 1 $g(x) = a_2$, a constant, so the particular part of the solution is

$$f(x)_p = \frac{a_2}{\alpha}$$

and

$$f(x) = \frac{a_2}{\alpha} + a_1 e^{-\alpha x}.$$

Note that as $\alpha \to 0$, $e^{-\alpha x} \to 1 - \alpha x$ and $f(x) \to c_1 x$ then $c_1 x = (a_2/\alpha) + a_1 - a_1 \alpha x$, and therefore $(a_2/\alpha) = -a_1$ and $c_1 = -a_1 \alpha$ therefore $c_1 = a_2$. Thus

$$f(x) = \frac{c_1}{\alpha}(1 - e^{-\alpha x}). \tag{3a}$$

For Case 2

$$g(r) = a_3 r$$

and

$$f(r)_p = \frac{a_3}{\alpha}\left(r - \frac{1}{\alpha}\right)$$

so

$$f(r) = \frac{a_3}{\alpha}\left(r - \frac{1}{\alpha}\right) + a_1 e^{-\alpha r}$$

as

$$\alpha \to 0, \quad e^{-\alpha r} \to 1 - \alpha r + \frac{\alpha^2 r^2}{2}$$

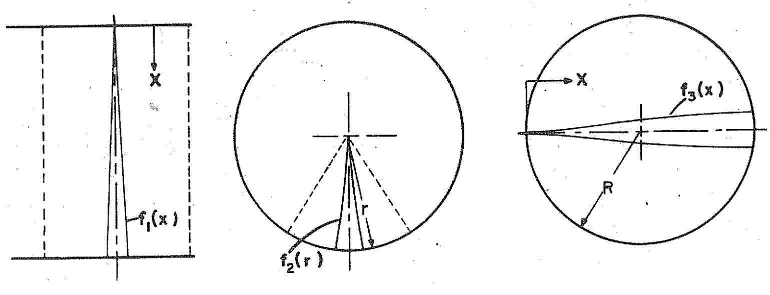

Fig. 2. Optimal grid line shapes for low shading factors ($f(x) \ll g(x)$) for three special cases. (a) Rectangular cell; (b) Circular cell with radial grid; and (c) Circular cell with diametric grid.

and

$$f(r) \to c_2 r^2.$$

Now,

$$c_2 r^2 = \frac{a_3}{\alpha}\left(r - \frac{1}{\alpha}\right) + a_1\left(1 - \alpha r + \frac{\alpha^2 r^2}{2}\right)$$

so

$$c_2 = \frac{a_1 \alpha^2}{2} \quad \text{or} \quad a_1 = \frac{2c_2}{\alpha^2}$$

and

$$\frac{a_3}{\alpha} = a_1 \alpha \quad \text{or} \quad a_1 = \frac{a_3}{\alpha^2}.$$

Therefore

$$a_3 = 2c_2$$

and

$$f(r) = \frac{2c_2}{\alpha}\left[r + \frac{1}{\alpha}(e^{-\alpha r} - 1)\right]. \quad (4a)$$

CONCLUSIONS

Shapes for optimally designed grid lines have been derived for both the case where shadowing losses are small and for the case where they become significant. The routine assumption of rectangular and linearly tapered grid lines is not generally optimal although manufacturing considerations may dictate suboptimal configurations.

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