

# Digital crossbar switch using nonlinear optical ring resonator

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## ABSTRACT

Optical elements are preferred to electronic ones for military computing and communications to reduce vulnerability to electromagnetic pulses from nuclear explosion, electromagnetic bombs or lightning. Equations are derived for an optical micro ring resonator and for a nonlinear ring resonator that uses Kerr material so that the resonant frequency changes with light intensity in the ring. The switch can be modulated at faster than 10 Gbps for compatibility with electronic switches and equipment. A two-by-two switch is described based on the nonlinear ring resonator. A Benes network is constructed using the two-by-two switches. This allows full permutations of the inputs by means of an algorithm for setting the switches. Several rings are used for each frequency with slightly different frequencies to allow switching of wavelength division multiplexed signals.

**Keywords:** Optical micro ring resonators Integrated optics, Nonlinear optics, Optical switches, Ring resonators, Optical two-by-two switch, Optical Benes network, Nonlinear optical switch

## 1. INTRODUCTION

Optical digital switches in telecommunications and computers are less vulnerable than electric ones to natural or deliberate electromagnetic interference, from lightning, nuclear explosions or electromagnetic bombs. Development, manufacturing cost, and integration with VLSI makes silica on silicon an attractive material for such optical crossbar switches. The ring resonator structure is selected because of its flexibility for creating many different optical components such as filters<sup>1,2</sup> and applied to a range of functionality<sup>3</sup> and applications<sup>4,5,6</sup>. In our earlier work<sup>7</sup> we investigated trade-offs in the design of ring resonators and improved performance with Bragg rings. In another of our earlier papers we show how switches are constructed by incorporating Kerr material into the ring<sup>8,9</sup>. Further the optical cross-bar switch uses very many identical rings and is sufficiently complex to lend itself well for optical integration.<sup>10</sup>

In section 2 we derive the basic equations for a ring resonator. In section 3 we describe the operation of a nonlinear ring resonator with Kerr nonlinear material. In section 4 a two by two nonlinear ring resonator switch is described. These switches are combined into a Benes interconnection network in section 5.

## 2. RING RESONATOR EQUATIONS

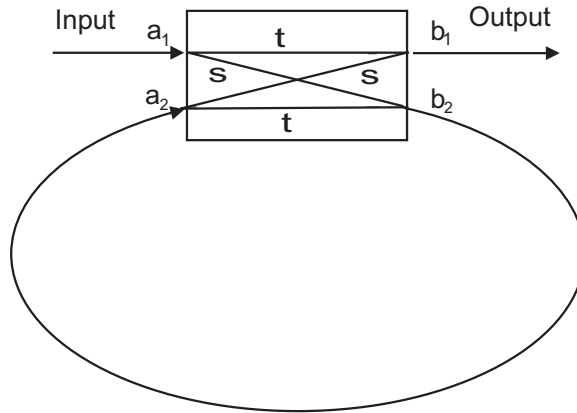
A ring resonator consists of a ring and straight waveguide close enough to form a directional coupler between them, figure 1.

### 2.1. Design of ring resonator as bandpass filter

A single mode waveguide is selected for the straight and ring waveguides. If the frequency is on the low side, close to cut-off, the field extends far from the waveguide and ring size must be larger than desired because otherwise too much power is lost due to the circular ring bend. The field in the cladding is known as the evanescent field. For a large evanescent field, coupling across the directional coupler between ring and straight waveguide is not lithographically demanding, allowing accurate control of ring resonant frequency and performance. If the frequency is on the high side, toward the start of the next higher mode, the field is tightly confined in the waveguide, radius of curvature can be small, ring size can be very small (microns are desirable for integrated optics) and ring bend losses small. But coupling across the directional coupler involves smaller distances, making lithography and parameter control more demanding. The ring resonance may then have to be tuned after construction.

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**Figure 1.** Optical waveguide coupled to a micro-ring

If the nominal length of the ring is such that the propagating field in the ring changes by multiples of  $2\pi$  during one revolution

$$\beta L = 2m\pi \quad (1)$$

where  $\beta$  is phase or propagation constant (radians/m) in ring,  $L$  is length of ring (m), and  $m$  an arbitrary integer. As  $\beta = 2\pi n_e/\lambda$ , with  $n_e$  the equivalent refractive index of the waveguide, we can write the length  $L$  of the circumference as an integer number of wavelengths,

$$L = \frac{m\lambda}{n_e} \quad (2)$$

Unfortunately  $n_e$  is difficult to compute because it depends on the percentage of light traveling in the core (the higher refractive index) versus that in the evanescent wave in the cladding (the lower refractive index). This in turn depends on the difference in refractive indices between core and cladding and the closeness to fundamental mode cut-off, as explained previously.

The directional coupler parameters, straight  $t$  and crossover  $s$ , figure 1, are controlled by the coupling coefficient  $\kappa$  determined by spacing between ring and coupler and coupling length  $l$ , the distance for which ring and coupler remain alongside each other. A small part  $s$  of the light entering at  $a_1$  is coupled to the microring at  $b_2$ , the rest (most of the input)  $t$  passes to the output at  $b_1$ . When the input is not at the resonant frequency, this is the output at  $b_1$ . However, at resonant frequency, the part coupled into the ring builds up to a high value in the ring so that it is large enough to cancel the input and reduce the output at  $b_1$  to zero.

Destructive cancellation is achieved at resonance by selecting the coupling length  $l$  and coupling coefficient  $\kappa$  to provide  $90^\circ$  phase shift on crossing from one side of the coupler to the other (through  $s$ ). In figure 1, light passing from  $a_1$  to  $b_2$  and then from  $a_2$  to  $b_1$ , crosses the coupler twice, developing a  $180^\circ$  phase shift to become out of phase with that reaching  $b_1$  from the straight waveguide  $a_1$ . The two fields will therefore cancel each other by destructive interference at resonance. However, for a complete cut-off bandstop filter at output  $b_1$  at resonance, in addition to being  $180^\circ$  out of phase, the two fields must have the same intensity to provide complete cancellation. The following equation (19) will show that the fields are the same intensity if, in the steady state, the transmission loss  $t$  from  $a_1$  to  $b_1$  equals the loss in traveling once around the ring times a factor for transmission insertion loss. When the input wavelength does not match that for ring resonance, power is not coupled into the ring and arrives at the exit at the top right, figure 1.

## 2.2. Equations for ring resonator bandstop filter

Next we derive the equations that verify the intuitive criteria discussed previously for complete cancellation at resonance. The steady state input-output equations for the directional coupler of figure (1) are<sup>2,1</sup>

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = (1 - \gamma)^{0.5} \begin{bmatrix} t & s \\ s & t \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \quad (3)$$

where  $t = \cos(\kappa l)$ ,  $s = -j\sin(\kappa l)$ ,  $\kappa$  is coupling coefficient,  $l$  is coupling length,  $\gamma$  is intensity insertion loss coefficient, and the  $j$  represents the  $90^\circ$  phase change in crossing an input to the other side output in the coupler.

The intensity loss around the ring is  $\exp\{-\rho L\}$ , where  $\rho$  is the intensity attenuation coefficient in the ring in Np/m. Therefore the amplitude loss is  $\exp\{-\rho L\}^{0.5} = \exp\{-\rho L/2\}$ . Adding the phase change due to propagation around the ring we can write the complex amplitude change around the ring as

$$E = \exp\left\{-\frac{\rho L}{2} - j\beta L\right\} \quad (4)$$

### 2.2.1. Equations for complete ring resonator: ring and coupler

We develop the equations from scratch and show that the results are similar in form to a Fabry-Perot oscillator with two mirrors<sup>7</sup>. For the light feeding back around the ring from the coupler output at port  $b_2$ , we can write the input to the left of the coupler at  $a_2$  as

$$a_2 = b_2 E \quad \text{or} \quad b_2 = \frac{a_2}{E} \quad (5)$$

We seek to solve equations (3) and (5) for the transmission through the ring resonator, figure 1

$$H = \frac{b_1}{a_1} \quad (6)$$

Substitute equation (5) into the lower equation of equation (3)

$$b_2 = \frac{a_2}{E} = (1 - \gamma)^{0.5} [s a_1 + t a_2] \quad \text{or} \quad a_2 \left( \frac{1}{E} - (1 - \gamma)^{0.5} t \right) = (1 - \gamma)^{0.5} s a_1 \quad (7)$$

Solving equation (7) for  $a_2$  gives

$$a_2 = \frac{(1 - \gamma)^{0.5} s a_1 E}{1 - (1 - \gamma)^{0.5} t E} \quad (8)$$

Substitute equation (8) into the top equation for the coupler, equation (3), gives

$$\begin{aligned} b_1 &= (1 - \gamma)^{0.5} \left[ t a_1 + s \left( \frac{(1 - \gamma)^{0.5} s a_1 E}{1 - (1 - \gamma)^{0.5} t E} \right) \right] = (1 - \gamma)^{0.5} a_1 \left[ \frac{t(1 - (1 - \gamma)^{0.5} t E) + s(1 - \gamma)^{0.5} s E}{1 - (1 - \gamma)^{0.5} t E} \right] \\ &= (1 - \gamma)^{0.5} a_1 \left[ \frac{t - t^2(1 - \gamma)^{0.5} E + s^2(1 - \gamma)^{0.5} E}{1 - (1 - \gamma)^{0.5} t E} \right] \end{aligned} \quad (9)$$

Hence using  $s^2 - t^2 = -1$  from definitions following equation (3),

$$b_1 = (1 - \gamma)^{0.5} a_1 \left[ \frac{t - (1 - \gamma)^{0.5} E}{1 - (1 - \gamma)^{0.5} t E} \right] \quad (10)$$

Hence the amplitude transmittance  $H$  and the power transmittance  $P$  through the ring resonator, figure 1 in equation (6) can be written from equation (10) as

$$H = \frac{b_1}{a_1} = (1 - \gamma)^{0.5} \left[ \frac{t - (1 - \gamma)^{0.5} E}{1 - (1 - \gamma)^{0.5} t E} \right] \quad \text{and} \quad P = \left| \frac{b_1}{a_1} \right|^2 = (1 - \gamma) \left| \frac{t - (1 - \gamma)^{0.5} E}{1 - (1 - \gamma)^{0.5} t E} \right|^2 \quad (11)$$

Substituting  $E$  from equation (4) into equation (11) and defining

$$x = (1 - \gamma)^{0.5} \exp\left(-\frac{\rho L}{2}\right) \quad (12)$$

gives transmitted power

$$P = (1 - \gamma) \left| \frac{t - x \exp\{-j\beta L\}}{1 - x t \exp\{-j\beta L\}} \right|^2 = (1 - \gamma) \frac{(t - x \exp\{-j\beta L\})(t - x \exp\{j\beta L\})}{(1 - x t \exp\{-j\beta L\})(1 - x t \exp\{j\beta L\})} \quad (13)$$

Using  $\exp\{j\beta L\} + \exp\{-j\beta L\} = 2\cos(\beta L) = 1 - 2\sin^2(\beta L/2)$  the denominator of equation (13) may be written

$$denom = (1 - x t \exp\{-j\beta L\})(1 - x t \exp\{j\beta L\}) = (1 - x t)^2 + 4x t \sin^2\left(\frac{\beta L}{2}\right) \quad (14)$$

The numerator of equation (13) may be written (by adding and subtracting  $(1 + x^2 t^2)$ )

$$\begin{aligned} num &= t^2 + x^2 - 2xt + 4x t \sin^2\left(\frac{\beta L}{2}\right) \\ &= (-1 - x^2 t^2) + (1 + x^2 t^2) + t^2 + x^2 - 2xt + 4x t \sin^2\left(\frac{\beta L}{2}\right) = denom + (1 - t^2)(1 - x^2) \end{aligned} \quad (15)$$

Substituting equations (15) and (14) into equation (13) gives a convenient form for the ring resonator, similar to the form for a parallel mirror Fabry-Perot resonator<sup>2,1</sup>,

$$T = \left| \frac{b_1}{a_1} \right|^2 = (1 - \gamma) \left[ 1 - \frac{(1 - x^2)(1 - t^2)}{(1 - tx)^2 + 4tx \sin^2(\beta L/2)} \right] \quad (16)$$

The maximum of equation (16) occurs when  $\sin^2(\beta L/2)$  is maximum at one and the minimum occurs when  $\sin^2(\beta L/2)$  is minimum at zero,

$$T_{max} = (1 - \gamma) \frac{(x + t)^2}{(1 + tx)^2} \quad (17)$$

$$T_{min} = (1 - \gamma) \frac{(x - t)^2}{(1 - tx)^2} \quad (18)$$

The output cancels to zero for a band stop filter at resonance when  $T_{min} = 0$  or from equation (18) and (12),

$$t = x \quad \text{or} \quad t = (1 - \gamma)^{0.5} \exp\left(-\frac{\rho L}{2}\right) \quad (19)$$

This specifies that the transmission loss  $t$  from  $a_1$  to  $b_1$  must equal the loss in traveling once around the ring times a factor for transmission insertion loss (as discussed in section 2.1).

Other useful characteristics of the microring resonator are full width at half maximum

$$\delta(\beta l) = \frac{2(1 - xt)}{\sqrt{xt}} \quad (20)$$

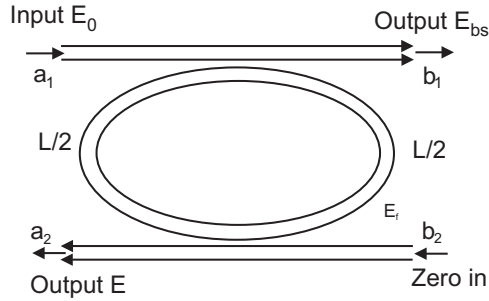
and finesse, indicative of the number of channels possible,

$$F = \frac{2\pi}{\delta(\beta l)} = \frac{\pi\sqrt{xt}}{(1 - xt)} \quad (21)$$

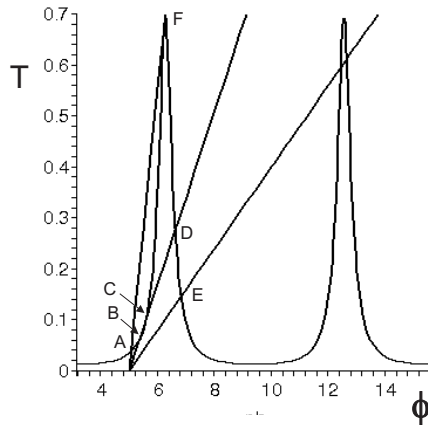
Simulations for optimizing ring resonator design are provided in the references.<sup>7</sup>

The equations can be extended to an open ring which involves two straight waveguides and an adjacent ring. Figure 2 shows a layout, like that from an integrated optic software package, for an open ring resonator in which two directional couplers are formed where the straight waveguides pass close to the ring<sup>2,9</sup>.

Input  $E_0$  at  $a_1$  is provided at the top left. When the input wavelength matches the ring resonant frequency, the bandstop output  $E_{bs}$  at  $b_1$  drops toward zero, as discussed in section 2.1 and section 2.2. At the same time, a bandpass output  $E$  is observed at  $a_2$  at the bottom left due to the high power in the resonator. When the



**Figure 2.** An open type microring resonator



**Figure 3.** Intensity transmission for open nonlinear ring resonator

wavelength of input  $a_1$  differs sufficiently from the resonant frequency of the microring, the outputs are reversed, the bandpass output switching from  $a_2$  to  $b_1$  and the bandstop from  $b_1$  to  $a_2$ . Hence, output  $E$  at  $a_2$  acts as a bandpass filter centered at the resonant frequency and output  $E_{bs}$  at  $b_1$  acts as a bandstop filter<sup>11</sup> at the resonant frequency. Such a filter on a bus from input  $a_1$  through output  $b_1$  is known as a drop-add filter in telecommunications because if a wavelength division multiplexed signal is input at  $a_1$  it will drop the frequency matching the ring resonance off at output  $a_2$ . At the same time a new channel with modulation of this frequency can be added at  $b_2$  and will join the bus out at  $b_1$ .

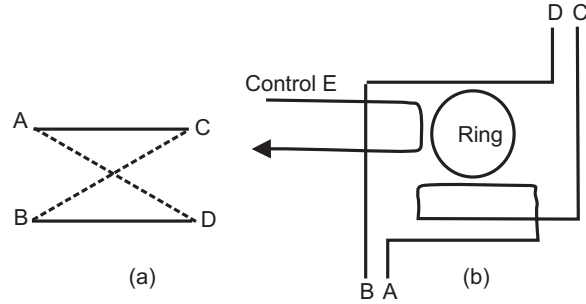
Previously, equation (16), we considered transmission to the bandstop output  $b_1$ . A similar procedure will give the transmission to the bandpass output  $a_2$

$$T_{bp} = \left| \frac{E}{E_0} \right|^2 = \frac{P}{P_0} = (1 - \gamma) \left[ \frac{x^2(1 - t^2)}{(1 - t^2x^2)^2 + 4tx \sin^2(\beta L/2)} \right] \quad (22)$$

The transmission as a function of phase in the microring  $\phi$  is plotted from equation (22) in figure 3 and shows periodic Airy functions. The straight lines in figure 3 were added to represent the nonlinear medium variation in  $\phi = \beta L$  with power and can be used to explain the presence of hysteresis<sup>9,1</sup>.

### 3. NONLINEAR RING RESONATOR SWITCHES

For a switch, the ring is doped with a nonlinear Kerr material that changes refractive index with power<sup>2</sup>. Thus adding power in the ring, even at a different frequency, controls resonant frequency. Kerr materials have weak nonlinearity, meaning that high power is normally needed to see an effect. However, by using resonance, the



**Figure 4.** Two-by-two switch, (a) Microring resonator two-by-two switch, (b) symbolic representation

power in the resonator is increased by the resonance factor  $Q$  (often hundreds of times), making the effective nonlinearity much greater. This lowers the power needed to switch the ring from one resonant frequency to another. However, too high a  $Q$  will make the device slower. One of the reasons for shrinking the microring resonator to a few micrometers is to simultaneously have a high  $Q$  and fast switching speed to allow 10GHz modulation.

Nonlinear Kerr material has a refractive index  $n$  that varies with intensity of light according to

$$n = n_0 + n_2 \frac{P_f}{A_{eff}} \quad (23)$$

where  $n_0$  is a reference refractive index,  $n_2$  is the nonlinear Kerr coefficient ( $3.18 \times 10^{-14}$  for semiconductor doped glass),  $P_f$  is the power in the ring and  $A_{eff}$  is the effective core area of the waveguide.

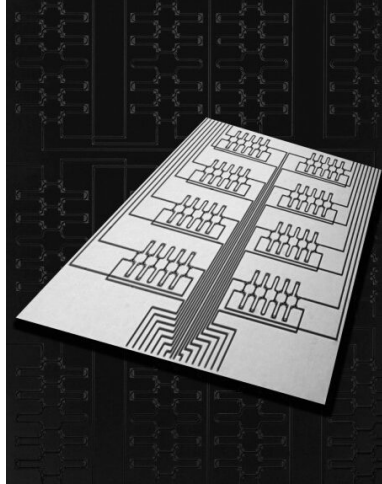
Therefore a microring with nonlinear material in the ring will change its resonant frequency with intensity in the ring. This permits an additional control frequency input to be used (at a different wavelength to avoid interference with the signal) to take a ring in or out of resonance at a signal frequency. A microring filter<sup>11</sup> may be used at the output to remove the control frequency. The filter could be avoided by rotating the control light in the ring in the opposite direction but then an isolator would be needed at the input to prevent interference back into the laser source.

In a typical case, the glass of the ring is doped with a nonlinear Kerr material: for example, a semiconductor doped glass  $CdS_xSe_{1-x}$  that has a refractive index depending on intensity  $I$ . In this case the Kerr coefficient of  $n_2 = 2 \times 10^{-14} \text{m}^2/\text{W}$  implies that refractive index varies from that of glass  $n_0$  by  $n_2 I = n_2 (P_f / A_{eff})$  where  $P_f$  is the power in the microring (figure 2) and  $A_{eff}$  is the effective area of the waveguide. The latter is larger than the cross section area of the core because of the evanescent wave spreading into the cladding; the spread is greater for smaller difference between core and cladding refractive indices.

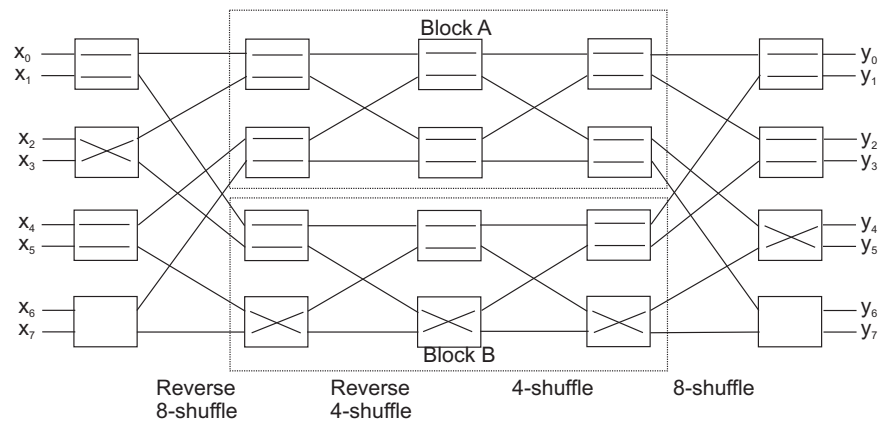
#### 4. TWO-BY TWO OPTICAL SWITCH WITH NONLINEAR-MICRORING RESONATORS

The basic circuit for many interconnection switches is the two-by-two switch shown in figure 4(a). The control signal switches the inputs from passing straight through (solid line) to crossing over (dashed line). A directional coupler could perform a similar function but would not have resonance to enhance the nonlinearity and would be larger. So we choose to construct the optical two-by-two switch from a single nonlinear-microring resonator, figure 4(b).

When there is no control signal at port  $E$ , the resonator resonates at less than wavelength  $\lambda_1$  even in the presence of signal at  $\lambda_1$ . In this case the inputs at  $\lambda_1$  pass straight through the switch:  $A$  to  $C$  and  $B$  to  $D$ . When a control signal at wavelength  $\lambda_2$  is placed at port  $E$ , the change in intensity in the ring causes its resonance to shift to the input wavelength  $\lambda_1$ . The inputs now cross over at the output:  $A$  to  $D$  and  $B$  to  $C$ .



**Figure 5.** Eight two-by-two optical switches of ring resonators by IBM



**Figure 6.** Benes network

In order to broaden the resonance bandwidth to reduce wavelength and temperature sensitivity, IBM used five rings in series in place of single rings.<sup>12</sup> Figure 5 shows IBM’s experimental device with eight sets of five rings on a single silicon chip using silicon on insulator (SOI). IBM indicates that 2000 such  $2 \times 2$  switches fit into a square millimeter.

### 5. BENES OPTICAL CROSSBAR SWITCH USING TWO-BY-TWO NONLINEAR MICRO-RING RESONATOR OPTICAL SWITCHES

Two-by-two switches can be connected to form an optical crossbar interconnected network<sup>13</sup>. A Benes network<sup>14</sup> is shown in figure 6. Inputs can be connected to any permutation of outputs on a one-to-one basis making the crossbar switch non-blocking. Each switch is set straight through or cross over to provide the desired connections. Many algorithms exist to ensure that all the desired connections are made correctly.<sup>15</sup> The network is broken into two subnetworks, labeled block *A* and block *B* in figure 6. For larger networks the subnetworks are recursively divided. The algorithm sets two-by-two switches to make the first connection from input to output by passing through submatrix block *A*. The next connection is made in reverse from output to input, but this time starting with the other input of the just used two-by-two switch at the network output and passing through block *B*. The next step starts with the other output of the two-by-two switch just used at the network input and passes

through block  $A$ . The back and forth process through alternating blocks is followed until all connections are made. The algorithm has to be followed carefully to make all the desired connections.

## 6. CONCLUSION

Equations for a ring resonator and an open ring resonator were developed. Kerr material was introduced into the ring to provide nonlinear operation, the resonant frequency changes with the light power in the ring. A  $2 \times 2$  switch was described based on a single open ring with an additional control signal for switching. This was assembled into a Benes network to form a digital optical crossbar switch. Many rings are used in parallel with slightly different resonant frequencies to provide sufficient bandwidth to pass wavelength division multiplexed signal used for telecommunications.

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