

## Spectrum of Turbulent Fluctuations of the Sea-Water Refraction Index<sup>†</sup>

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The behavior of the spectrum of fluctuations  $E_n$  of refraction index  $n$  in turbulent sea water when the fluctuations of  $n$  are controlled by fluctuations  $T'$  of the temperature and  $S'$  of the salinity is analyzed. The spectrum of  $E_n$  is found for homogeneous and isotropic turbulence as a function of the rate of dissipation (equalization) of temperature and salinity fluctuations which, together with the dissipation rate of the energy of turbulence, control the statistical behavior of fluctuations of  $n$ . These rates were parametrized by employing the gradient hypothesis. It was found that, depending on the contributions of temperature and salinity fluctuations to fluctuations of  $n$ , one may observe domains of anomalous behavior which may include appearance of local extrema.

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### 1. Introduction

Propagation of light in a medium with random inhomogeneities involves changes in the wave amplitude, phase and arrival angle. These perturbations are random. One of the reasons for their appearance is turbulence which in stratified atmosphere, for example, is responsible for the existence of random temperature and humidity fluctuations which control the fluctuations of the refraction index. This highly distorts the optical wave fields. The investigation of fluctuations of optical waves in the turbulent atmosphere is important as a part of obtaining exact quantitative data on the characteristics of visible and infrared radiation, in interpreting data of radiation observations from artificial satellites, in obtaining data of astrophysical observations free of the effect of the terrestrial atmosphere, etc. The particular interest in such studies is associated with the use of lasers in communications, data-transmission, telemetering, location and similar systems. In fact, the information capacity of optical communication lines, the space-time resolution of laser locators, the precision of geodetic laser optical instruments and other technical parameters of optical systems can be assessed only with allowance for fluctuations of optical fields [1, 2].

The theoretical principles of investigating the properties of a light wave traveling through a turbulent atmosphere have been developed by a number of investigators [3–6]. They showed that the

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<sup>†</sup> Translated from *Prikladna Gidromekhanika*, 1999, 1 (73), No. 1, pp. 52–63.

statistical behavior of fluctuations of light-wave parameters (phase, arrival angle, amplitude) are a function of temperature fluctuations. As a rule, they employed a model of the spectrum of temperature fluctuations [4] which, for large scales, corresponds to the Obukhov-Corrsin  $E_T \approx k^{-5/3}$  law (the inertia interval), whereas for small scales the spectrum damps out exponentially (dissipation interval); here  $k$  is the wave number. Comparison with data of full-scale measurements points to rather satisfactory agreement of results [1–4].

Similar aspects of the propagation of light in the turbulent water medium, in particular, in sea water, have been much less explored. The refraction index  $n$  for sea water is determined under actual conditions basically by the temperature  $T$  and salinity  $S$ . In the inertia-convective spectral range the temperature fluctuations  $E_T$  and salinity fluctuations  $E_S$  are, as in the atmosphere, described by the Obukhov-Corrsin law, i.e.,  $E_T \sim k^{-5/3}$  and  $E_S \sim k^{-5/3}$  [4]. For small scales (large  $k$ ) the situation becomes much more complicated as compared with the case of atmosphere. It is specific to the water medium that the kinematic viscosity  $\nu$  is much higher than the molecular thermal diffusivity  $\chi_T$  and the more so than the molecular salinity transport coefficient  $\chi_S$ :  $\nu \gg \chi_T \gg \chi_S$ . As a result, at small scales, where viscosity has a marked effect on velocity fluctuations, the effect of thermal diffusivity and diffusion of salinity on temperature and salinity fluctuations is still insignificant. The spectra of temperature and salinity fluctuations in this viscous-convective spectral interval (Batchelor interval) are proportional to  $k^{-1}$ :  $E_T \sim k^{-1}$  and  $E_S \sim k^{-1}$  [4], whereas the spectrum of velocity fluctuations decays exponentially. Accordingly, the spectrum of fluctuations of temperature and of other impurities in the water medium differs significantly from that in the atmosphere. An analysis of the statistical properties of a light wave traveling in a turbulent water medium when the index of refraction  $n$  is controlled by a single component (temperature  $T$  or salinity  $S$ ) was performed by Hill [7]. The dependence of the fluctuations  $n'$  on temperature fluctuations  $T'$  and salinity fluctuations  $S'$  causes the spectrum of  $E_n$  to behave in a rather specific manner.

This paper is concerned with the spectral characteristics of turbulent temperature and salinity fluctuations. A new approximating formula describing the spectrum of fluctuations of the index of refraction  $n$  for the case when the fluctuations of the latter are controlled by those the salinity and temperature is obtained and the mutual effect of these fluctuations is analyzed. The relationship between the statistical characteristics of fluctuations of  $n$  and of those of the averaged temperature and salinity distributions is examined. The coefficients of turbulent transfer are parametrized on the basis of the gradient hypothesis. Results of calculations of the spectrum on the basis of oceanographic observations are presented.

## 2. Spectrum of Turbulent Fluctuations of the Temperature (Salinity)

Turbulence is a complex vortical flow with a large number of scales of motion. It is the result of instability of the flow of liquid relative to inevitable flow perturbations. At high Reynolds numbers  $Re = U_0 L_0 / \nu$ , which defines the intensity level of turbulent flows, the effect of free-stream variables such as anisotropy, inhomogeneity and unsteadiness weaken with reduction in the scales of the vortices as a result of the chaotic nature of energy transfer from large- to small-scale vortices by fragmentation. Here  $U_0$  and  $L_0$  are the characteristic velocities and dimension of the domain occupied by the fluid. As a result, the statistical mode of small-scale velocity fluctuations becomes homogeneous, isotropic and steady and also independent of the specifics of the free stream. It is controlled only by the flow of energy from large to small vortices (cascade energy transfer). Viscosity primarily affects very small vortices. Kinetic energy is converted into heat precisely in this small-scale motion. Here the quantity of energy transmitted from the free stream to locally homogeneous and isotropic motions which are not yet perceptibly affected by viscosity is equal to the

quantity of energy that is dissipated in small-scale vortices. The magnitude of the cascade energy flux corresponds to the rate of dissipation of the energy of turbulence. This means that the mean flow affects the mode of small-scale fluctuations only indirectly, by way of the magnitude of the energy flux which is transmitted in cascade manner and in the end is dissipated into heat.

We shall examine the temperature fluctuations on the assumption that the temperature inhomogeneity does not perceptibly affect the behavior of the turbulent flow. It is natural to expect that in this case also the mode of temperature fluctuations caused by the translation of liquid lumps at different temperatures will be locally homogeneous and isotropic. Note that actually inhomogeneities in the temperature (and salinity) distributions may perceptibly affect the motion of liquid in the gravity field. It is known [4, 8] that this effect is felt primarily by the large-scale motion. As a result, the buoyancy of sea water does not prevent the existence of regions with developed turbulence, in particular, inertia and inertia-convective intervals of velocity and temperature fluctuation spectra were actually recorded [8].

The breakup of large vortices is accompanied by fragmentation of the macrostructure of temperature inhomogeneities into increasingly smaller units. The mean temperature  $\langle T \rangle$  in small-scale regions can be regarded as virtually constant for which reason the inhomogeneity in such regions is represented by the temperature fluctuation  $T' = T - \langle T \rangle$ . Following Obukhov [9], we shall take as the measure of the temperature inhomogeneity of a liquid volume the quantity

$$H = \frac{1}{2} \rho \int_V \langle (T')^2 \rangle dV$$

where  $\rho$  is the density of the fluid, i.e., we shall assume that the degree of temperature inhomogeneity is represented by the quantity  $(T')^2/2$ . It is known that the mode of turbulent temperature fluctuations is not determined solely by the dynamic parameters of the fluid ( $\varepsilon$  and  $\nu$ ), but also by the rate of reduction of the measure of temperature inhomogeneity  $(T')^2/2$ , i.e., by the rate of dissipation (equalization) of fluctuations  $\varepsilon_T$  of the temperature field, and also by the molecular thermal diffusivity  $\chi_T$ .

In analyzing the spectral characteristics of turbulent temperature fluctuations we introduce the following characteristic designations of wave numbers  $k$  (of the spectral variable):  $k_K$  – the Kolmogorov wave number representing the effect of viscosity;  $k_B^T = \varepsilon/\nu\chi_T^2$  and  $k_B^S = \varepsilon/\nu\chi_S^2$  – the Batchelor wave numbers representing the effect of thermal diffusivity and diffusion of salinity, respectively;  $k_L$  is the wave number of energy conveying vortices, i.e., of large-scale vortices which contain the bulk of the flow energy.

We write the equation of heat transfer in a viscous, thermally conducting incompressible fluid in the form [3]

$$\frac{\partial T}{\partial t} + v_i \frac{\partial T}{\partial x_i} = \chi_T \frac{\partial^2 T}{\partial x_i^2} \quad (1)$$

where  $t$  is the time;  $v_i$  ( $i = 1, 2, 3$ ) are components of the velocity vector and  $T$  is the temperature. Introducing the averaged values of velocity  $\langle v_i \rangle$  and temperature  $\langle T \rangle$  which in this section are regarded as constant, we will write  $v_i = \langle v_i \rangle + v_i'$ ,  $T = \langle T \rangle + T'$  where  $v_i'$  and  $T'$  are respectively the fluctuating parts of the velocity and temperature. Then we obtain from Eq. (1)

$$\frac{\partial T'}{\partial t} + \langle v_i \rangle \frac{\partial T'}{\partial x_i} + v_i' \frac{\partial T'}{\partial x_i} = \chi_T \frac{\partial^2 T'}{\partial x_i^2} \quad (2)$$

Analyzing this equation in points A and B we obtain, on the assumption of statistical homogeneity and isotropy, for the correlation function of temperature fluctuations

$$\Psi_{TT}(t, \xi_i) = \langle T'_A T'_B \rangle$$

the equation

$$\frac{\partial}{\partial t} \Psi_{TT}(t, \xi_i) - 2 \frac{\partial}{\partial \xi_i} \Omega_{T_i, T}(t, \xi_i) = 2\chi_T \frac{\partial^2}{\partial \xi_i^2} \Psi_{TT}(t, \xi_i) \quad (3)$$

where the variable  $\xi_i = (x_i)_B - (x_i)_A$ .

We introduce the scalar

$$\Omega_{TT}^* = 2 \frac{\partial}{\partial \xi_i} \Omega_{T_i, T}$$

Since functions  $\Psi_{TT}$  and  $\Omega_{T_i, T}$  depend only on the distance between points A and B, we convert to variables  $t$  and  $r$ . As a result, we obtain an expression describing the variation in the correlations of fluctuations of the temperature field [4]:

$$\frac{\partial}{\partial t} \Psi_{TT}(t, r) - \Omega_{TT}^*(t, r) = 2\chi_T \frac{1}{r} \frac{\partial^2}{\partial r} \left[ r^2 \frac{\partial}{\partial r} \Psi_{TT}(t, r) \right] \quad (4)$$

We now subject this equation to a three-dimensional Fourier transformation with respect to space variables. With reference to the isotropic nature of the temperature fluctuations and, having evaluated the integrals over the angular variables, we find the spectral analog of Eq. (4):

$$\frac{\partial}{\partial t} E_T(t, k) - W_T(t, k) = -2\chi_T k^2 E_T(t, k) \quad (5)$$

where  $E_T(t, k) = 4\pi k^2 E_{TT}(t, k)$  is the three-dimensional spectral density. Equation (5) describes the variation in time of the spectral distribution of the intensity of temperature fluctuations. Function  $W_T(t, k)$  describes the redistribution of the perturbations of the temperature field over the spectrum of wave numbers brought about by eddy mixing. Integrating of Eq. (5) over the entire spectrum of wave numbers we obtain

$$\frac{\partial}{\partial t} \int_0^\infty E_T(t, k) dk = -2\chi_T \int_0^\infty k^2 E_T(t, k) dk \quad (6)$$

The left-hand side of this equation is the rate of change of the total energy of fluctuations, i.e., the rate of dissipation of energy  $\varepsilon_T$  of temperature fluctuations. From this follows the condition of norming of the spectrum of turbulence [4]

$$2\chi_T \int_0^\infty k^2 E_T(t, k) dk = \varepsilon_T \quad (7)$$

It is known that the set equations that describe turbulent flow is not closed. Closure of this set of equations is attained by resorting to various hypotheses, many of which are described in the book [4]. One of the ways for attaining closure is the use of spectral equations and working out suitable approaches to solving this problem. This applies in equal measure to equations describing the behavior of turbulent fluctuations of the temperature field. Equation (5) is the principal spectral equation employed for this purpose. An analysis of hypotheses which may be employed for attaining closure of Eqs. (5) is presented in the book [4]. One of the fruitful ways of doing so is by introducing the scalar spectral flow function  $F_T$ :

$$W_T(t, k) = -\frac{\partial F_T(t, k)}{\partial k} \quad (8)$$

and establishing the relationship between  $F_T(t, k)$  and  $E_T(t, k)$ .

One of such models suggested for the steady-state case is the Corrsin-Pao model [10, 11] which involves the assumption that the spectral element represented initially by wave number  $k$  and time  $\tau$  is shifted over time  $d\tau$  to the higher wave number  $k + dk$ . Here  $\tau(k)$  is the time scale obtained from dimensional analysis. Corrsin and Pao introduce the velocity  $B(k)$  at which the spectral element is transported through wave number  $k$ ,  $B(k) = dk/d\tau \approx k/\tau(k)$ . Then

$$F_T(k) = B(k)E_T(k) \quad (9)$$

This model is based on the assumption of local nature of energy transport over the spectrum. The model was validated by the results of modeling of energy transfer in homogeneous isotropic turbulence by numerically solving the Navier-Stokes equation, written in the wave-number space, on the basis of triadic interactions between spectral components. These interactions, which satisfy the condition  $k_1 = k_2 + k_3$ , where  $k_i$  ( $i = 1, 2, 3$ ) are the wave numbers of spectral components, are the principal process of energy transfer over the spectrum [14]. It was shown [12, 15] that the main contribution to energy transfer in the inertia range is made by local interactions, whereas in the dissipation interval it is nonlocal interactions that make a significant contribution. However, in all cases transport is predominantly local. The situation when nonlocal interactions induce local energy transfer comes about because energy transfer occurs primarily between two spectral components with high wave numbers (the pattern of such an interaction was described by Frisch [17]), whereas the contribution of the low-wave number component is small [14]. Similar conclusions were drawn also by others [13, 15]. We hence see that numerical modeling confirms the principal assumption underling the Corrsin-Pao model of local energy transfer over the spectrum.

In the inertia-convective spectral interval  $k_L \ll k \ll k_K$  (the Obukhov-Corrsin interval) we have from dimensional considerations  $\tau \approx \varepsilon^{-1/3} k^{-2/3}$ , then by solving Eq. (5) we obtain  $E_T \approx k^{-5/3}$  – the Obukhov-Corrsin law. In the viscous-convective interval  $k_K \ll k \ll k_B^T$  (Batchelor's interval)  $\tau \approx \gamma^{-1}$ , where  $\gamma$  is the deformation rate,  $\gamma = (\varepsilon/\nu)^{1/2}$ , which yields  $E_T \approx k^{-1}$ , i.e., the Batchelor spectrum. A generalized model was worked out by Hill [20], who wrote the above rate as

$$B(k) = \frac{C_0 k}{\tau_1 + \tau_2} \quad (10)$$

Here  $C_0$  is the Obukhov-Corrsin constant,  $\tau_1 \approx \varepsilon^{-1/3} k^{-2/3}$ ,  $\tau_2 = C_1 \gamma^{-1}$ ; the free parameter  $C_1$  is determined from comparison with experimental data. Upon substitution of Eqs. (8)–(10), Eq. (5) for the steady case can be written as

$$A_T^{-1} \frac{d}{dk} \left[ \frac{y^{5/3}}{1 + y^{2/3}} E_T(y) \right] = -2y^2 E_T y \quad (11)$$

where  $y = C_1^{3/2}$ ,  $A_T = C_0 C_1^{-2} \text{Pr}^{-1}$  and  $\text{Pr} = \nu/\chi_T$ . The solution must satisfy norming condition (7). As a result, we obtain the following expression for the spectrum [20]:

$$E_T(y) = \frac{C_0 C_1^{-5/2} \varepsilon_T}{k_\kappa^3 \nu} y^{-5/3} \left( 1 + y^{2/3} \right) \exp \left[ -A_T \left( \frac{3}{2} y^{4/3} + y^2 \right) \right] \quad (12)$$

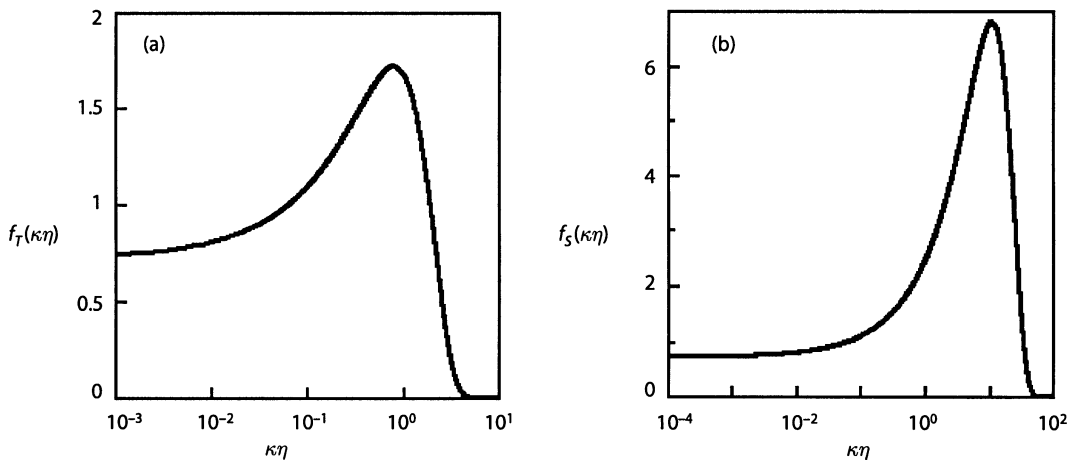
Returning to variable  $k$ , we write

$$E_T(k) = C_0 \varepsilon^{-1/3} \varepsilon_T k^{-5/3} \left[ 1 + C_1 (k\eta)^{2/3} \right] \exp \left\{ -A_T \left[ \frac{3}{2} C_1^2 (k\eta)^{4/3} + C_1^3 (k\eta)^2 \right] \right\} \quad (13)$$

It was shown by comparison with a large volume of experimental data [20] that this expression describes rather well the spectrum of fluctuations of the temperature field; here  $C_1 \approx 2.35$ . In addition, it is known [4, 7] that  $C_0 \approx 0.72$ . Note that the spectrum of salinity fluctuations is described by a similar formula. The only difference consists in replacing  $\varepsilon_T$  by  $\varepsilon_S$  and  $A_T$  by  $A_S = C_0 C_1^{-2} \chi_S/\nu$ . Since  $\chi_S \ll \chi_T$ , molecular effect affect turbulent temperature fluctuations at larger spatial scales than is the case for salinity fluctuations. This brings about perceptible differences between these spectra at high wave numbers.

As an illustration Fig. 1 shows plots of function  $f_T(k\eta) = \varepsilon_T^{-1} \varepsilon^{1/3} k^{5/3} E_T(k)$  (Fig. 1a) and  $f_S(k\eta) = \varepsilon_S^{-1} \varepsilon^{1/3} k^{5/3} E_S(k)$  (Fig. 1b), where  $E_T(k)$  and  $E_S(k)$  are the spectra of temperature and salinity fluctuations.

It is seen that in the inertia-convective interval where  $E_T(k) \approx k^{-5/3}$  and  $E_S(k) \approx k^{-5/3}$  functions  $f_T(k\eta)$  and  $f_S(k\eta)$  are close to horizontal lines and tend to the Obukhov-Corrsin constant  $C_0 \approx 0.72$ . At high values of  $k\eta$  the spectrum of temperature and salinity fluctuations has viscous-con-



**Fig. 1.** Spectra of temperature and salinity fluctuations.

vective (Batchelor) intervals where the spectra are proportional to  $k^{-1}$ , i.e.,  $f_T(k\eta) \approx (k\eta)^{2/3}$  and  $f_S(k\eta) \approx (k\eta)^{2/3}$  and the figures show a rise in the curves. When  $k\eta$  is increased further, the exponent in Eq. (13) starts exerting its effect and the curves tend rapidly to zero. These parts correspond to viscous-diffusion spectral intervals. Note that there is no viscous-convective interval in the atmosphere. As a result, the graphs of spectra do not show rising curves and the inertia-convective interval is followed by a rapid exponential drop of the curves.

### 3. Spectrum of Turbulent Fluctuations of the Refraction Index

Unlike the atmosphere, where changes in the refraction index are in many cases caused solely by temperature fluctuations, in sea water these changes are additionally caused by fluctuations of salinity. The familiar expressions for  $n$  as a function of  $T'$  and  $S'$  are described by high-power polynomials and are rather cumbersome [21]. Limiting ourselves to the linear approximation, we write for the fluctuating part of the refraction index

$$n = -\alpha T' + \beta S' \quad (14)$$

where  $\alpha = 2.6 \cdot 10^{-4}$  liter/deg and  $\beta = 1.75 \cdot 10^{-4}$  liter/gram. We shall now derive an expression that describes the spectrum of turbulent fluctuations of the refractive index for sea water. We shall analyze the spectrum of fluctuations of the field of some quantity  $J$ , which is a function of scalar quantities  $P$  and  $Q$  which have the respective molecular transfer coefficients  $\chi_P$  and  $\chi_Q$ .

Note that the statistical behavior of a multicomponent mixture in locally homogeneous and isotropic turbulence was investigated rather extensively in applications to combustion, operation of chemical reactors, etc. Thus Pao [18] obtained expressions for the spectra of fluctuations of the concentration of a mixture in the presence chemical reactions of the first kind. The behavior of the concentrations spectrum at high wave numbers for the case when the molecular coefficients of diffusion of the mixture's component were equal was investigated. Such a case was also analyzed by Patel [23] who investigated the spectrum of concentration fluctuations of a two-component mixture. Komori et al. [19] employed the Lagrange stochastic model for computing the correlation between two initially segregated components with identical molecular diffusion coefficients. A direct numerical simulation of the diffusion of passive scalar quantities with different molecular diffusion coefficients in steady isotropic turbulence was performed by Yeung and Pope [12]; this was followed by Yeung's study [16] who included the effect of the mean gradients of scalar quantities. These studies focused upon the role of different classes of triadic interactions in the cascade process of energy transfer. As a rule, the Schmidt numbers pertaining to individual mixture components did not exceed unity in these studies and differed from one another not more than fourfold. It was demonstrated that initially identical scalar fields gradually lose their correlation because of diffusion; the effect of difference in diffusion coefficients here first manifests itself at small scales and then extends to the domain of large scales. In other words, what is observed is a peculiar reverse cascade process. It was shown that a statistically steady mode gradually sets on in the liquid in the presence of a mean gradient of the scalar quantities.

Returning to derivation of an expression for the spectrum of  $E_n$ , we shall assume that  $J = P + Q$ . Then the correlation function of fluctuations of  $J$  can be written as

$$\langle J'_A J'_B \rangle = \langle P'_A P'_B \rangle + \langle Q'_A Q'_B \rangle + 2\langle P'_A Q'_B \rangle \quad (15)$$

since in locally homogeneous and isotropic turbulence  $\Psi_{PQ} = \langle P_A' Q_B' \rangle = \langle P_B' Q_A' \rangle$ . Repeating the procedure employed in the previous section, one can easily obtain expressions describing the behavior of correlations  $\langle P_A' P_B' \rangle$  and  $\langle Q_A' Q_B' \rangle$ . Applying then Fourier transformations, we can find expressions for  $E_P(k)$  and  $E_Q(k)$ .

Let us now determine the reciprocal spectrum  $E_{PQ}(k)$  following the procedure described in the previous section in finding  $E_T(k)$ . By analogy with Eq. (2) we write the equation of transport of fluctuations of scalar quantity  $P'$  in point  $A$  and multiply it by the value of  $Q'$  in point  $B$ . Then the equation for  $P'$  written in point  $B$  is multiplied by  $Q_A'$ . A similar procedure is performed for fluctuations of scalar quantity  $Q'$ . Then we add the four equations and average them; this yields

$$\begin{aligned} & \frac{\partial}{\partial \xi_i} \langle [(v'_i)_B - (v'_i)_A] P'_A Q'_B \rangle + \frac{\partial}{\partial \xi_i} \langle [(v'_i)_B - (v'_i)_A] P'_B Q'_A \rangle \\ &= 2\chi_P \frac{\partial^2}{\partial \xi_i^2} \langle P'_A Q'_B \rangle + 2\chi_Q \frac{\partial^2}{\partial \xi_i^2} \langle P'_A Q'_B \rangle \end{aligned} \quad (16)$$

where  $\xi_i = (x_i)_B - (x_i)_A$ . Transforming further with reference to the fact that

$$\langle (v'_i)_B P'_A Q'_B \rangle = -\langle (v'_i)_A P'_B Q'_A \rangle$$

we find

$$-\frac{\partial}{\partial \xi_i} (\Omega_{Q_i, P} \Omega_{P_i, Q}) = (\chi_P + \chi_Q) \frac{\partial^2}{\partial \xi_i^2} \Psi_{PQ} \quad (17)$$

Introducing the scalars

$$\Omega_{PQ}^* = \frac{\partial}{\partial \xi_i} \Omega_{P_i, Q} \quad \Omega_{QP}^* = \frac{\partial}{\partial \xi_i} \Omega_{Q_i, P} \quad (18)$$

and making use of the fact that functions  $\Psi_{PQ}$ ,  $\Omega_{PQ}^*$  and  $\Omega_{QP}^*$  depend solely on the distance between these two points, we convert in Eq. (17) to the variable  $r$ . As previously, we apply the three-dimensional Fourier transformation of the spatial variables to the resulting equation, then convert to spherical variables and evaluate the integrals with respect to angular variables; all this yields the spectral analog of Eq. (17)

$$\tilde{W}_{PQ}^* + \tilde{W}_{QP}^* = (\chi_P + \chi_Q) k^2 \tilde{E}_{PQ} \quad (19)$$

Converting to three-dimensional spectral density  $E_{PQ} = 4\pi k^2 \tilde{E}_{PQ}$ , we introduce the analog of function  $W_T(k)$ :  $W_{PQ} = 8\pi k^2 (\tilde{W}_{PQ}^* + \tilde{W}_{QP}^*)$  and then Eq. (19) becomes

$$W_{PQ} = (\chi_P + \chi_Q) k^2 E_{PQ} \quad (20)$$

Please note that when  $P=Q$  Eq. (20) becomes Eq. (5) (the steady-state case). By analogy with Eq. (8) we introduce now the scalar spectral stream function  $F_{PQ}$ :

$$W_{PQ} = -\frac{\partial F_{PQ}}{\partial k} \quad (21)$$

which, on the basis of the Hill model, is written as

$$F_{PQ}(k) = B(k)E_{PQ}(k) \quad (22)$$

where  $B(k)$  is the conditional rate of transport of perturbations through spectral point  $k$ , written as (10). Substituting Eqs. (21) and (22) into Eq. (20) and making use of Eq. (10), we obtain

$$A_{PQ}^{-1} \frac{d}{dy} \left[ \frac{y^{5/3}}{1 + y^{2/3}} E_{PQ}(y) \right] = -2y^2 E_{PQ}(k) \quad (23)$$

where  $y = C_1^{3/2} k / k_K$  and  $A_{PQ} = 0.5 C_0 C_1^{-2} (\chi_P + \chi_Q) \nu^{-1}$ . Solving this equation with reference to the norming condition

$$(\chi_P + \chi_Q) \int_0^\infty k^2 E_{PQ}(k) dk = \varepsilon_{PQ} \quad (24)$$

which is an analog of Eq. (7) (Eq. (24) becomes identical to Eq. (7) at  $P=Q$ ), we find

$$E_{PQ}(y) = R \varepsilon_{PQ} y^{-5/3} (1 + y^{2/3}) \exp(-A_{PQ} Y) \quad (25)$$

where  $R = C_0 C_1^{-5/2} k_K^{-3} \nu^{-1}$  and  $Y = 1.5 \cdot y^{4/3} + y^2$ . In the final analysis, making use of Eq. (15), we obtain an expression for the spectrum of temperature fluctuations of  $J$

$$E_J = R y^{-5/3} (1 + y^{2/3}) (G_P + G_Q + 2G_{PQ}) \quad (26)$$

where

$$\begin{aligned} G_P &= \varepsilon_P \exp(-A_P Y) & G_Q &= \varepsilon_Q \exp(-A_Q Y) \\ G_{PQ} &= \varepsilon_{PQ} \exp(-A_{PQ} Y) & A_{PQ} &= C_0 C_1^{-2} \\ A_P &= C_0 C_1^{-2} \chi_P \nu^{-1} & A_Q &= C_0 C_1^{-2} \chi_Q \nu^{-1} \end{aligned}$$

We shall now examine certain limiting cases.

I.  $y \ll 1$ ,  $A_P, A_Q, A_{PQ} \ll 1$ . This corresponds to the inertia-convective interval  $k_L \ll k \ll k_K$  (the Obukhov Corrsin interval). Expanding the exponents in Eq. (26) in a series and retaining the first term of each of the expansions, and utilizing the fact that  $y$  is a small quantity, we obtain

$$E_J = C_0 \varepsilon^{-1/3} (\varepsilon_P + \varepsilon_Q + 2\varepsilon_{PQ}) k^{-5/3} \quad (27)$$

which is the Obukhov-Corrsin law. Here  $\varepsilon_J = \varepsilon_P + \varepsilon_Q + 2\varepsilon_{PQ}$  is the rate of dissipation (equalization) of fluctuations of the field of a quantity [22].

II.  $y \gg 1$ ,  $A_P, A_Q, A_{PQ} \ll 1$ . Here  $k_K \ll k$ , i.e., we are in the viscous-convective interval (the Batchelor interval). After pertinent expansions we find

$$E_J = C_0 C_1 \tau_\eta (\varepsilon_P + \varepsilon_Q + 2\varepsilon_{PQ}) k^{-1} \quad (28)$$

which is the Batchelor spectrum).

III.  $A_P \gg 1$ ,  $A_Q \gg 1$ ,  $A_{PQ} \gg 1$ . As expected, the spectrum damps out exponentially. This is the viscous-diffusion interval.

Returning to the variable  $k$ , we write an expression for the spectrum of fluctuations of the refraction index. In the final analysis, making use of Eq. (14), we have

$$E_n(k) = C_0 \varepsilon^{-1/3} k^{-5/3} \left[ 1 + C_1 (k\eta)^{2/3} \right] [G_T(k) + G_S(k) - 2G_{TS}(k)] \quad (29)$$

where

$$\begin{aligned} G_T(k) &= \alpha^2 \varepsilon_T \exp(-A_T \delta) \\ G_S(k) &= \beta^2 \varepsilon_S \exp(-A_S \delta) \\ G_{TS}(k) &= \alpha \beta \varepsilon_{TS} \exp(-A_{TS} \delta) \\ A_T &= C_0 C_1^{-2} \chi_T \nu^{-1} \quad A_S = C_0 C_1^{-2} \chi_S \nu^{-1} \\ A_{TS} &= 0.5 C_0 C_1^{-2} (\chi_T + \chi_S) \nu^{-1} \\ \delta &= \frac{3}{2} C_1^2 (k\eta)^{4/3} + C_1^3 (k\eta)^2 \end{aligned}$$

When the molecular transport coefficients are equal, i.e.,  $\chi_T = \chi_S$ , Eq. (29) becomes the expression for the spectrum found by Patel [23].

The local structure of fields of scalar quantities (density, refraction index, etc.) determined by fluctuations of two other scalar fields with different molecular transport coefficients, in locally-homogeneous and isotropic turbulence in the inertia-convective and viscous-convective intervals was analyzed by the one of the present authors with his coworkers [31, 32]. They showed that the rate of dissipation (equalization) of the field of such a scale quantity can be written as

$$\varepsilon_n = \alpha^2 \varepsilon_T + \beta^2 \varepsilon_S - 2\alpha \beta \varepsilon_{TS} \quad (30)$$

#### 4. Relationship with the Parameters of Averaged Fields

The above expression that described the spectrum of locally-homogeneous and isotropic temperature fluctuations of the refraction index contains quantities representing dissipation of the temperature and salinity fluctuations, i.e.,  $\varepsilon_T$ ,  $\varepsilon_S$  and  $\varepsilon_{TS}$ . We shall now examine their relationship with the outer parameters of the medium, in particular with the averaged temperature and salinity distributions. First we consider the temperature distribution. Fluctuations of temperature arise in the turbulent water medium when the mean temperature is a function of position, most frequently of the

vertical coordinate. Turbulent mixing causes particles to move through some distance where the ambient liquid temperature differs from the intrinsic temperature of the particles, i.e., there arises a temperature fluctuation. When a mean-temperature gradient exists, a statistically steady condition can be maintained only in the presence of external heat sources  $U_T$  that would sustain this distribution. Then the transport equation for temperature, with correction for incompressibility can be written as (compare with Eq. (1))

$$\frac{\partial T}{\partial t} + v_i \frac{\partial T}{\partial x_i} = \chi_T \frac{\partial^2 T}{\partial x_i^2} + U_T(x_i) \quad (31)$$

We are assuming here that  $\langle v_i \rangle$  and  $T = \langle T \rangle + T'$ , however, now  $\langle T \rangle = T_0(x_i)$ . We average Eq. (31) and then subtract the resultant expression from Eq. (31):

$$\frac{\partial T'}{\partial t} + \frac{\partial}{\partial x_i} (v'_i T_0 + v'_i T' - \langle v'_i T' \rangle) - \chi_T \frac{\partial^2 T'}{\partial x_i^2} = 0 \quad (32)$$

We multiply this equation by  $T'$  and average. For steady conditions, which is under consideration here, we obtain

$$\left\langle T' \frac{\partial}{\partial x_i} v'_i T_0 \right\rangle + \left\langle T' \frac{\partial}{\partial x_i} v'_i T' \right\rangle - \left\langle T' \frac{\partial}{\partial x_i} \chi_T \frac{\partial}{\partial x_i} \right\rangle = 0 \quad (33)$$

After some transformations, we can write this equation as

$$\langle T' v'_i \rangle \frac{\partial T_0}{\partial x_i} + \chi_T \left\langle \left( \frac{\partial T'}{\partial x_i} \right)^2 \right\rangle - \frac{\partial}{\partial x_i} \left[ \left\langle \left( \chi_T T' \frac{\partial T'}{\partial x_i} \right) \right\rangle - \frac{1}{2} \langle v'_i \langle T'^2 \rangle \right] = 0 \quad (34)$$

In locally homogeneous and isotropic turbulence the correlation of  $v'_i$  of the solenoidal field and  $\langle T'^2 \rangle$  of the scalar field is equal to zero [3, 4] and, by virtue of homogeneity of temperature fluctuations,  $\langle \langle T'^2 \rangle \rangle = \text{const}$ . It follows from this that the last term (the divergence of the random field) is equal to zero. This, in spite of the fact that the temperature distribution in our case is not homogeneous, since  $T_0$  is a function of position. However, when  $\text{grad } T_0 = \text{const}$ , in the case of local isotropic turbulence everything remains valid [3]. Then Eq. (34) becomes

$$\varepsilon_T = \left\langle \chi_T \left( \frac{\partial T'}{\partial x_i} \right)^2 \right\rangle = - \langle T' v'_i \rangle \frac{\partial T_0}{\partial x_i} \quad (35)$$

where  $\varepsilon_T$  is the rate of dissipation of the fluctuations of the temperature field [4]. Equation (35) can also be obtained from physical considerations on the assumption that the production of temperature fluctuations by the turbulent flow is balanced by molecular dissipation of these fluctuations [8]. A similar equation can also be written for the rate of dissipation of salinity fluctuations. Let us now analyze the relationship between  $\varepsilon_{TS}$  and the gradients of average values of temperature and salinity. First we write for salinity an equation similar to Eq. (31):

$$\frac{\partial S}{\partial t} + v_i \frac{\partial S}{\partial x_i} = \chi_s \frac{\partial^2 S}{\partial x_i^2} + U_s(x_i) \quad (36)$$

We average it, subtract from the original equation and then multiply the result by  $T'$ . We proceed similarly with Eq. (31), i.e., we subtract from the latter the averaged equation, multiply the result by  $S'$  and then add the resulting equations. Regrouping, we obtain

$$\begin{aligned} & \frac{\partial}{\partial t} T' S' + S' \frac{\partial}{\partial x_i} v'_i T_0 + T' \frac{\partial}{\partial x_i} v'_i S_0 + S' \frac{\partial}{\partial x_i} v'_i T' + T' \frac{\partial}{\partial x_i} v'_i S' \\ & - S' \frac{\partial}{\partial x_i} \langle v'_i T' \rangle - T' \frac{\partial}{\partial x_i} \langle v'_i S' \rangle - \chi_T S' \frac{\partial^2 T'}{\partial x_i^2} - \chi_S T' \frac{\partial^2 S'}{\partial x_i^2} = 0 \end{aligned}$$

We average this equation with reference to the fact that we are dealing with the steady case. As a result we have

$$\begin{aligned} & \langle S' v'_i \rangle \frac{\partial T_0}{\partial x_i} + \langle S' v'_i \rangle \frac{\partial S_0}{\partial x_i} + \left\langle v'_i \frac{\partial T' S'}{\partial x_i} \right\rangle \\ & - \chi_T \left\langle \frac{\partial}{\partial x_i} S' \frac{\partial T'}{\partial x_i} \right\rangle - \chi_S \left\langle \frac{\partial}{\partial x_i} T' \frac{\partial S'}{\partial x_i} \right\rangle + (\chi_T + \chi_S) \left\langle \frac{\partial T'}{\partial x_i} \frac{\partial S'}{\partial x_i} \right\rangle = 0 \end{aligned} \quad (37)$$

We note that in the case of  $T = S$  Eq. (37) reduces to Eq. (34). Regrouping again, we obtain the following expression for constant gradients of the mean temperature and mean salinity

$$\varepsilon_{TS} = \frac{\chi_T + \chi_S}{2} \left\langle \frac{\partial T'}{\partial x_i} \frac{\partial S'}{\partial x_i} \right\rangle = -\langle S' v'_i \rangle \frac{\partial T_0}{\partial x_i} - \langle T' v'_i \rangle \frac{\partial S_0}{\partial x_i} \quad (38)$$

To parametrize the turbulent fluxes of heat  $\langle T' v'_i \rangle$  and salt  $\langle S' v'_i \rangle$ , one usually introduces into expressions for  $\varepsilon_T$ ,  $\varepsilon_S$  and  $\varepsilon_{TS}$  coefficients of eddy thermal diffusivity  $K_T$  and diffusion of the salt  $K_S$  [4]:

$$\langle T' v'_i \rangle = -K_T \frac{\partial T_0}{\partial x_i} \quad \langle S' v'_i \rangle = -K_S \frac{\partial S_0}{\partial x_i} \quad (39)$$

i.e., these fluxes are respectively proportional to the gradients of the mean values of temperature and salinity fields and are directed oppositely to these gradients. Note that the aforementioned diffusion coefficients highly exceed their molecular analogs, i.e.,  $K_T \gg \chi_T$  and  $K_S \gg \chi_S$ . Employing Eqs. (39) we can express the dissipation rates as

$$\begin{aligned} \varepsilon_T &= K_T \left( \frac{\partial T_0}{\partial x_i} \right)^2 & \varepsilon_S &= K_S \left( \frac{\partial S_0}{\partial x_i} \right)^2 \\ \varepsilon_{TS} &= \frac{K_T + K_S}{2} \left( \frac{\partial T_0}{\partial x_i} \right) \left( \frac{\partial S_0}{\partial x_i} \right) \end{aligned}$$

These expressions are convenient for estimating the above rates because the terms making them up can be found by analyzing hydrological data. In the ocean usually all the average parameters of temperature and salinity fields are a function of the vertical coordinate. Then

$$\begin{aligned}\varepsilon_T &= K_T \left( \frac{dT_0}{dz} \right)^2 & \varepsilon_S &= K_S \left( \frac{dS_0}{dz} \right)^2 \\ \varepsilon_{TS} &= \frac{K_T + K_S}{2} \left( \frac{dT_0}{dz} \right) \left( \frac{dS_0}{dz} \right)\end{aligned}\tag{40}$$

### 5. Analysis of the Spectra of Fluctuations of the Refraction Index

Using Eqs. (40) and (30), we shall write Eq. (29) for the spectrum of fluctuations of the refraction index distribution as

$$E_n(k) = C_0 \varepsilon_n \varepsilon^{-1/3} k^{-5/3} \left[ 1 + C_1 k \eta^{2/3} \right] \Phi(k, \omega)\tag{41}$$

where

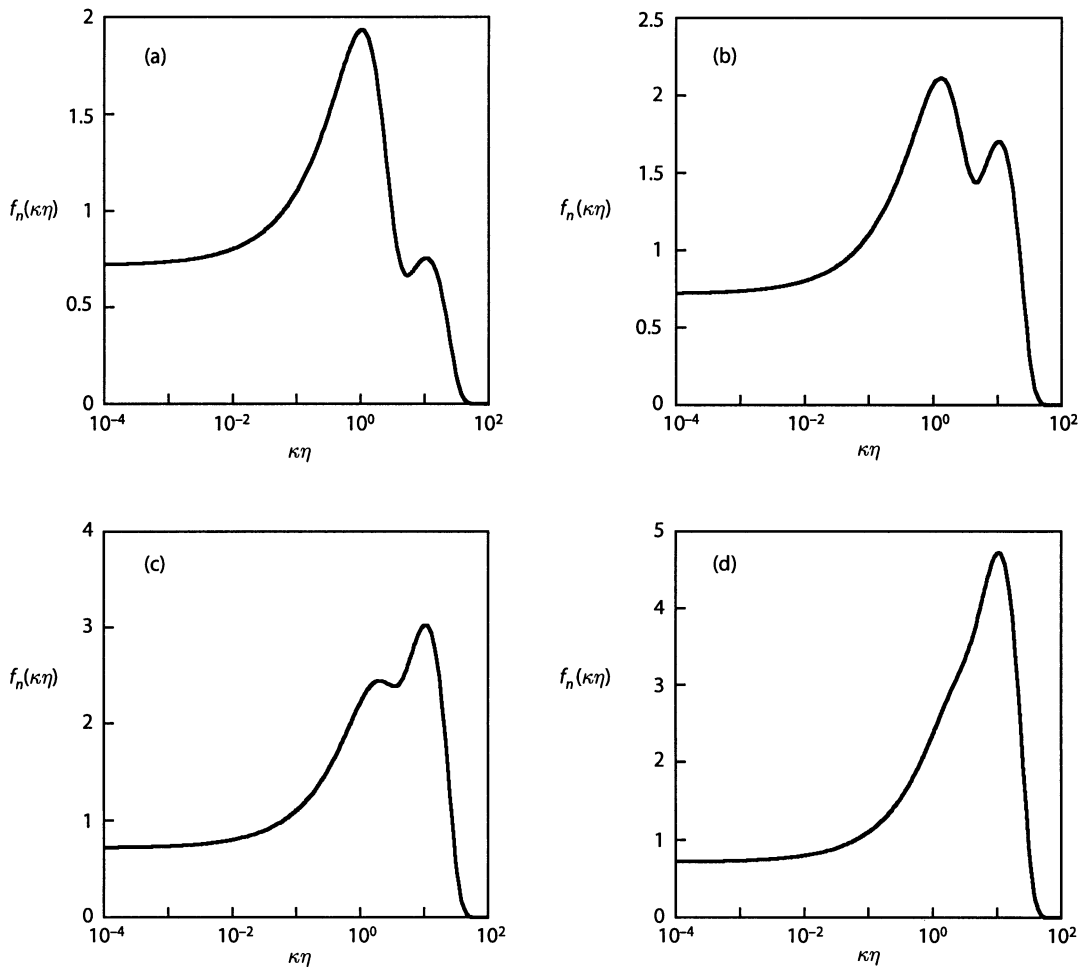
$$\begin{aligned}\Phi(k\omega) &= \frac{\omega^2 \theta \varkappa_T + \varkappa_S - \omega(1 + \theta) \varkappa_{TS}}{\omega^2 \theta + 1 - \omega(1 + \theta)} \\ \varkappa_T &= \exp(-A_T \delta) & \varkappa_S &= \exp(-A_S \delta) \\ \varkappa_{TS} &= \exp(-A_{TS} \delta) & \theta &= \frac{K_T}{K_S} \\ \omega &= \frac{\alpha(dT_0/dz)}{\beta(dS_0/dz)}\end{aligned}$$

For constant gradients of the mean temperature and salinity we can write  $\omega = (\alpha \Delta T_0) / (\beta \Delta S_0)$ , where  $\Delta T_0$  and  $\Delta S_0$  are respectively the differences in temperature and salinity between the top and bottom boundaries of the domain under study. Quantity  $\omega$  defines the contributions of the temperature and salinity distributions to the distribution of the refraction index. A similar quantity, known as the density rate, which represents the contributions of the temperature and salinity to the density field, is one of the principal parameters in investigating differential-diffusion convection [24]. Note that as  $\omega \rightarrow 0$  Eq. (41) takes the form of Eq. (13), i.e., it becomes identical to the expression describing the spectrum of fluctuations of the temperature field, whereas at  $\omega = 0$  it becomes the spectrum of fluctuations of the salinity field.

The spectra of fluctuations of the refractive index distribution were calculated from Eq. (41) by employing actual oceanographic data. The bulk of the calculations was performed for the case of stable vertical temperature and salinity distributions, i.e., reduction in temperature with depth and rise in salinity. This situation is entirely typical for the ocean. Thus, detailed measurements of gradients of the mean temperature and salinity performed in the North Pacific showed [25] that such an absolutely stable state prevailed in 35% of the cases. It follows from data on temperature and salin-

ity distributions presented by Fedorov [26] and Karlin et al. [27] that the value of  $\omega$  lies between 0 and  $-5$ . These values are in satisfactory agreement with data of Gargett [25].

The previously introduced eddy coefficients of thermal diffusivity and diffusion of salt depend in general on the rate of dissipation of the energy of turbulence and on the vertical density distribution. Monin showed [28] that, at stable stratification, the eddy momentum transfer coefficient (analog of the molecular kinematic viscosity) can be regarded as constant. By virtue of similarity of the physical mechanism of heat and salt transfer, which occurs only by means of mixture of water masses, the eddy coefficient of thermal diffusivity and diffusion of salt can also be treated as constant quantities and, moreover, should be equal to one another, i.e.,  $\theta = 1$ . However, analysis of data on the mixing of liquids that are inhomogeneous with respect to temperature and salinity shows that the rates of entrainment (meaning also the eddy heat and salt streams) of nonturbulent liquid into the domain with turbulent motions depends on the kind of stratification, i.e., entrainment is different for the temperature and salinity stratifications. In other words, turbulent fluxes are also a function of the molecular diffusion coefficient, the effect of which manifests itself primarily at small



**Fig. 2.** Spectra of  $E_n$  for different  $\omega$ : a)  $-2$ ; b)  $-1$ ; c)  $-0.5$ ; d)  $-0.2$ .

scales. Questions of parametrization of the effect of molecular processes on turbulent heat and salt fluxes requires further study [29, 30].

The calculated spectra of fluctuations of the refraction index distribution are plotted in Fig. 2. The vertical coordinate represents the values of the normed spectrum of  $f_n(k\eta) = \varepsilon_n^{-1} \varepsilon^{1/3} k^{5/2} E_n(k)$ , whereas the horizontal axis is the wave number  $k\eta$ , nondimensionalized according to the Kolmogorov scale. It is seen that the shape of the spectra differ significantly from the spectrum of fluctuations of the temperature distribution shown in Fig. 1a or the salinity spectrum shown in Fig. 1b. The spectrum of  $E_n$  acquires a double local peak, the location of which shifts depending on the contributions of temperature and salinity, i.e., on the value of  $\omega$ . The suggestion that there may exist an anomalous form of the spectrum of fluctuations of the refractive index was first put forward by Hill [7].

Note that in the case of  $\omega > 0$  the contributions of temperature and salinity mutually compensate. This causes an abrupt drop in the spectrum of  $E_n$  at high  $k$ , i.e., at small space scales. It is known [24] that molecular effects manifest themselves primarily in the fact that the rate of entrainment, meaning also the magnitude of the eddy thermal diffusivity, becomes higher than the eddy diffusivity of salt. Then the spectrum of  $E_n$  will not contain the above drops which are explained by the assumed approximation of turbulent fluxes. Moreover, at  $\omega > 0$  conditions arise for the inception of differential-diffusion convection and the question of parameterization of the fluxes becomes highly complicated; this case is not considered here.

It is clear that the appearance of these local extrema in the spectrum should have a corresponding effect on the statistical parameter of a light wave that travels through a turbulent water medium.

## 6. Conclusion

This paper is concerned with the statistical parameters of fluctuations of the refraction index. In the ocean medium, these fluctuations are controlled fluctuations of temperature and salinity. Unlike the atmosphere, the molecular thermal diffusivity in water is markedly lower than the kinematic viscosity. This causes the appearance of a new viscous-diffusion interval, in which the spectrum of temperature fluctuations is proportional to  $k^{-1}$ , i.e.,  $E_T \approx k^{-1}$ . The spectrum of salinity fluctuations has the same form. The mutual effect of temperature and salinity fluctuations may cause the spectrum of fluctuations of the refraction index to behave anomalously, with corresponding changes in the statistical characteristics of the traveling light waves. This assertion was first put forward by Hill [7].

In this paper we analyzed the behavior of the spectrum of fluctuations  $E_n$  of the distribution of the index of refraction  $n$  in turbulent water medium in the case when the fluctuations of this index are controlled by those of temperature and salinity. For local homogeneous and isotropic turbulence a new expression was found for the spectrum which is a function of the rates of dissipation (equalization) of temperature and salinity fluctuations which, together with the rate of dissipation of the energy of turbulence, control the statistical mode of fluctuations of  $n$ . The above rates of dissipation were parametrized which made it possible to express these rates in terms of the gradients of averaged temperature and salinity distributions and the corresponding eddy diffusivities. Specific calculations were performed using oceanographic measurements of temperature and salinity distributions in different parts of the ocean; all these distributions were stable. It was shown that depending on parameter  $\omega$ , which represents the contributions of fluctuations  $T'$  of the temperature and  $S'$  of the salinity, a double local extremum arises in the spectrum of  $E_n$ . Its appearance is caused by differences in the contributions of  $T'$  and  $S'$  to  $n$  and differences in the molecular thermal and salinity diffusivities. Note that the expression for the spectrum of  $E_n$  given by Hill [20] for

the case when the fluctuations are controlled either by fluctuations in  $T'$  or in  $S'$  follows from that obtained by us as a particular case.

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