Field Guide to

Optical Thin Films

Ronald R. Willey

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Field Guide to Optical Thin Films

The principles of optical thin films are reviewed and applications shown of various useful graphical tools (or methods) for optical coating design: the reflectance diagram, admittance diagram, and triangle Diagram. It is shown graphically how unavailable indices can be approximated by two available indices of higher and lower values than the one to be approximated. The basis of ideal antireflection coating design is shown empirically. The practical approximation of these inhomogeneous index profiles is demonstrated. Much of the discussions center on AR coatings, but most other coating types are seen in the perspective of the same graphics and underlying principles. **Reflection control** is the basis of essentially all dielectric optical coatings; and transmittance, optical density, etc., are byproducts of reflection (and absorption). The best insight is gained by the study of reflectance. It is also shown that AR coatings, high reflectors, and edge filters are all in the same family of designs. The graphical tools described are found to be useful as an aid to understanding and insight with respect to how optical coatings function and how they might be designed to meet given requirements.

Ronald R. Willey

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Frequently used variables, symbols, and terms:

A	Absorptance intensity
absentee layer	Layer of an even number of QWOTs thickness at the design wavelength
angle matched	PT of layer adjusted to be QWOT at an angle
AOI	Angle of incidence to surface normal
AR	Antireflection (coating)
BBAR	Broad band AR coating
BS	Beamsplitter
BW	Bandwidth
c	Speed of light in vacuum
cavity	A spacer of HWOT (or multiples) with high reflectors on either side
cm^{-1}	Wavenumbers, number of waves per cm.
dB	Decibel (= −10 OD)
DOE	Design of experiments methodology
DWDM	Dense wavelength division multiplexing
GHz	Gigahertz, 10^9 cycles per second
HEAR	High-efficiency AR coating
HWOT	Half-wave optical thickness
i	Imaginary, in complex numbers
IR	Infrared
k	Extinction coefficient
LP	Layer pair (H and L)
LWP	Long wavelength pass (filter)
MDM	Metal-dielectric-metal coating
MIR	Multiple internal reflection
MLAR	Multilayer antireflection (coating)
ML	Matching layer(s)
n	Real part of the index of refraction
N	Complex refractive index. $N = n - ik$
n_e	Effective index of refraction
NBP	Narrow band pass (filter)
OD	Optical density (= $\log_{10}(1/T)$)
ODBWP	Optical density bandwidth product
OT	Optical thickness
PD	Prism diagram

PT	Physical thickness
QHQ	Quarter-half-quarter wave OT design of AR coating
QWOT	Quarter wave optical thickness
r	Reflectance amplitude coefficient
R	Reflectance intensity ($R = rr^*$)
Rabbit ears	High reflectance in the pass band of a NBP the spectral appearance is like rabbit ears
R_{ave}	Average reflectance over a spectral region
rugate	Index versus thickness profile of rippled or corrugated form rather than square wave
SLAR	Single-layer antireflection (coating)
SWP	Short wavelength pass (filter)
t	Thickness
T	Transmittance, intensity
TIR	Total internal reflection
UV	Ultraviolet
v	Speed of light in medium
wavenumber	Number of wavelengths per centimeter in vacuum
Y	Admittance
z	Optical axis
α	Absorption coefficient
θ	Angle of incidence, refraction, or reflection
λ	Wavelength
ν	Frequency of the light
σ	Number of wavelengths per centimeter
φ	Phase

Optical Basic Concepts

Optical thin films are primarily the study of interference between the multiple reflections of light from optical interfaces between two different media. Some principles and terminology are drawn from the basics of geometrical optics.

Index of refraction n:

$$n \equiv c/v, \quad v = c/n \quad c = 2.99792458\times10^{8}\,\text{m/s},$$

where c is the speed of light in vacuum and v is the speed of light in a medium.

Wavelength λ and **frequency** ν:

$$\lambda = v/\nu, \quad \text{in vacuum: } \lambda = c/\nu$$

The **wave number** σ is the number of wavelengths per centimeter:

$$\sigma = 1/\lambda \text{ units of cm}^{-1}.$$

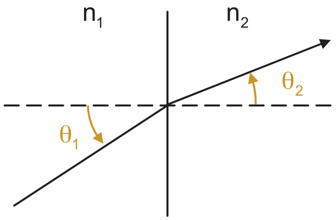
Optical path difference (OPD) is proportional to the time required for light to travel between two points. In homogeneous media:

$$\text{OPD} = nd,$$

where d is the physical distance between the two points.

Snell's law of refraction:

$n_1 \sin\theta_1 = n_2 \sin\theta_2.$

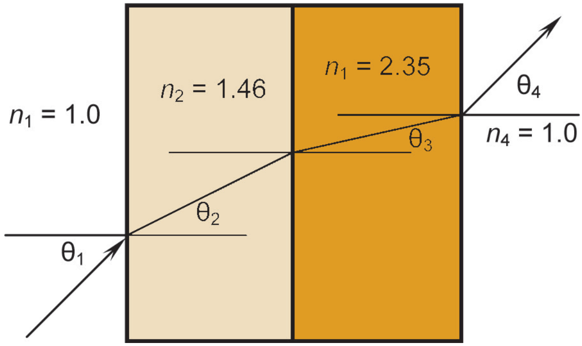


The incident ray, the refracted ray, and the surface normal are coplanar. When propagating through a series of parallel interfaces, the quantity $n \sin\theta$ is conserved.

Internal Angles in Thin Films

In a stack of plane parallel thin-film surfaces of various indices, the angle of a ray within a given film can be calculated using Snell's law. Because the law applies at each interface and follows from the previous interface, the internal angle in the film is independent of the order of the films or where that film is within the stack:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 = n_3 \sin \theta_3 = n_4 \sin \theta_4.$$



The immerging ray from the stack refracts back to an index of 1.0 and is parallel to the entering ray because the index is the same.

Optical coatings on lenses are not strictly plane parallel thin-film surfaces, but on the scale of the thickness of these films (~1μm), the approximation is valid.

On a more microscopic scale (~1 nm), the surfaces of real coatings are not usually very smooth or flat. However, at wavelengths of one or two orders of magnitude greater than that, the approximation of "flat" is still valid.

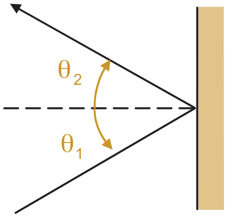
In the deep ultraviolet (UV) or x-ray region, these things can be of concern because of the shorter wavelengths.

Reflection

Law of reflection:

$$\theta_1 = -\theta_2$$

The incident ray, the reflected ray, and the surface normal are coplanar.



Total internal reflection (TIR) occurs when the angle of incidence of a ray propagating from a higher-index medium to a lower-index medium exceeds the **critical angle:**

$$\sin\theta_C = n_2/n_1 .$$

At the critical angle, the angle of refraction θ_2 equals 90°.

The **reflectance amplitude** r of an interface between n_1 and n_2 at normal incidence ($\theta_1 = 0$), with no absorption, is given by the **Fresnel reflection equation:**

$$r = (n_1 - n_2)/(n_1 + n_2).$$

When **absorption** is included:

$$N_1 = n_1 - ik_1 ,$$

where

$$r \text{ (complex)} = (N_1 - N_2)/(N_1 + N_2).$$

When the **angle of incidence** (AOI) θ_1 is not 0°, the *s*- and *p*-polarizations have different **effective indices:**

$$N_S = n \times \cos\theta \text{ and } N_P = n / \cos\theta ;$$

In these cases $r_S = (N_{1S} - N_{2S})/(N_{1S} + N_{2S})$

and $r_P = (N_{1P} - N_{2P})/(N_{1P} + N_{2P})$.

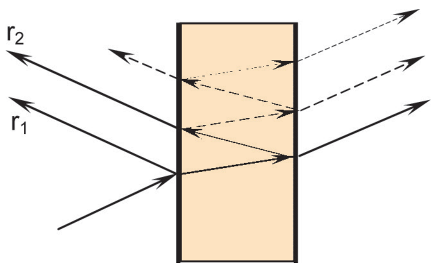
Normally, thin-film calculation/design software takes care of all of these details.

Reflections

Reflectance intensity R is what is measured with a photometer. It is the product of the **reflectance amplitude** r and its complex conjugate r^*:

$$R = rr^*.$$

Each interface between two media of differing index has **Fresnel reflection**. As seen in the figure, a ray falling on the first surface has r_1 reflected from it. The transmitted part has r_2 reflected when it falls on the second surface of the shaded medium. Part of this second reflection is reflected when it falls back on the first surface. Part of that reflection is reflected when it falls again on the second surface, etc.



When all of the reflections from the first and second surfaces of a thin film are considered, the resulting reflectance r is rigorously given by

$$r = (r_1 + r_2 e^{-i\varphi})/(1 + r_1 r_2 e^{-i\varphi}).$$

Here, $e^{-i\varphi}$ is the complex phase factor that accounts for the phase difference between r_1 and r_2 caused by the optical thickness of the film:

$$e^{-i\varphi} = \cos\varphi - i\sin\varphi.$$

These equations properly account for the multiple internal reflections in the medium.

Example Reflection Calculations

A soap bubble might have an index of refraction of 1.4. The reflections in air (n = 1.0) from the first and second surfaces would be

$$r_1 = (1 - 1.4)/(1 + 1.4) = -1/6.$$

and

$$r_2 = (1.4 - 1)/(1.4 + 1) = +1/6.$$

If the bubble is infinitesimally thin, the two reflections cancel each other:

$$r_1 + r_2 = -1/6 + 1/6 = 0.0\,,$$

where φ = 0, so that

$$r = (r_1 + r_2 {*}1)/(1 + r_1 r_2 {*}1) = 0.0\,.$$

If the thin film of the bubble were one **quarter-wave optical thickness** ($\text{QWOT} = \lambda/4 = nd$) at the wavelength under consideration, the path of the ray from the first surface to the second and back to the first would be one half wavelength. In this case φ = 180 deg. Here, the two reflections would add to each other for maximum effect:

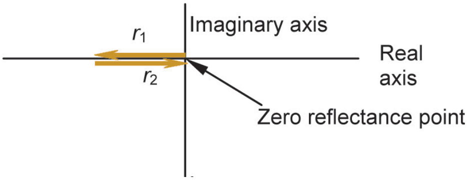
$$r = \left[r_1 + r_2 \ast (-1)\right] / \left[1 + r_1 r_2 \boxtimes (-1)\right] = -0.3243\,,$$
$$r \approx r_1 + r_2 \approx -1/6 + (-)1/6 \approx -1/3\,.$$

The reflectance R would be rr^* or ~1/9, which is ~11%. The appearance of the reflection when the bubble has a thickness of one QWOT would be white. As the bubble became thinner, its appearance would change toward no reflectance or "black." In the normal case, a bubble has a thickness of many QWOTs. For a given physical thickness, there are more QWOTs in blue light (short wavelength) than in the red (long wavelength). Therefore, the blue and red "rays" are at different phases and therefore have different reflectance. This causes the rainbow that we see in soap bubbles.

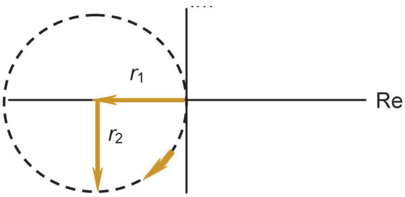
Reflectance as Vector Addition

The case of the soap bubble is illustrated from the viewpoint of vectors.

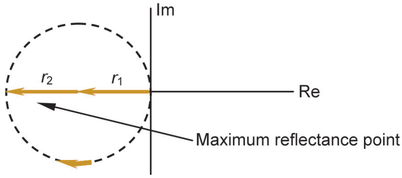
When φ = 0, r_1 = –1/6, and r_2 = +1/6.



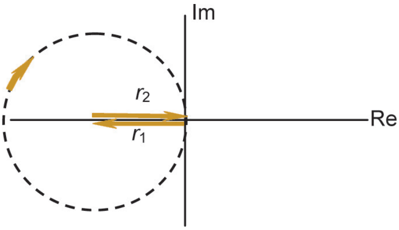
When φ = 90°:



When φ = 180° (one QWOT):

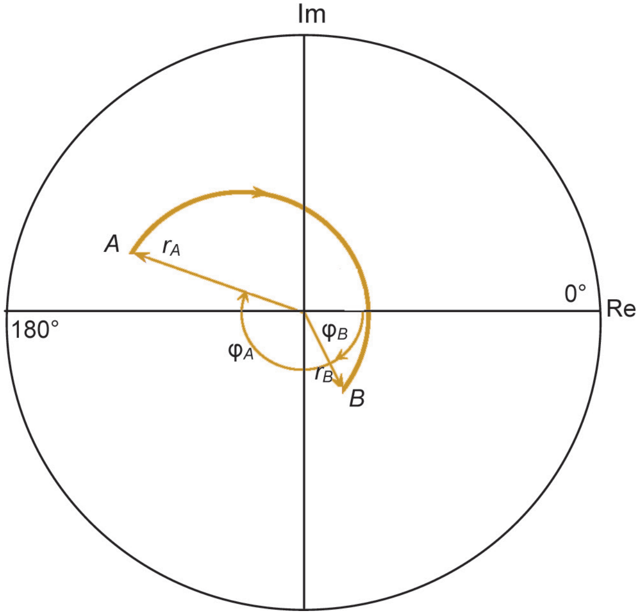


When φ = 360° (two QWOTs or one half-wave OT):



Reflectance Amplitude Diagram

The **reflectance amplitude diagram**, often referred to as a **circle diagram**, follows from the foregoing vector diagrams (p.6).

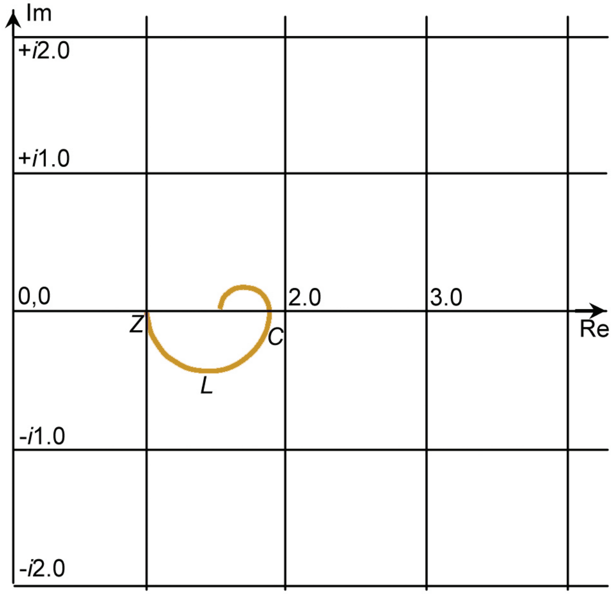


The outermost circle represents $r = 1.0$ and also $R = rr^* = 1.0$ or $R = 100\%$ reflectance. The origin of the real and imaginary axes is where $r = 0.0$ or zero reflectance.

Any new layer starts from the point of the reflectance amplitude of whatever lies beneath it, whether that is a substrate or a stack of coating layers on a substrate. This point is represented by point *A* in the figure, where the starting reflectance amplitude is r_A and its phase is φ_A. After the addition of a given physical thickness (PT = d) and optical thickness (OT) of a layer of given index, the resulting reflectance reaches point *B*. The new reflectance amplitude is r_B and its phase is φ_B. Point *B* would then be the starting reflectance for the next layer.

Admittance Diagram

The **admittance** (Y) of a medium, as applied to optical thin films, is an electrical quantity, which is normalized to be equivalent to the index of refraction. When the effective index of a thin-film stack is plotted as a function of increasing thickness from the substrate to the end of the last layer, an **admittance diagram** is produced. This is a conformal mapping of the reflectance amplitude diagram. For many cases the diagrams look quite similar, except that they have been rotated 180° about a point. In the locus of homogeneous layers, nonabsorbing layers are circles.



The range of an admittance diagram is the semi-infinite plane from 0 to ∞ to the right on the real axis and $\pm i\infty$ on the imaginary axis. The two-layer case of a QWOT of M and L is plotted here for comparison. Note that point S is at 1.52 $-i$0.0, the index of the substrate; C is at 1.90132 $-i$0.0; and Z is at $Y = N_e = 1.0 - i0.0$, where $r = (1.0 - 1.0) / (1.0 + 1.0) = 0$.

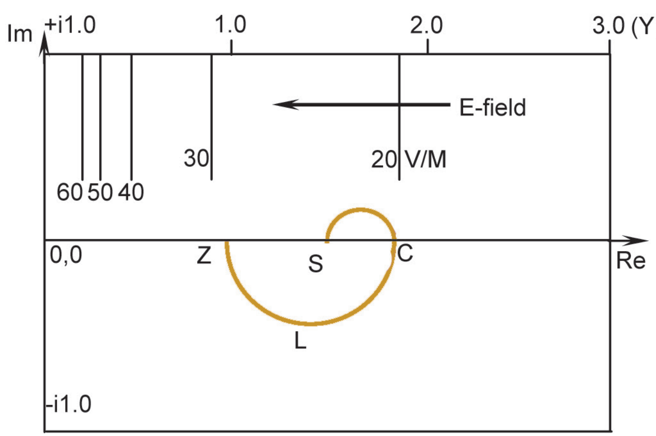
Electric Field in a Coating

The electric field within a coating layer is of importance when laser damage thresholds are considered and also when working with absorbing layers. In the latter case, the amount of energy absorbed in the layer depends upon the relative value of the electric field within that layer. One aspect of the laser damage issue seems to be that the interface between real deposited layers has some absorption and defects that are more vulnerable if the electric field is high at that interface.

The **relative volts/meter** is a function of the real value of the admittance and can be calculated as follows:

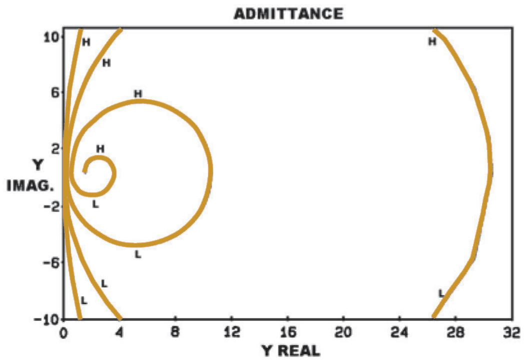
$$E = 27.46 / [\mathrm{Re}(Y)]^{0.5}\,.$$

This is very convenient to visualize on an **admittance diagram**. The closer the locus of the coating gets to $Y = 0$, the higher the electric field, and therefore the more vulnerable the coating is to high energy flux. A laser stack is sometimes designed with non-QWOT layers that do not terminate toward the left on the real axis. Instead, the locus stops short of, or continues past, the crossing of the real axis until it is further away from the risks of high electric field at $Y = 0$.

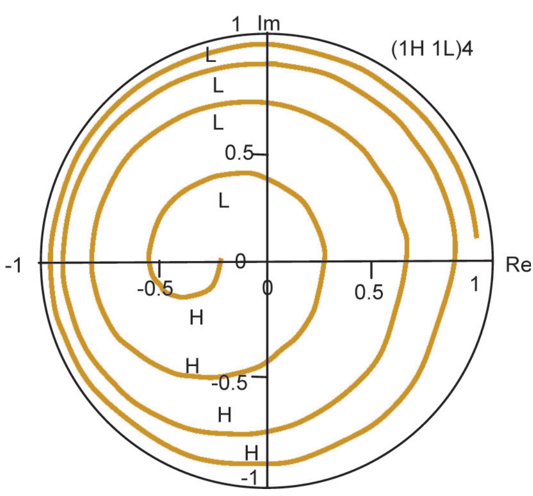


Admittance versus Reflectance Amplitude Diagrams

The admittance diagram is generally as useful as the **reflectance amplitude** or **circle diagram**, particularly in the realm of low reflectance (and thereby low admittance). However, the admittance diagram is not as useful for high reflectors such as a stack like (1H 1L)4, as can be seen in the figure below. For high reflectors of many layers, the admittance tends to go off of whatever scale is chosen for the plot, whereas the reflectance amplitude diagram is constrained to the unit circle.



Reflectance Amplitude Diagram



Triangle Diagram

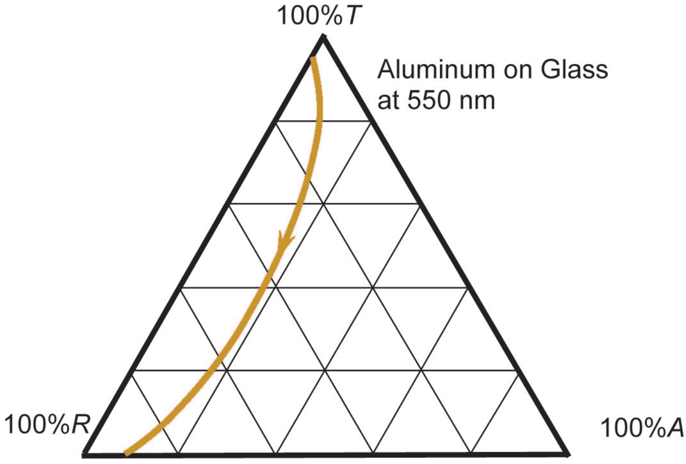
The energy falling on an optical thin film will be either reflected (R), transmitted (T), or absorbed (A). Scattering is ignored for the purposes of this section. It can then be stated that

$$R + T + A = 1 \,.$$

A convenient way to visualize the properties of materials with absorption, such as metals, is using a **triangle diagram**. When there is no absorption (A), the transmittance (T) is simply $1 - R$ and the diagram is not particularly useful.

This figure shows the path in R, T, and A for a coating of aluminum on glass. It starts at 96% T, 4% R, and 0% A on the bare glass surface. As the aluminum thickness increases, the locus moves downward. This happens to pass through a point of about 40% T, 43% R, and 17% A. When the film becomes opaque, it is at a point of 0% T, 90% R, and 10% A.

These triangle diagrams are useful when a material has a significant value of k, the imaginary index, as in metals and semiconductors.



Single-Layer Antireflection Coating

The most common **single-layer antireflection coating** (SLAR) on glass is a QWOT of MgF_2. If the index of the glass is 1.52 and that of the MgF_2 is 1.38:

$$r_1 = (1.0 - 1.38)/(1.0 + 1.38) = -0.1597\,,$$

$$r_2 = (1.38 - 1.52)/(1.38 + 1.52) = -0.0483\,.$$

When the film is infinitesimally thin, $\varphi = 0$, so that

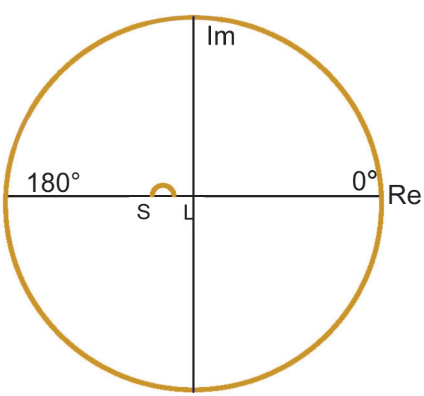
$$r = \left[(r_1 + r_2 * 1)/(1 + r_1 r_2 * 1)\right] = -0.2064\,;$$
$$R = 4.260\% \text{ (same as bare substrate)}.$$

When the thin film is one QWOT, at the wavelength under consideration, then $\varphi = 180°$:

$$r = \left[r_1 + r_2 * (-1)\right]/\left[1 + r_1 r_2 * (-1)\right] = -0.1123\,;$$
$$R = 1.260\%\,.$$

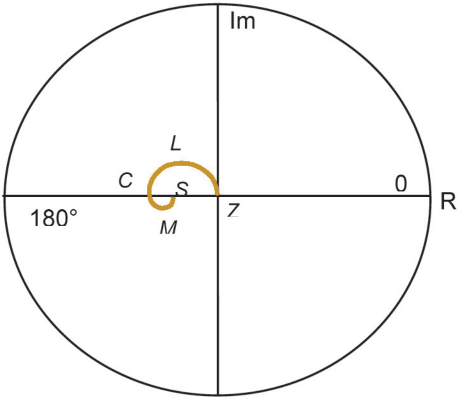
Thus, this SLAR reduces the reflection of the glass surface from 4.26% to 1.26% (only at the QWOT wavelength, and 0° AOI; at other wavelengths and angles, R is modified).

The locus of this coating as seen on a reflectance amplitude or circle diagram as it grows from zero to one QWOT is a semicircle moving clockwise from point S to L.



Two-Layer AR Amplitude Diagram Example

In this example (in air), the substrate is of index 1.52 and its **reflectance amplitude** point is at S. The first QWOT layer is of index $M = 1.70$. The second QWOT layer is of index $L = 1.38$. This two-layer coating results in zero reflectance at the origin point, Z, for this design wavelength (only).



The supporting calculations are

$$r_{\text{substrate}} = (1.0 - 1.52)/(1.0 + 1.52) = -0.20635; R_S = 4.528\% .$$

After the deposition of the first layer:

$$r_1 = (1.0 - 1.70)/(1.0 + 1.70) = -0.25926$$

$$r_2 = (1.70 - 1.52)/(1.70 + 1.52) = +0.05590; \quad \varphi = 180°$$

$$r_C = \{[-0.25926(-)(+0.05590)]/[1.0 + (-) - 0.25926 * (+0.05590)]\}$$

$$= -0.310658; n_{\text{effective}} = 1.90132; \quad R_C = 9.651\%$$

After the deposition of the second layer:

$$r_1 = (1.0 - 1.38)/(1.0 + 1.38) = -0.15966$$

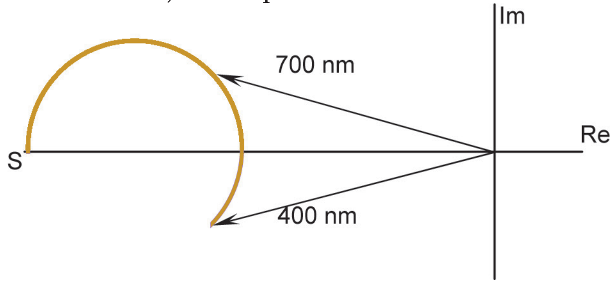
$$r_2 = (1.38 - 1.90132)/(1.38 + 1.90132) = -0.15888; \varphi = 180°$$

$$r_Z = \{[-0.15966(-)(-0.15888)]/[(1.0 + (-) - 0.15966) * (-0.15888)]\}$$

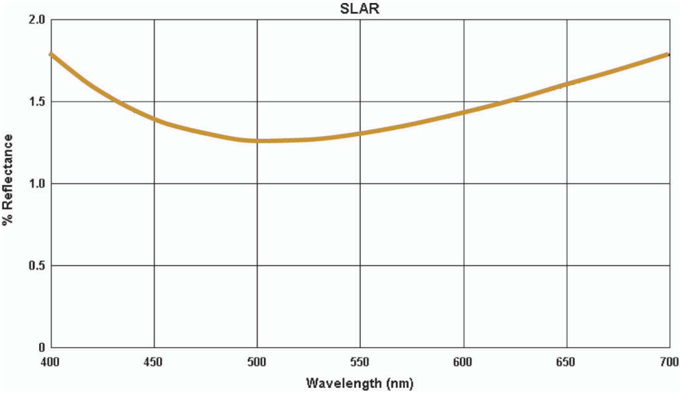
$$= -0.00080; \quad R_C = 0.000064\%!$$

Wavelength Effects

If the SLAR were designed to be just one QWOT at a 510-nm wavelength, that same physical thickness (PT) would be 0.729 QWOT at a wavelength of 700 nm, and 1.275 QWOT at 400 nm. This means that the semicircle shown previously (p. 12) would be less than a semicircle for 700 nm and more than one for 400 nm. The 400 and 700 nm terminations of the layer would be further from the origin than QWOT at 510 nm, and therefore would have a higher reflectance. Specifically, the reflectance would be 1.799% at 400 nm and 1.786% at 700 nm, as compared to 1.260% for 510 nm.

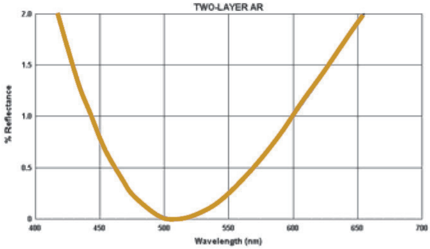


The resulting reflectance versus wavelength would appear as the figure below.



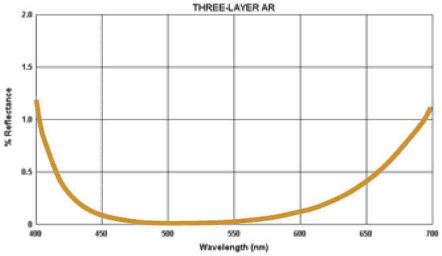
Wavelength Effects (cont.)

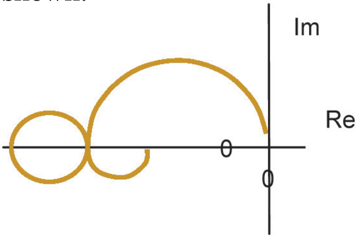
The two-layer AR design shown previously had one QWOT of index 1.70 and one QWOT of index 1.38 to bring the



reflectance to essentially zero at the design wavelength. If this was designed for 510 nm, both layers would be 1.275× thicker at 400 nm and 0.729× thinner at 700 nm than one QWOT. The **wavelength effects** would be even more exaggerated than the SLAR case as shown.

Coatings such as these are referred to as "V-coatings" because of the V shape of the reflectance spectrum. However, if a layer of index 2.2 and thickness of two QWOTs were added between the two layers, the result is shown.

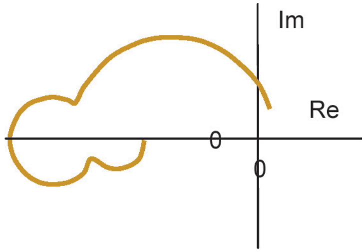




This additional half- wave OT layer is referred to as an **achromatizing layer** because it minimizes the changes in reflectance with color or wavelength. This might best be understood in the frame-work of the reflectance amplitude or circle diagrams at different colors/ wavelengths. At the design wavelength of **510 nm**, the first QWOT of index 1.7 moves the locus from the substrate to the left and on the negative real axis. The first QWOT of index 2.2 moves the locus from this point again to the left and on the negative real axis.

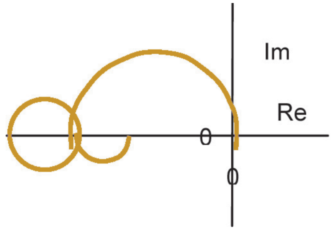
Wavelength Effects (cont.)

The second QWOT of index 2.2 moves clockwise back to the beginning of the 2.2 layer. The half-wave layer is called an **absentee layer** because it does not change the reflectance, and therefore acts as though it were absent. The last layer of index 1.38 moves the locus from this point to the origin of the coordinates, $r = 0$.



At a wavelength of **650 nm**, the layers are not as thick as a QWOT, and therefore are not full semicircles. All of the layers are too short, but the two QWOTs of the 2.2 index layer compensate for this by opening in such a way as to push the end point of the last layer toward the origin and closer to $r = 0$.

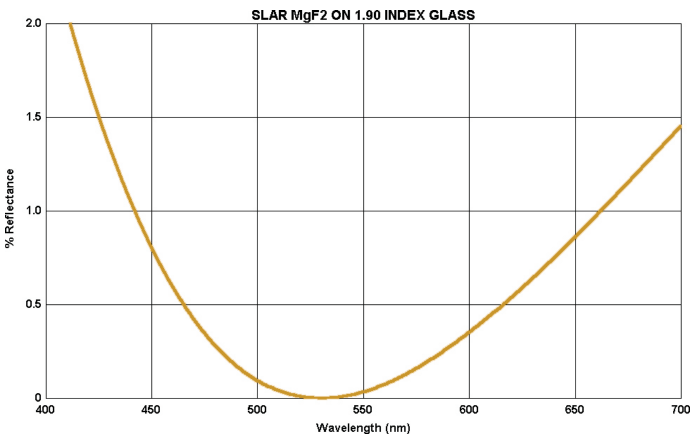
At a wavelength of **450 nm** the layers are thicker than a QWOT, and therefore are more than full semicircles on such a diagram. All of the layers are too long, but the two QWOTs of the 2.2 index layer compensate for this by closing in such a way as to pull the end point of the last layer toward the origin.



Broad-Band AR Coating

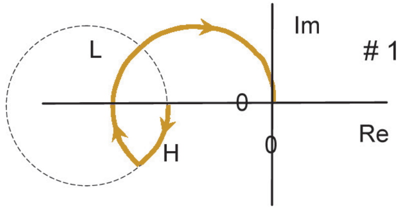
The three-layer AR coating described on p. 15 is the basis of the **broad-band AR** (BBAR) coating, sometimes referred to as a **high-efficiency AR** (HEAR) coating. It has low reflection over a broad spectral range, particularly as compared to the V-coat. It has significantly lower reflection than the SLAR of MgF_2 on 1.52 index glass. It should be noted, however, that MgF_2 on a glass of index 1.90 (or near that) would have zero reflectance at the design wavelength and a moderately broad spectral AR band, as seen in the figure below. When high-index glasses receive an AR coating, a SLAR of MgF_2 may be a viable alternative to the BBAR.

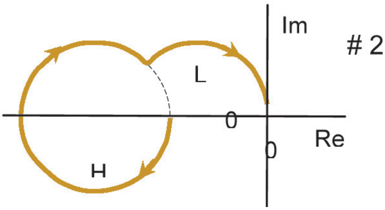
The three materials for the BBAR coating might be Al_2O_3 for the medium index layer, TiO_2 or Ta_2O_5 for the high-index layer, and MgF_2 for the low-index layer. The indices do not have to be exactly as illustrated here, and small adjustments from exact QWOTs can often balance the results in production for best spectral performance. This type of coating is probably the next most common production coating in the world, after the SLAR with MgF_2.



Two V-Coat Possibilities

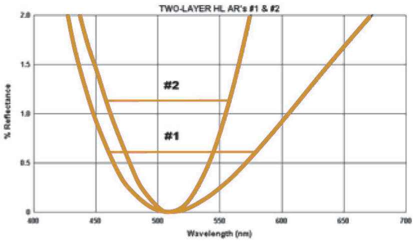
There are two possibilities for a V-coat with indices of 2.2 and 1.38. One (#1) is an H-layer with much less than a QWOT and an L-layer that is more than a QWOT.

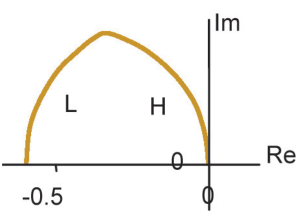




The second (#2) has a first H-layer that starts out the same, but is much thicker before it terminates. A smaller L-layer completes the AR coating.

However, #1 is less sensitive to changes of wavelength because it is a "flatter V." The first has a smaller locus due to fewer QWOTs of total thickness. As a result, it changes less as the wavelength changes. The broader design implies that production should be easier since it would be more tolerant to small changes of wavelength and AOI.





V-coat ARs can be designed for all wavelength regions as long as transparent materials are available for those regions. They can be designed from almost any materials if they have enough difference in index of refraction. The example is in the infrared (IR) at 10.6 μm, where the substrate and the H-coating material is germanium of index 4.0, and *L* is a fluoride of index 1.35.

Effective Index of Refraction

The last surface of any coating stack behaves as though it were a slab of homogeneous material of a specific **effective index of refraction** $n_{\text{effective}}$, or n_{e}, the value of which depends on all the layers of the stack and the substrate. It can be found from the last reflectance amplitude and phase computed for the coating stack. The final r and φ could be of any value from 0.0 to 1.0 and 0° to 360°, respectively.

The effective index can be found using the Fresnel reflection equation of a surface in air (or vacuum):

$$r = (1 - n)/(1 + n)\,.$$

Using simple algebra, this can be rearranged to give

$$n = (1 - r)/(1 + r) = n_e\,.$$

When the phase of the final reflectance amplitude is on the real axis, so that φ is either 0° or 180°, then this effective index is real and has no imaginary component. However, in the general case, r is a complex number and has an imaginary component. Therefore, n_{e} is also a complex number and has an imaginary component. This means that it behaves like a general semiconductor or metallic material at that wavelength with absorption in terms of the resulting reflectance amplitude, which generates

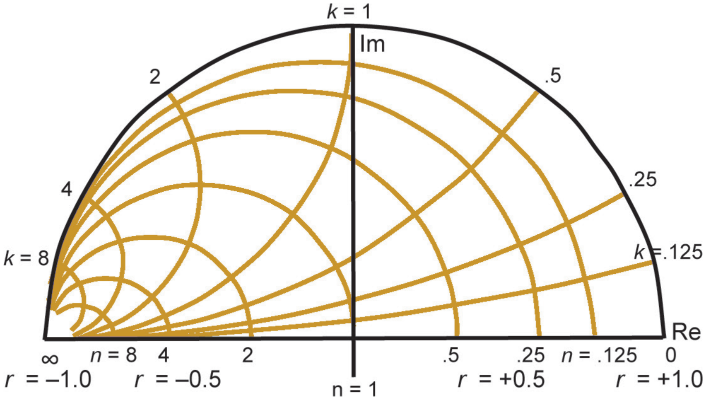
$$N_e = n_e - ik_e\,.$$

However, if there have been no absorption losses in the coating stack, such losses are not implied. The reflection just looks this way, but the rest of the flux has been transmitted without absorption.

Recall that $R = rr^*$, where r^* is the complex conjugate, so that the reflected intensity does not give evidence of the phase.

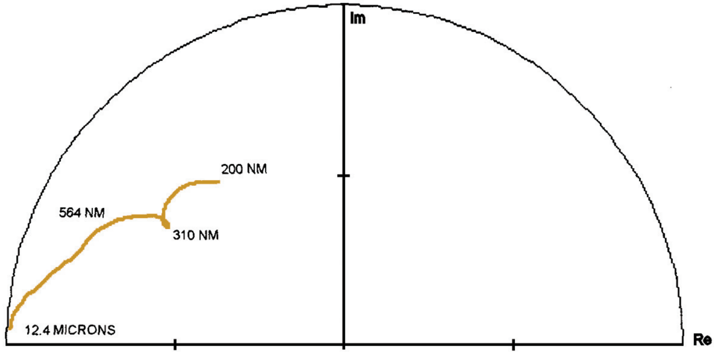
Complex Effective Index Plot

When the **complex effective indices** are calculated for general reflectance amplitudes and phases, we find n_e and k_e values as shown in the figure. This figure is symmetric about the horizontal (real) axis.



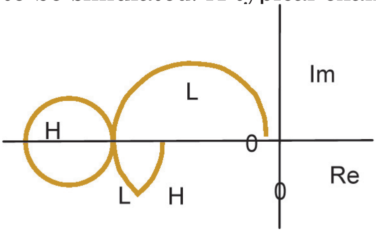
Note that r = 1.0 corresponds to an effective index of N_e = 0.0 $-i$0.0, and r = –1.0 corresponds to an index of N_e = ∞ $-i$0.0. Also, r = i1.0 corresponds to an index of N_e = 1.0 $-i$1.0.

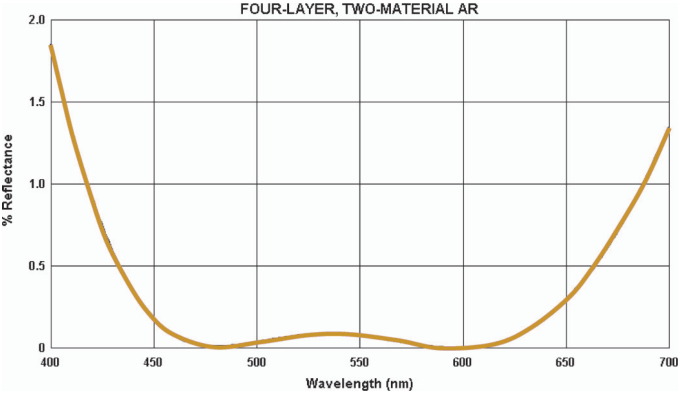
The following figure is the application of the above diagram to plot the n and k of the opaque point nickel (Ni) versus wavelength.



Simulating One Index with Two Others

It is possible, at one wavelength and AOI, to simulate a layer of any thickness and index by two or more layers of materials whose indices are greater and less than the index to be simulated. A typical example is the case of the three-layer BBAR discussed earlier. The materials used there were $H = 2.2$, $M = 1.7$, and $L = 1.38$. The M-layer can be replaced or simulated by a layer of H and L. These short H- and L- layers bring the locus to the same point as that of the replaced M-layer. The reflectance versus wavelength is similar to the three-layer design.

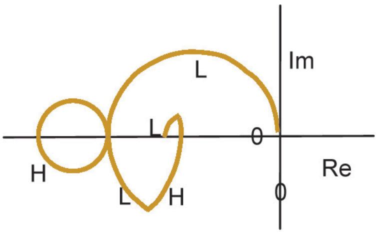




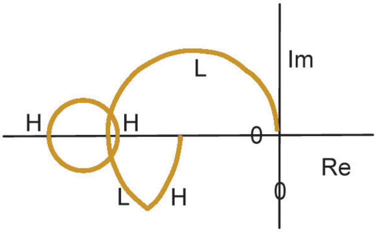
Note that a V-coat similar to the one previously shown will result if the second H-layer (two QWOTs) is eliminated. This would then be a two-layer AR with H and L materials instead of M and L. However, the layers are not QWOTs in the new case.

Herpin Equivalent Layers

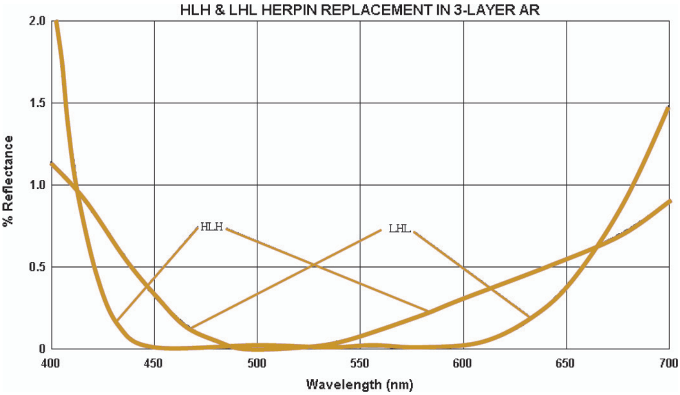
M. A. Herpin showed mathematically that any intermediate index layer could be simulated by a symmetric three-layer combination of HLH or LHL where the outer layers are of the same thickness. This gives a matrix representation that is the same, at one wavelength and AOI, as the layer being simulated or replaced. Although the first and third L-layers appear to be of differing sizes, they do have the same OT.



The LHL version with spectrum.

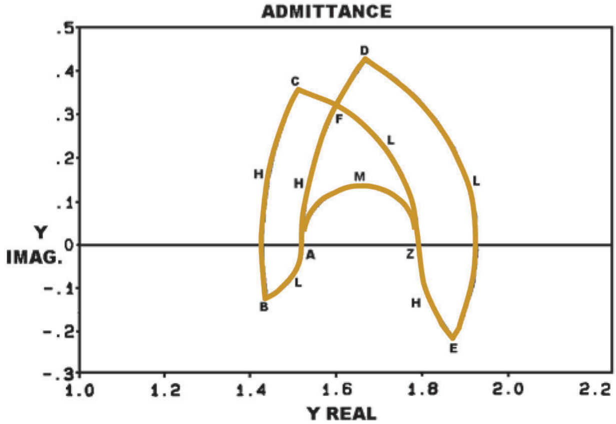


The HLH version with spectrum.



Approximations of One Index with Others

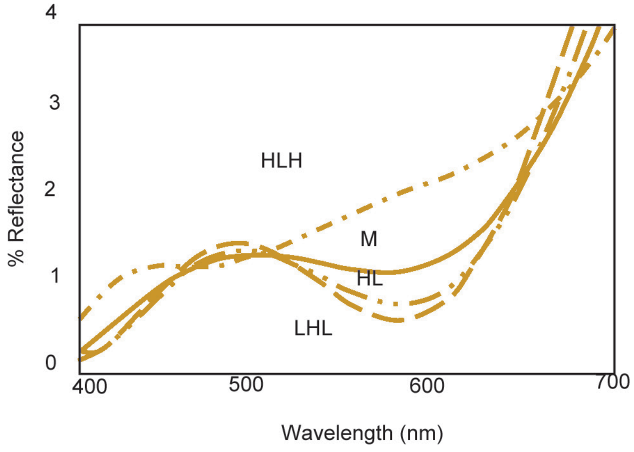
Herpin equivalent index approximation is a useful tool. Some more general cases are examined here. The admittance locus of a QWOT layer *M* of index 1.65 (such as in a three-ayer AR) that moves from a substrate point *A* of $Y = 1.52$ to a termination point at *Z* is illustrated. At this wavelength and angle (0°), any combination of layers that moves the locus from *A* to *Z* gives the same reflectance. The Herpin HLH travels the locus *ABCZ*, while the LHL solution travels the locus *ADEZ*. *AFZ* is another path that accomplishes the same thing.



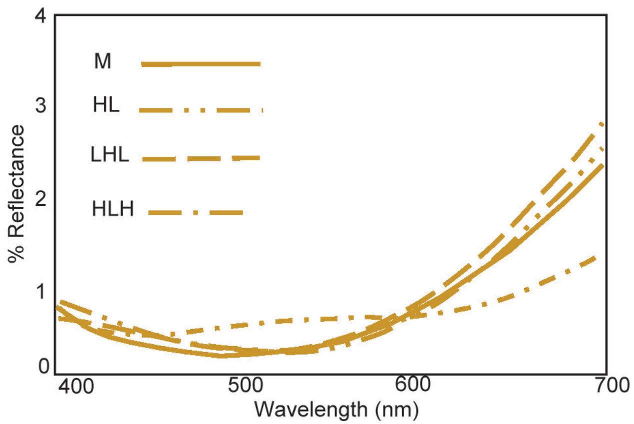
These *A-Z* paths are the first "layer" of the classic three-layer AR design of QWOT-HWOT-QWOT (QHQ) or the **1M-2H-1L** design. The following figures show how these choices change the reflection as a function of wavelength for *s*- and *p*-polarizations at 45° AOI.

The closer the approximating locus stays to that of the layer being approximated, the closer the reflection versus wavelength and angle will be to that of the design with the M-layer being approximated or simulated.

Approximations of One Index with Others (cont.)



S-polarization at 45° for this “QHQ” of design 1M 2H 1L.

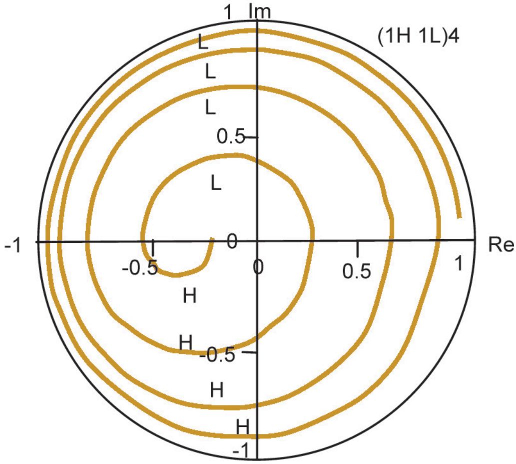


P-polarization at 45° for this “QHQ” of design 1M 2H 1L.

Note that the HL approximation gives the closest spectral simulation and the HLH is the worst. These are seen in the admittance diagram to be the closest and furthest from the locus of the M-layer.

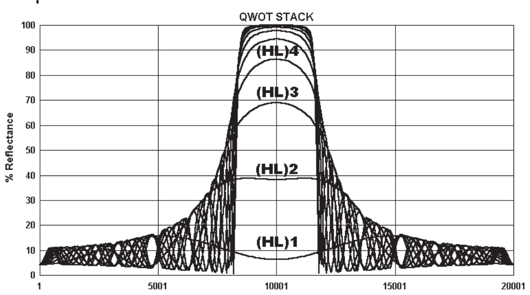
QWOT Stack Reflectors

If a sequence of H (2.2) and L (1.38) QWOT layers are deposited on a glass substrate (1.52), the reflection, at the design wavelength, increases with each pair of H and L layers. The circle diagram shows 4 layer-pairs or 8 layers.



Reflectance Amplitude Diagram

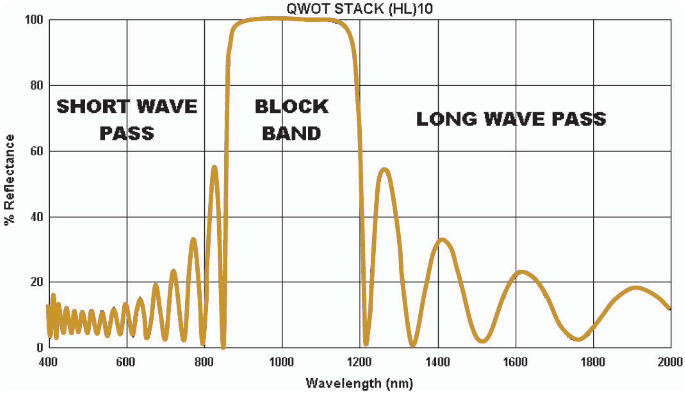
The figure shows the effect of each layer-pair up to 10. The reflection approaches a "square-top" shape for a large number of layers. These reflectance spectra are plotted here on a wave number scale which is 10,000/λ in μm. This provides a symmetric result. The design wavelength in this case is 1.0 μm.



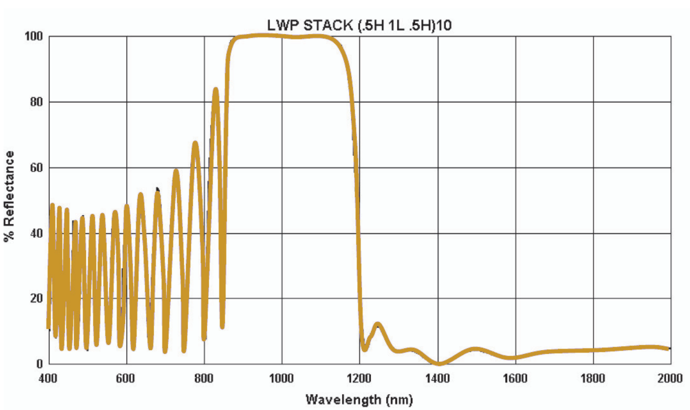
Note the ripples in the lower reflectance side bands. These are of finer pitch as the number of layers increases.

QWOT Stack Properties

The (HL)10 design is shown here on a wavelength scale, which loses its symmetric appearance. The high-reflectance band is called the **block band** because it blocks the transmittance at those wavelengths. The left region, where the wavelengths are transmitted, is called the **short wave pass band** (SWP), and the right-side region, where the wavelengths are transmitted, is called the **long wave pass band** (LWP).

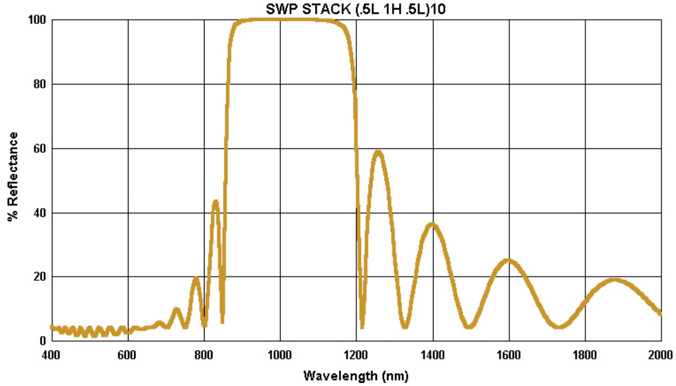


If a LWP is needed, a small change in the design from (1H 1L)10 to (.5H 1L .5H)10 reduces the ripples in the pass band to a more useful level. However, the ripples on the other side of the block band are increased.

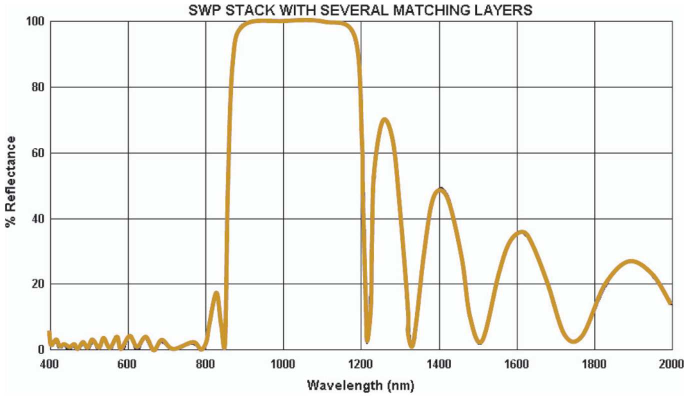


QWOT Stack Properties (cont.)

If a SWP is needed, a small change in the design to (.5L 1H .5L)10 reduces the ripples in the pass band. These first and last layers of ½ QWOT are called **matching layers** (ML) to match the impedance or admittance of the stack to that of the media that contact it.



These could also be thought of as AR layers. The single ML in the SWP case above leave higher ripples thanmight be acceptable. The use of more MLs can reduce the ripple further. These MLs are usually not of QWOT. The example below uses four layers on each side of the stack. Five layers are usually the point of diminishing returns for reducing ripple.



Width of the Block Band

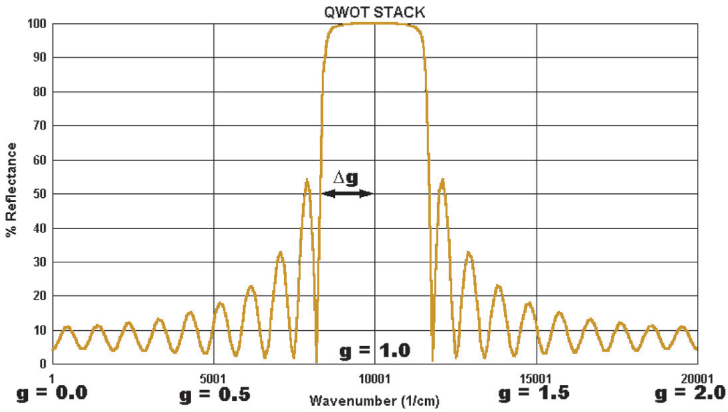
The width of the block band is best viewed on the wave number scale for symmetry. The **g-value** is the design wavelength (λ_0) divided by the wavelength (λ) under consideration, or the frequency under consideration (σ) (wave number) divided by the design frequency (σ_0):

$$g = \lambda_0 / \lambda \text{ or } g = \sigma / \sigma_0 .$$

The half width of the high reflectance zone of the block band is called **Δg**. It depends on the indices of the layers in the stack and can be calculated by

$$\Delta g \approx (2/\Pi)\arcsin\left[(n_H - n_L)/(n_H + n_L)\right].$$

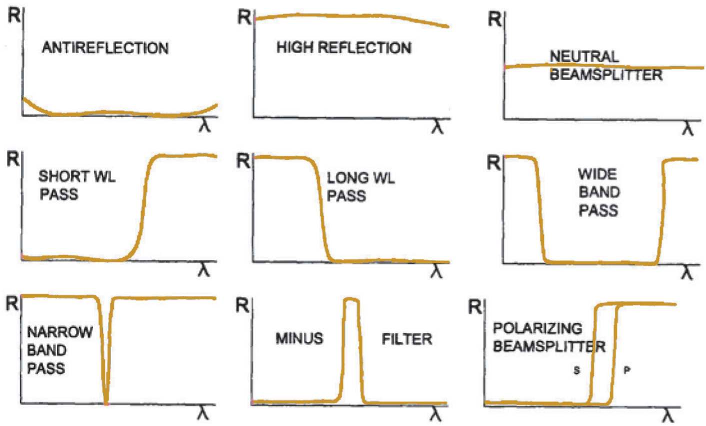
In the figure below where the design is (1H 1L)10, $n_H = 2.2$ and $n_L = 1.38$. This produces a $\Delta g \approx 0.147$ (however, the measured value here at R = 50% is actually 0.170).



It can be seen that the width of the block band depends on the difference in the indices of H and L. For Ge and a fluoride at 4.0 and 1.35 index, Δg would be 3.30, or about 2X the typical visible spectrum stack given above. On the other hand, with 1.46 and 1.38, Δg would be 0.0179, which is quite narrow. This will be mentioned later as a **minus filter**.

Applications of the QWOT Stack

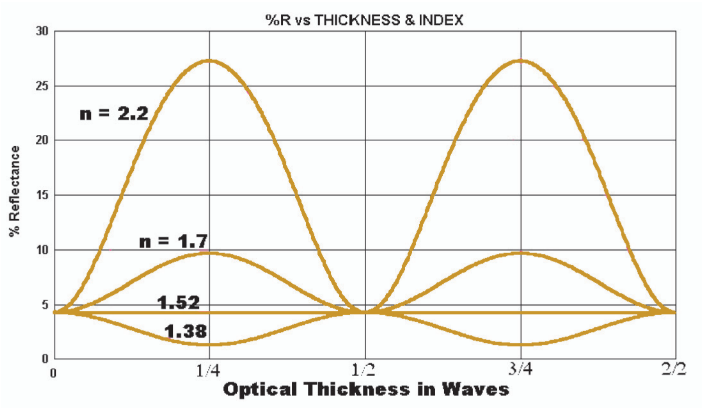
The QWOT stack might be called the **building block** for most optical coatings. The figure illustrates the many types of coatings based on the building blocks that have been discussed. The **AR** is derived from the pass band (preferably the LWP side) of a stack where the MLs are adjusted for AR. The **high reflector** is one or more stacks that have been juxtaposed to give the necessary width of the reflection band. The **neutral beamsplitter** has enough layer pairs to reach the required reflection and is further designed for band width. The **SWP** and **LWP** types have been discussed. A wide band pass filter can be made by combining a SWP and LWP filter. A **narrow bandpass filter** consists of stacks and half-wave spacers (which will be discussed in more detail on pp. 31–36). The **minus filter** has a block band that is usually narrow due to a small difference in indices. **Polarization beamsplitters** use the changes of effective indices of the *s*- and *p*-polarizations with AOI to reflect one polarization and transmit the other (will be discussed in more detail on pp. 40).



The great variety of optical coatings that are built from the building blocks of the QWOT stack can be readily seen here. This can facilitate the understanding of optical thin films.

Absentee Layer

The reflection of a layer of material deposited on a substrate changes with thickness from an initial value equal to that of the substrate to some minimum or maximum at one QWOT. When the thickness increases to two QWOTs, the reflectance returns to the same as at zero thickness. This would be called an **absentee layer** because it is the same as if it were not there (at the design wavelength). At three QWOTs, the reflectance is the same as at one QWOT. At four and all even multiples of a QWOT, there is an absentee layer again. This pattern repeats indefinitely with increasing thickness.

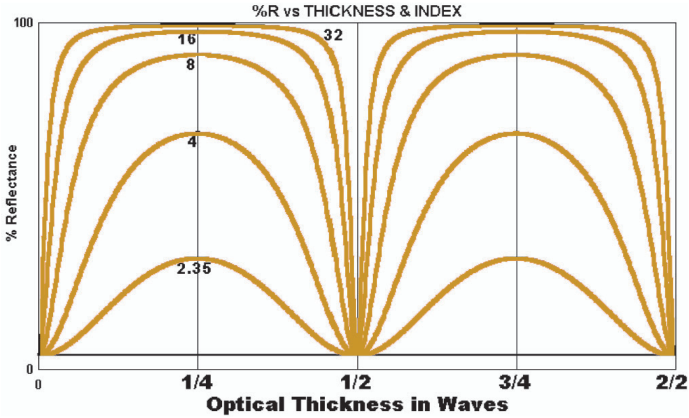


If the index of the layer is less than the substrate, the reflection is reduced by the coating. If the index is higher, the reflection is increased; and the maximum effect is at odd multiples of one QWOT. As can be seen in the figure, a coating of the same index as the substrate can be of any thickness without changing the reflectance of the surface.

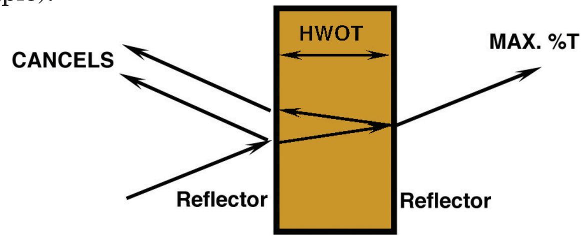
It can be worked out that a QWOT of 1.233 index on a substrate of 1.52 would reduce the reflectance to zero, but a practical version of such a material is not readily available. It can also be seen that a simple QWOT of index 2.2 or higher might produce a reasonable beamsplitter with more than 25% reflectance.

Narrow Band Pass Filter

As the index of refraction of a coating layer increases, the reflection at odd multiples of a QWOT increases. At even multiples of a QWOT, the reflection remains the same as the bare substrate. As can be seen in the figure, the high-reflectance area becomes broader and the low-reflectance (high transmittance) area becomes narrower. This “pass” band is the basis of the **narrow band pass filter** (NBP). It represents a high reflectance at two interfaces that are separated by two QWOTs or some multiple of one **half wave of optical thickness** (HWOT).



The higher the effective index, the higher the reflectance except at the HWOT multiples, and also the narrower the pass band. Therefore, the essence of a NBP is two high reflectors (mirrors) spaced by a HWOT. Early versions of this were made by semitransparent layers of silver on the two sides of a dielectric spacer. Most current versions use QWOT stacks to make high reflectors on a HWOT (or multiple).



Optical Density and Decibels (dB)

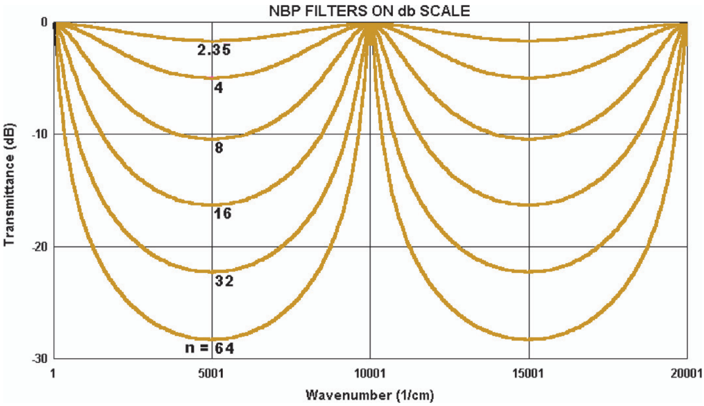
Optical density (OD) is defined as

$$OD = \log_{10}(1/T).$$

Transmittance of 10% would be $T = 0.1$, $OD = 1.0$. One percent would be $T = 0.01$, OD = 2.0, etc. **Decibels (dB)**, which is a unit used in the communications industry, is defined as

$$\text{dB} = -10 \log_{10}(1/T) = -10\, OD.$$

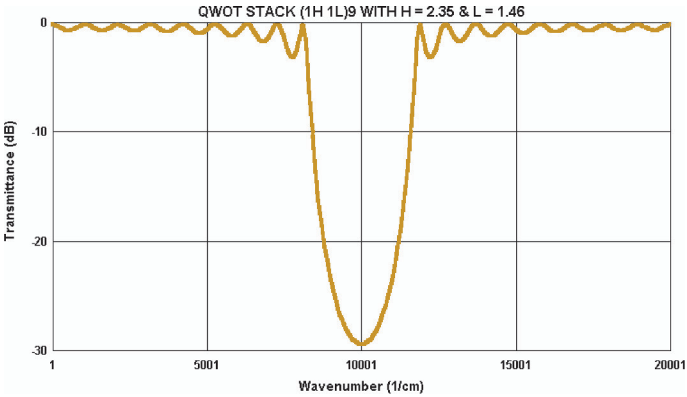
Therefore, –10 dB would be 10%*T*, and –20 dB would be 1%*T*. The figure illustrates how this would look for the various high-index slabs discussed on the previous page. Note also that the ordinate scale is in wavenumbers instead of OT, although they are linearly related. The blocking bands centered on 5,000 and 15,000 cm^{-1} (2.0 and 0.667 μm) are sometimes referred to as the **skirts**.



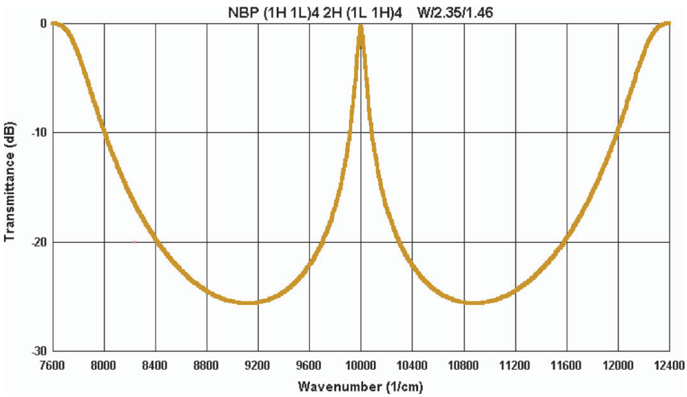
The NBP is at 10,000 cm^{-1} or 1.0 μm, and it can be seen how its bandwidth decreases and the blocking depth (in dB) increases with increasing reflector strength. These factors are important to the communications technology with lasers and fiber optics, and are also important to other applications of NBP filters.

NBP Filter Design

A reflector design that uses H = 2.35 and L = 1.46 which has a dB value similar to the previous slab with fictitious index 64 requires about 18 layers, or (1H 1L)9, as seen in the figure.

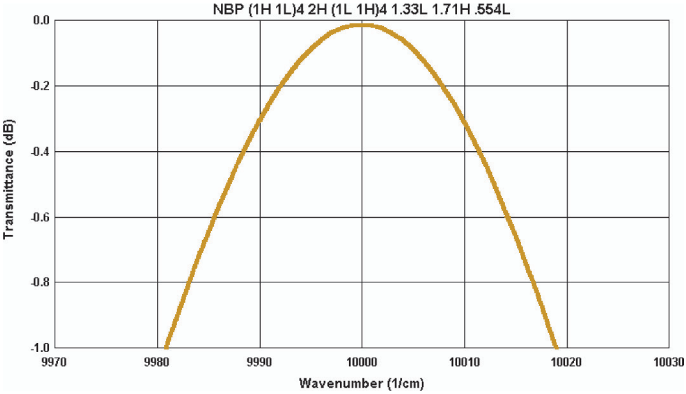


When this is split into two parts separated by 2H, the design is (1H 1L)4 2H (1L 1H)4. These (1H 1L)4 parts are the reflectors/mirrors that are separated by the 2H spacer layer. The transmittance spectrum for this shows some similarity with the NBP filter of index 64 on the previous page. The major difference is due to the **Δg** in each case; the H/L of 64/1 gives a much wider blocking band than the 2.35/1.46 of real-world materials.

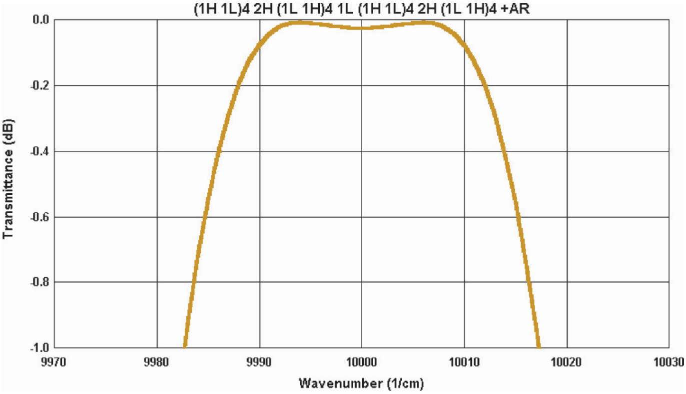


Multiple-Cavity NBP

The figure below plots the portion of the previous filter that transmits more than 80% (0.1 dB). Because there is one spacer that is some multiple of a HWOT, this is called a **single-cavity filter**. Three layers have been added to the basic design to be an AR for the reflection of the substrate.

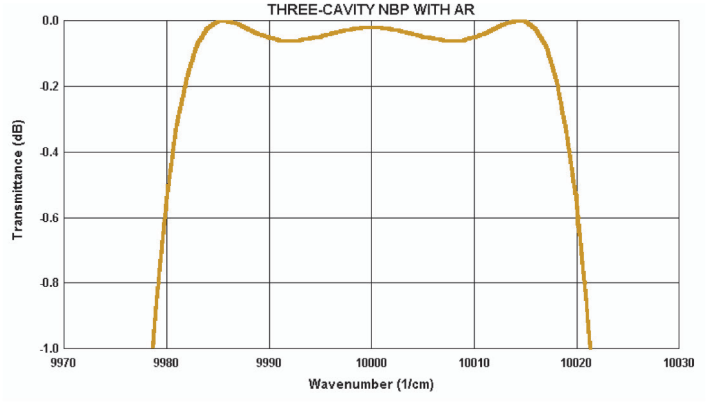


If two such designs are merged using a QWOT coupler layer, this is called a **two-cavity filter**. There are three benefits. The high transmittance area is broader and the skirts are steeper and have more dB of blocking.

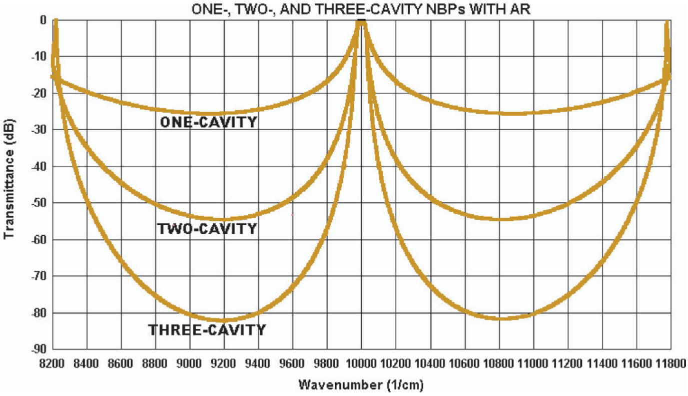


Multiple-Cavity NBP (cont.)

A **three-cavity** design would extend the mentioned benefits at the expense of more layers/complexity. It can be seen that the top is broader and flatter while the sides are steeper. This is a benefit in isolating the wavelengths passed by one filter from those of an adjacent band.

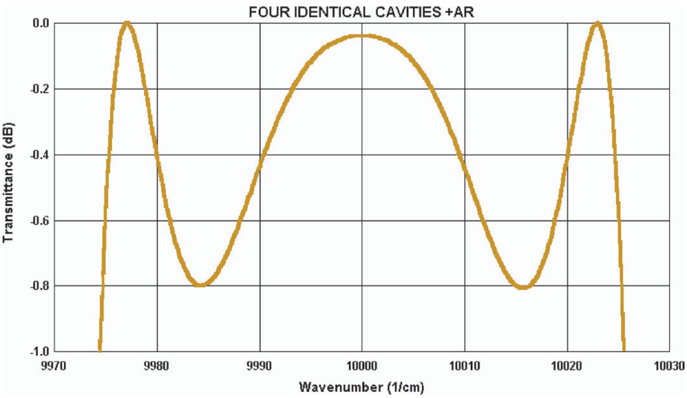


The figure below shows the steepness and depth of the skirts for one-, two-, and three-cavity designs. On the other hand, the **dB** of the peak blocking bands can be adjusted by the number of layers in the reflector stacks of any of these designs.

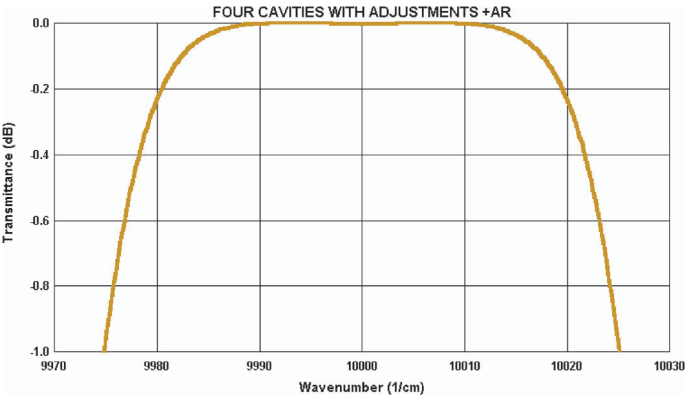


Rabbit Ears

When the foregoing multiple-cavity approach is extended to four cavities, an unpleasant surprise occurs, called **rabbit ears**. If four identical cavities are coupled by a QWOT between each, the figure below is the result.



In order to correct this problem, it is necessary to adjust the design. One option is to reduce the number of layer pairs in the reflectors of the outer cavities. The design below can be seen to have a very flat region over a fairly broad spectrum.



Polarization Effects

When the **angle of incidence** (AOI) θ_1 is not 0°, the ***s*-** and ***p*-polarizations** have different **effective indices**.

$$N_S = n \times \cos\theta \text{ and } \quad N_P = n / \cos\theta$$

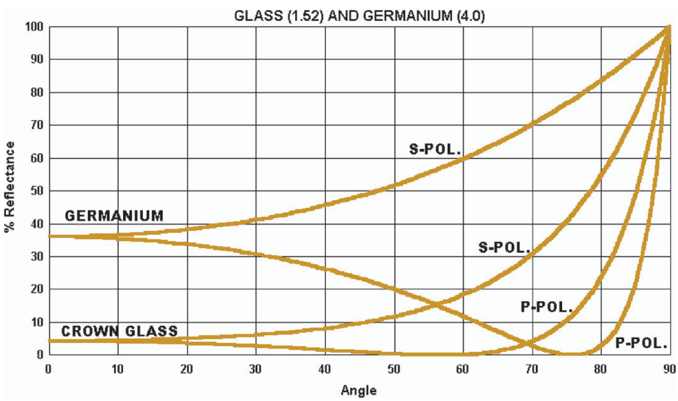
In these cases:

$$r_S = \left(N_{1S} - N_{2S} / N_{1S} + N_{2S}\right),$$

and

$$r_P = (N_{1P} - N_{2P})/(N_{1P} + N_{2P}).$$

When this is applied to the examples of an uncoated surface of glass at index 1.52 or germanium at 4.0, the reflectance versus AOI is plotted below.

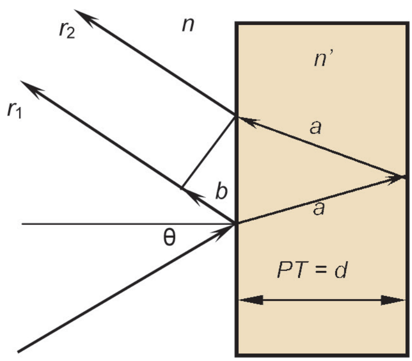


Note that for each index there is an angle where the reflection of the *p*-polarization goes to zero. This is called the **Brewster angle** where there is no *p*-reflection. A plane-parallel window placed at this angle to the light is referred to as a **Brewster window**. Such has been useful in gas laser cavities to contain the gas and pass the beam without attenuation in the *p*-polarization. Lasers with Brewster windows are linearly polarized because the *s*-polarization is much more "lossy" due to reflections.

Optical coatings show all of these same effects at interfaces.

Wavelength Shift with Angle of Incidence

If the physical thickness (PT) of a film is d and the light is incident at an angle θ, the physical path a in the medium is greater than d. The OT of a is an', and the path from the first surface back to the first surface is $2a$, which is greater than $2d$. Intuitively, it might be expected that the optical path difference of the film would be longer at increasing angles, and thereby the spectrum of the coating would be shifted to longer wavelengths.



However, a more detailed investigation shows that the extra optical path traveled by the r_1 ray (nb), while the r_2 ray travels $2an'$, more than compensates for this angle effect. In fact, the optical path difference of the film is shorter at increasing angles, and thereby the spectrum of the coating is shifted to shorter wavelengths.

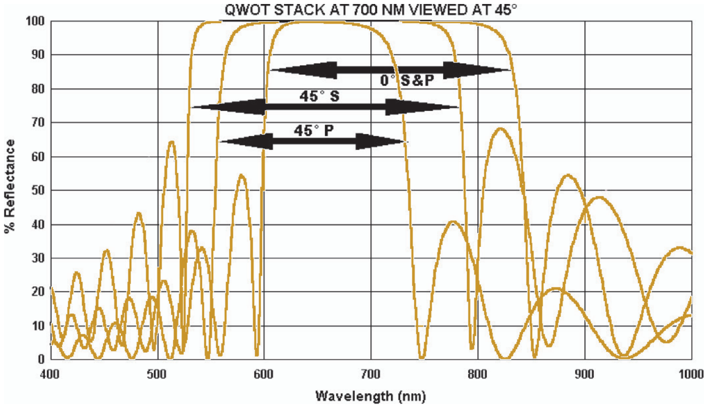
If it is desired that a layer act as a QWOT at a specific angle θ, the layer needs to be made thicker by a factor or "matched" in accordance with its indices of refraction and the angle. The incident medium is n and the layer is n'.

$$\text{Factor} = \cos\left\{\arcsin\left[\left(n/n'\right)\sin\theta\right]\right\}/\left\{1-\left[\left(n/n'\right)\sin\theta\right]^2\right\}$$

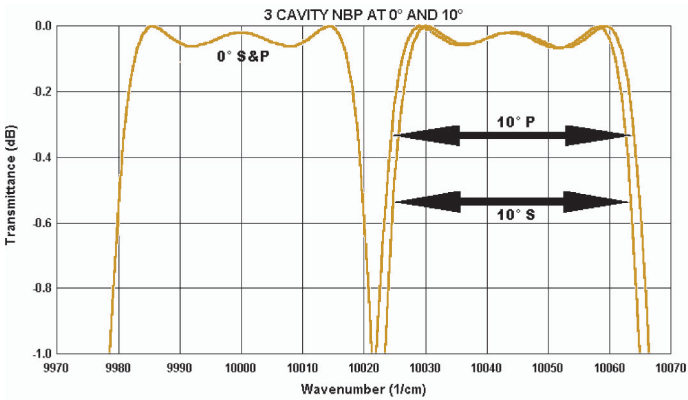
When the QWOT for normal incidence is multiplied by this factor, it will behave as though it were a QWOT when used at the new angle θ.

Angle of Incidence Effects in Coatings

When the **angle of incidence** is not at 0° or normal to the coating surface, Snell's law and polarization effects come into play. There is a general shift to shorter wavelengths, and the width of block bands are influenced by the different Δg's that result from different effective indices for *s*- and *p*-polarizations. The plot of the reflectance of a simple QWOT stack illustrates these effects.

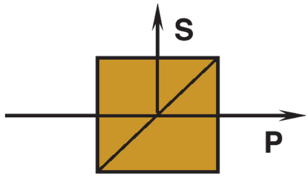


The plot below shows the effects on a NBP filter. Note that the shift is still to shorter wavelengths or higher frequencies, but is to the right on a wavenumber scale. The *s* and *p* do not split much at 10°, and the Δ*g* for *s* is smaller than *p* for this design. This ability to tune the wavelength shorter by changing the AOI is an asset to the DWDM technology.



Polarizing Beamsplitters

The previous page suggests how a polarizing beamsplitter might be designed. The change of the blocking bandwidth of *s* and *p* with angle can be used to transmit *p* while reflecting *s* at certain wavelengths. The cube beamsplitter shown below illustrates this.

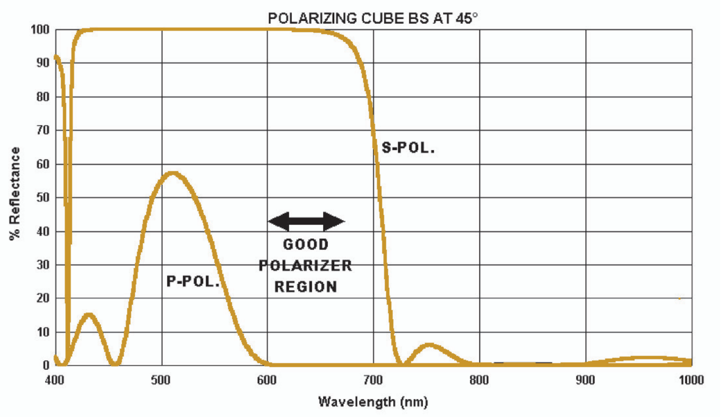


The following design has been adjusted in the matching layers to reduce the *p* reflection to zero from 600–700 nm:

.61212H .87231L (1H 1L)5 1H .88257L .61931H

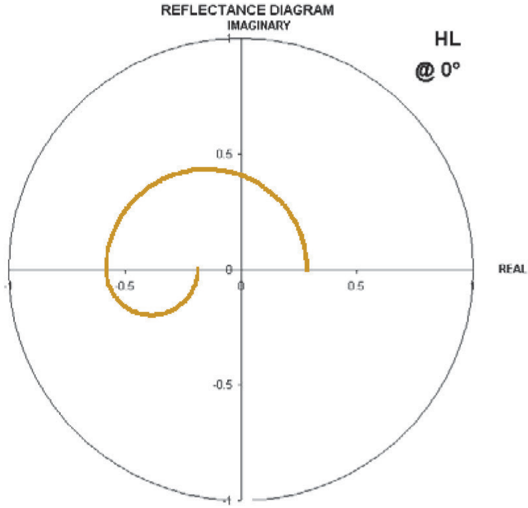
at 700 nm, substrate = 1.52, H = 2.35, L = 1.38.

With the high reflection of the *s*-polarization, this provides a good polarizer over a region of 50 nm or more. Under very specific conditions, the reflection of the *p*-polarization can be designed to be near zero over the 400–700 nm band. This would be a **MacNeille polarizer** and provide a polarizer of high efficiency over the whole visible spectrum.

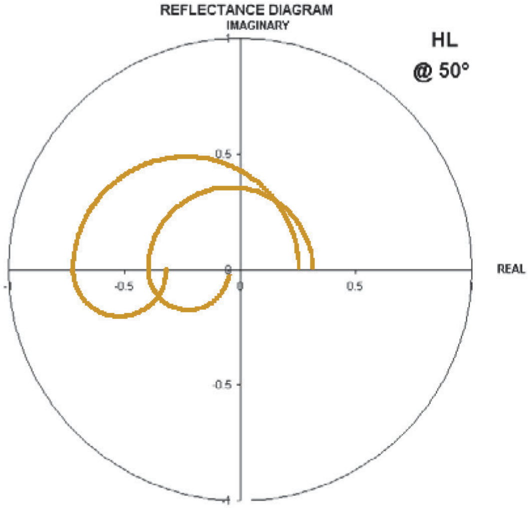


Polarization as Viewed in Circle Diagrams

The figures below show the circle diagrams of a 1H plus 1L coating at 0°, 50°, and 89°. At 0°, the *s* and *p* are the same and terminate at $r \sim 0.28$, $R \sim 8\%$.

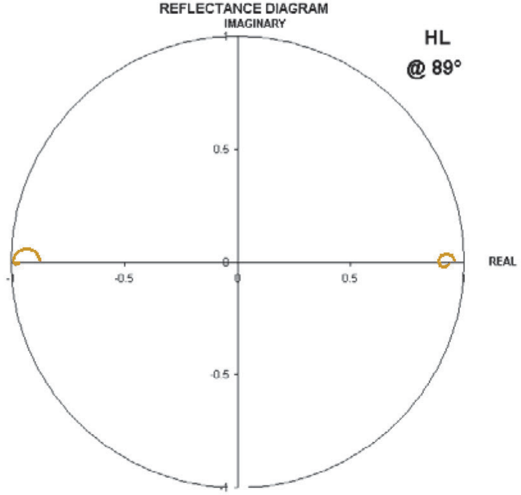


At 50° and 89°, the *s* has shifted to the left and *p* to the right.

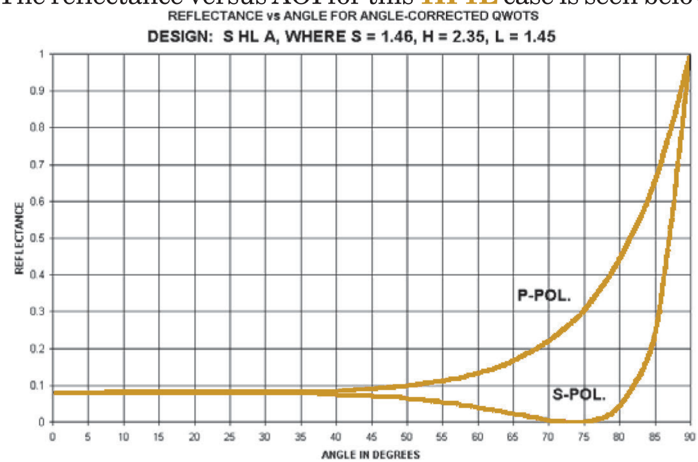


Polarization as Viewed in Circle Diagrams (cont.)

It can be seen at 89° that *s* and *p* are both approaching 100%R, but with 180° difference in phase.



The reflectance versus AOI for this **1H 1L** case is seen below.



It is interesting to see that the *s* and *p* are reversed, and this coating would provide a Brewster window at ~73° for *s*-pol!

Non-Polarizing Beamsplitters in General

The optical coating design of beamsplitters that are non-polarizing—that is, those that have the same reflection for both *s*- and *p*-polarizations at specified angles—is a particular challenge. This is because the effective indices of refraction for the media and coating layer materials have a different function of the angle of incidence for each polarization, as shown before. That limits what can be achieved. Macleod has commented that, “The techniques which are currently available operate only over very restricted ranges of wavelength and angle of incidence (effectively over a very narrow range of angles).” Specific materials can be used to design reasonable non-polarizing coatings at only certain angles, whereas other materials can be made to serve for other angles. Since the range of practical coating materials and substrates is limited for most applications, non-polarizing solutions tend to be quantized in reflectance and angle.

This section is further confined to “non-immersed” designs as opposed to coatings surrounded by glass, such as cube beamsplitters, which offer broader possibilities when they can be employed.

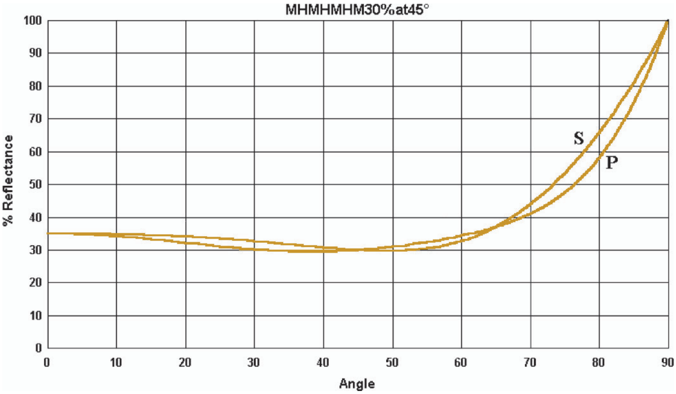
The foregoing figures and those that follow on polarization are cases where the thicknesses were adjusted to produce QWOT for each layer and index at the angle being evaluated. This is referred to as angle-matched thicknesses and is discussed in the early pages on polarization and angle effects (see pp. 38).

It will be seen that two advantages accrue when only angle-matched QWOT layers are used: 1) the change in reflectance with wavelength is minimum at the design wavelength because it goes through zero (as can be seen in a circle diagram), and 2) the sensitivity to layer errors can be minimal.

A Non-Polarizing Beamsplitter Design Procedure

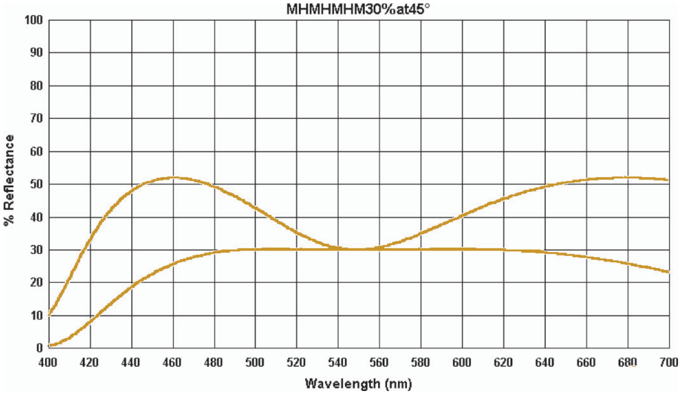
A systematic search can be done of all possible QWOT coating designs having up to a certain number of layers. A substrate index (such as 1.46) and the coating material indices (such as $L = 1.45$, $M = 1.65$, and $H = 2.35$) are chosen to suit the problem at hand. All of the possible combinations of layers for four layers of three materials can be screened for practicality. Where two QWOTs of the same material occur, the case can be dropped because the layer would be absentee or HWOT. As five or more layers are considered, the number of possibilities tends to become so large as to need the exercise of further limiting judgment.

Each design is evaluated from 0° to 90° with the necessary angle matching adjustments of thickness with angle. If the reflectance of the *s*- and *p*-polarizations intersect at other than 0° or 90°, there is a non-polarizing design with all QWOTs at that angle and reflectance. The figure below is an example of such a case where, in the design MHMHMHM, the *s* and *p* reflectances intersect near 45°.

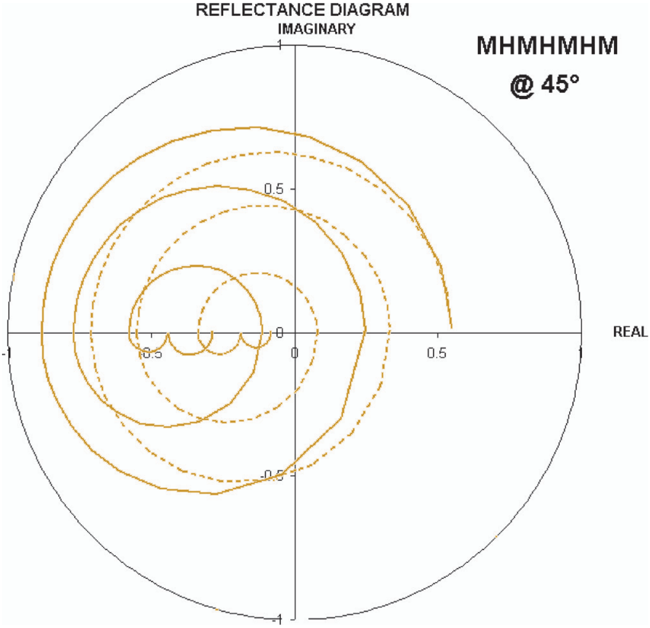


The reflectance at 45° is near 30%. The change in reflectance for *s* and *p* can be seen from the figure, e.g. from 40° to 50°. This design also has a non-polarizing angle near 65°. The change with wavelength at 45° is seen on the next page.

Non-Polarizing Beamsplitter Design (cont.)

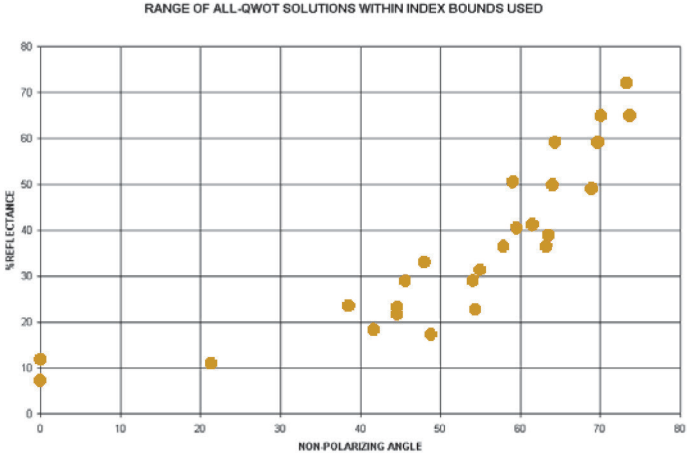


It can be seen that the non-polarizing spectral range is quite limited. The figure below shows this design on a circle diagram at 45°. The *s* is the solid curves and the *p* is the dotted curves. Note that the two sets of curves meet on the positive real axis at a phase of 0°.



Non-Polarizing BS's Found & Rules-of-Thumb

The procedure described was used for *all* 5-layer cases and a selected range of 6- and 7-layer cases. The figure below shows the various **non-polarizing solutions** found.



Final design computer optimizations can be used to balance the needs for spectral and angular sensitivity where one can be traded for the other, but no gain over both is likely. This will generally lead to non-QWOT layers, but the QWOT procedure is a good starting point.

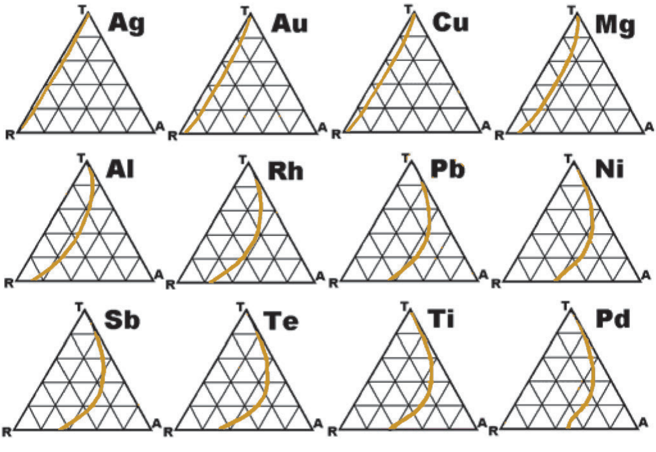
Design rules-of-thumb for this family of coatings might be: 1) determine the low, medium, and high indices that can be reliably produced at the wavelength needed, 2) search for the possible combinations of layers using these materials with QWOT solutions (intersections) near the desired angle and reflectance (being sure to include the proper angle-matched thicknesses), 3) adjust the indices to gain the angle and reflectance as needed (within the ranges that can be produced), and 4) adjust the design with optimization as necessary, using non-QWOT layers to achieve the most acceptable compromise for the application.

Various Metals on Triangle Diagrams

The figure below shows the triangle diagrams for 12 metals as thin films, arranged from generally the least absorbing (Ag) to the most (Pd). The locus of the ideal metal on a triangle diagram would move from maximum $\%T$, straight along the edge of $0\%A$ to the $100\%R$ point at the lower left. The locus of the perfect absorber, on the other hand, would move from maximum $\%T$, straight along the edge of $0\%R$ to the $100\%A$ point at the lower right.

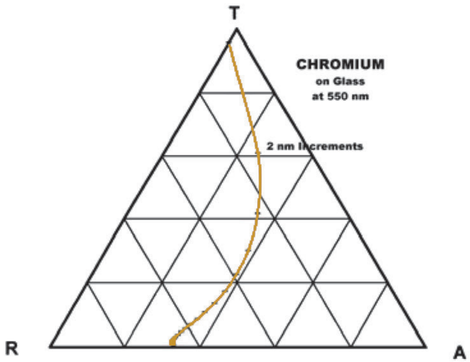
Silver (Ag) is the most nearly ideal metal of those shown, and does make a very good reflector in an opaque film. **Gold** (Au) and **copper** (Cu) are also seen to have high reflection and low absorption. All three metals are nearly ideal metals in the infrared part of the spectrum (2–20 μm), and even **aluminum** (Al) is quite good in that region. The choice among those materials for IR applications will usually depend more on the physical and environmental factors than the high reflectance that they all can have.

Nickel (Ni) and **chromium** (Cr) have similar triangle diagrams, and a mixture of the two in **nichrome** is useful for neutral density filters that absorb some of the flux and designs where absorptance is needed .

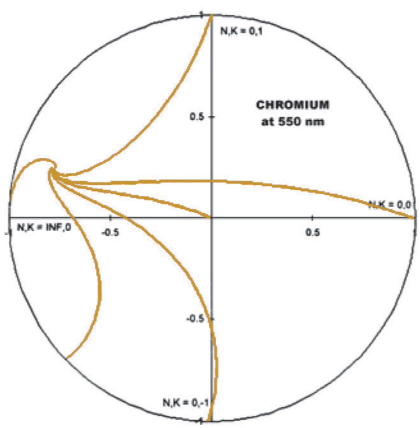


Chromium Metal Details

The figure below shows the triangle diagram for chromium. The locus starts at 96%T, 4%R, and 0%A for the bare glass substrate. It moves downward and is marked in 2-nm increments of PT for the film until the opaque point is reached at 0%T.

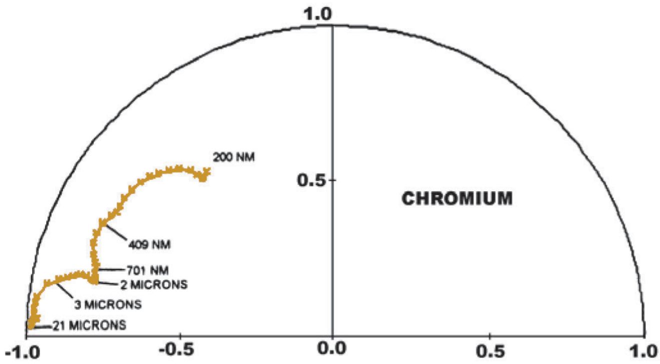


The reflectance diagram below shows how the reflection of a chromium film would change with thickness from five different starting points. All these loci converge on the same opaque point in the upper left quadrant. Note that these loci are quite different from the circle loci seen with dielectric films. Paths from intermediate starting points can be interpolated between the lines shown.

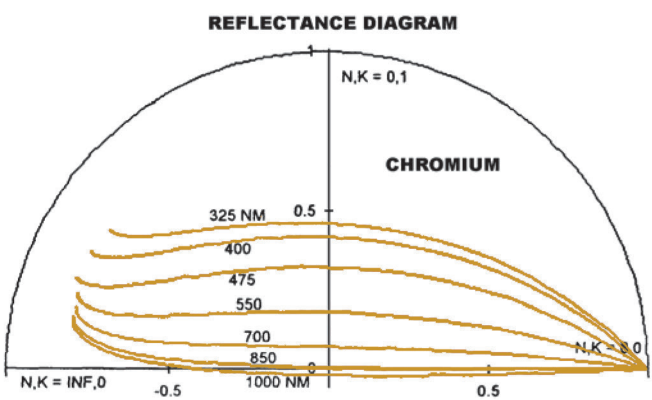


Chromium Metal Details (cont.)

For each metal, the opaque point and index values is a function of wavelength. The first figure below shows how the positions of the opaque points for chromium vary.



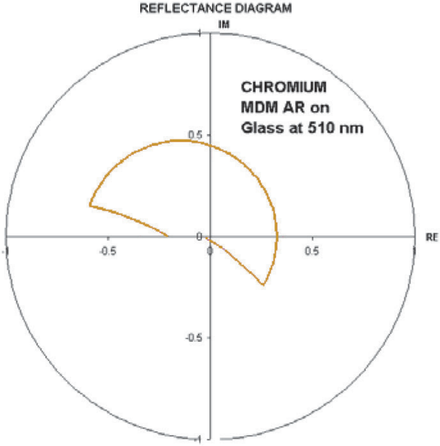
The next reflectance diagram below shows how the loci for chromium will vary from the same starting point on the right ($N=0 -i0$) to the opaque point at a given wavelength in the upper left quadrant.

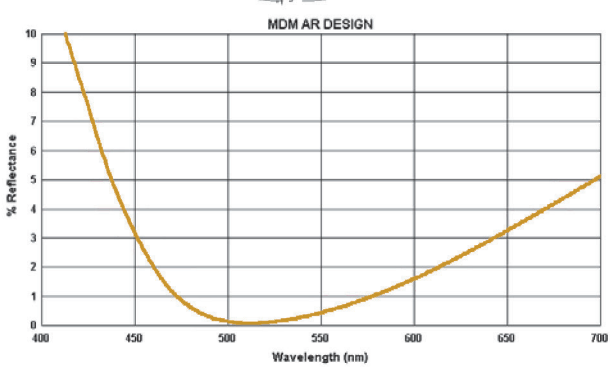


The somewhat different behavior from dielectrics of thin absorbing films suggest that there could be design possibilities that may not have yet been explored.

A Design Example Using Chromium

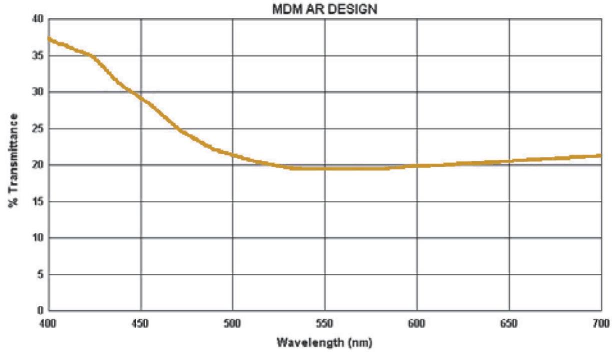
An AR coating can be designed with Cr and a dielectric layer. Here, SiO_2 is chosen as the dielectric layer and the substrate is of index 1.52. As Cr is deposited on the glass, the locus moves from $r = \sim -0.2$ toward the opaque point of Cr to the left. Before the opaque point, the SiO_2 layer begins, and it travels a bit more than one QWOT before the last Cr starts. The last Cr layer brings the locus close to the origin of the coordinates, or $r = R = 0$. This then is an AR coating at the design wavelength. The reflectance versus wavelength is plotted at the bottom of the page. This might be an interesting concept for sunglasses, but otherwise somewhat academic. There would be a dominant blue reflectance and transmittance (next page).



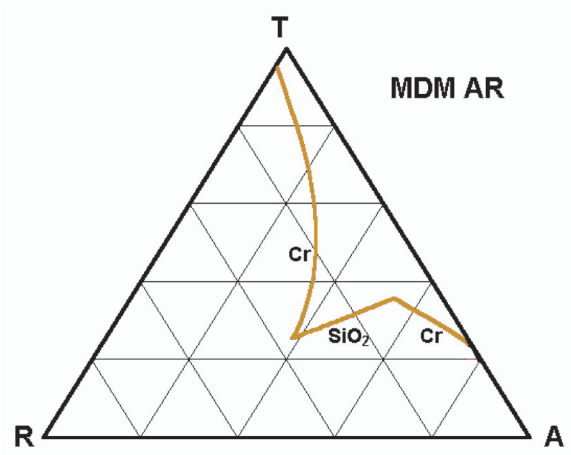


A Design Example Using Chromium (cont.)

The transmittance spectrum here is about 20% in the green and red, but would transmit quite a bit more in the blue. This is the opposite of what might be desirable for sunglasses for a hazy day.



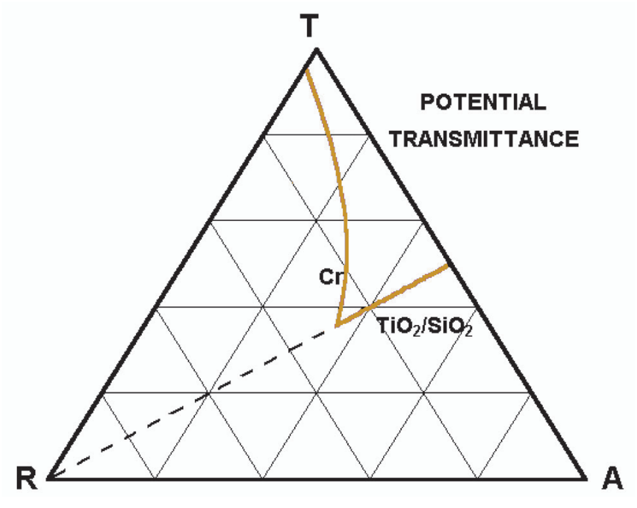
The triangle diagram below shows how this coating would appear in that format. The locus of the first Cr layer begins on the substrate at the top, moves downward to about 25%T where the SiO_2 takes over. At about 8%*R* and 35%*T*, the last Cr begins and takes the %*R* to zero at about 20%*T*. Note that the SiO_2 locus is a straight line radiating from the 100%*R* point.



Potential Transmittance

The **potential transmittance** represents the maximum that might be achieved if there were no reflection losses. This is illustrated by the figure below. The Cr layer shown, when taken alone, would transmit approximately 36%. If a dielectric AR coating were applied, such as TiO_2 and SiO_2, the reflection losses could be reduced to zero over some spectral range (depending on the complexity of the dielectric design). The locus of these dielectric layers would be on a line from the 100%R point through the end of the Cr layer. In the case shown, the resulting transmittance is 50%, the maximum potential for that given Cr layer.

On a triangle diagram, the loci of dielectric layers lie on lines radiating from the 100%R point. If there were no absorption in a film stack, the locus would all lie on the line between 100%R and 100%T. This view therefore does not show any useful information concerning the dielectric part of the coating. However, when the R, T, and A are plotted as a function of coating thickness perpendicular to the plane of the triangle, the locus of the dielectric layers yields more information.



Estimating What Can Be Done Before Designing

Before designing a coating it is helpful to have some idea of whether the **goal of the design** is achievable. The ability to estimate **performance limits** can avoid fruitless design efforts and avoid the neglect of potential performance gains or simplifications. Some information is provided for estimating the number of layers and other properties of AR, dichroic, bandpass, and blocking filters.

Broad-band AR coatings are the first and major subject considered for estimation. The variables are bandwidth (B), index of refraction of the last layer (L), overall optical thickness of the coating (T), and the difference (D) between the highest and lowest indices used (except for the last layer). It has been found that any more than the minimum number of layers increases the average reflectance. For smaller bandwidths, designs tend to require a number of layers equal to 6 times the parameter of overall thickness (discussed below) divided by the minimum thickness, whereas 10 times that parameter are needed for larger bandwidths. It has further been found that the optimization of a starting design, other than a design of only a few layers, tends to reach a conclusion at a local minimum that is not always the best that can be achieved in the general region of available variables. The best **overall thickness parameter** for a design has a tendency to be quantized.

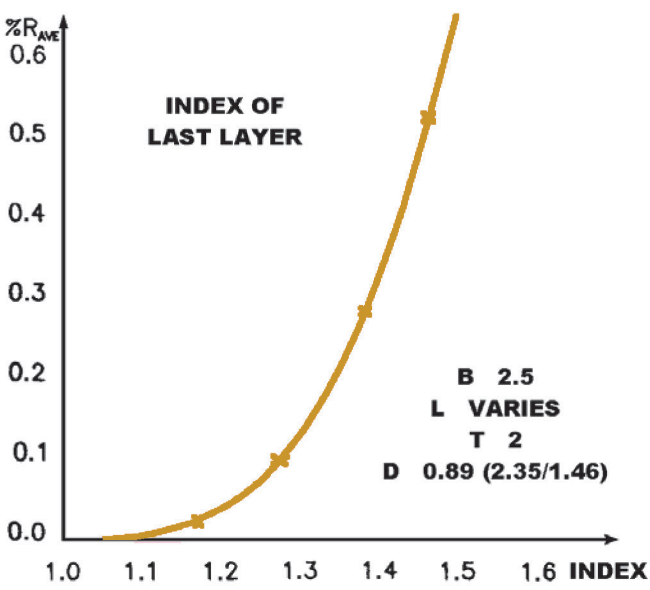
The **bandwidth** (B) is defined in this work as the ratio of the highest frequency to the lowest (or the longest wavelength to the shortest) in the low-reflectance band. The choice of materials is limited by the spectral range of interest and the environmental resistance desired; this study has principally confined the materials used to indices of 2.35 and 1.46. It will be emphasized that there are definite lower limits on the overall thickness and number of layers for an optimal design, and that it is undesirable to exceed a certain upper range of overall thickness and the minimum number of layers for a given design problem.

Effects of Last Layer Index on BBAR Coatings

The index of refraction of the last layer of a BBAR stack before the air or a vacuum has a strong influence on how small R_{ave} can be made in a design. It can be demonstrated that, for a very large bandwidth, the lowest R_{ave} that can be achieved is approximately the reflectance of a slab of that same last low-index material.

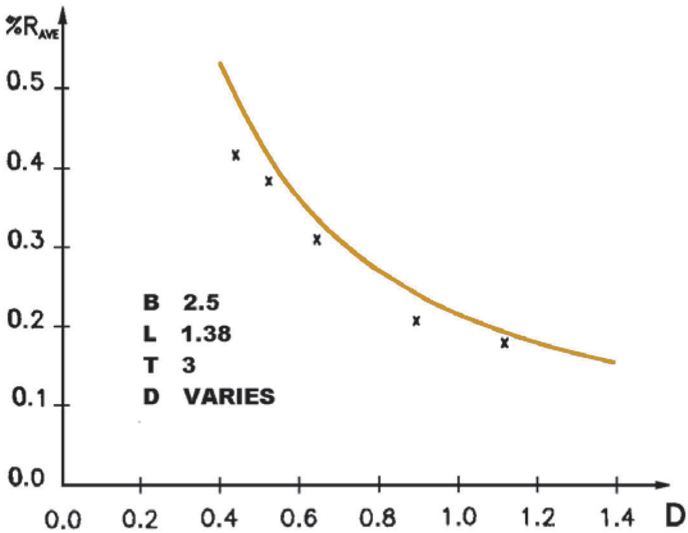
The figure shows the empirical findings where the bandwidth, overall thickness, number of layers, and materials of all other layers were held constant. The highest cross mark is for an index like SiO_2 at 1.46. The next mark down is for a material like MgF_2 at 1.38. The other marks are fictitious materials of lower index.

The point to note here is that using MgF_2 as opposed to SiO_2 for the last layer of a visible spectrum AR should make it possible to achieve a design with about half the minimum R_{ave} in a broadband application. This becomes less critical as the bandwith is less, as in a V-coat.



Effects of Index Difference (H–L) on BBAR Coatings

The index of refraction difference between the high and low materials of the stack, other than the last layer, also has an influence on the lowest R_{ave} that can be achieved. In most thin-film design problems, the larger index difference is an advantage. If nothing else, it reduces the number of layers needed to achieve a given effect.



The combined effect of the last layer index *L* and the difference in the index of the other layers *D* can be stated as follows:

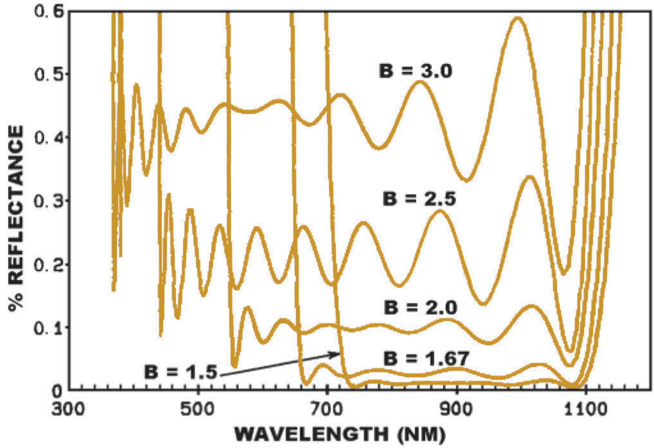
$$R_{ave} = \text{Factor} \times (L - 1)^{3.5}/D$$

The ranges of *L* and *D* over which this formula has been tested and given reasonable estimates is

L from 1.1 to 2.2
D from 0.4 to 2.8

Bandwidth Effects on BBAR Coatings

In the figure below, an AR coating design of a given overall thickness *T* with specified materials and number of layers was optimized over the spectral region from 733 to 1100 nm, or a bandwidth *B* of 1.5. The design targets were then broadened to the range of 660 to 1100 nm, or a *B* of 1.67. Upon reoptimization, the minimum average reflectance (R_{ave}) over the new band was somewhat greater than for the narrower band. When this step was repeated for larger and larger bandwidths, the R_{ave} in the band increased progressively, as seen in the figure.

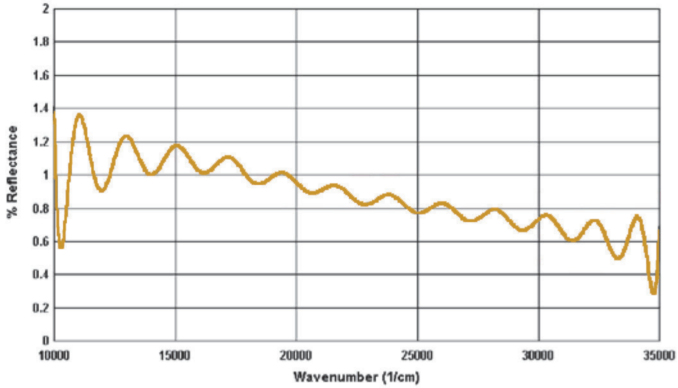


If the bandwidth were made very narrow, as in the case of a V-coating, there is no design limit on how low the R_{ave} could be made.

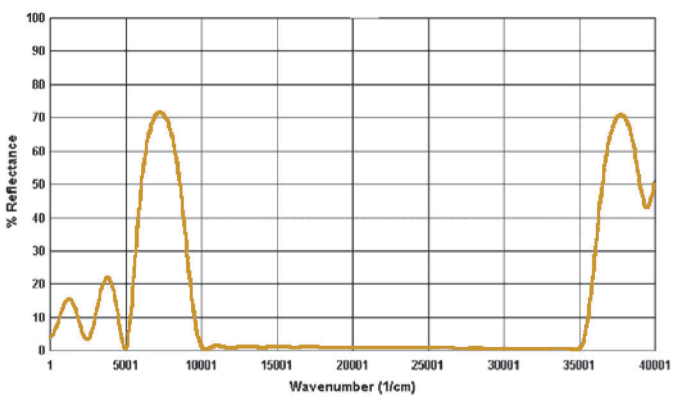
The above plot is on a linear wavelength scale; however, the wavenumber scale (linear in frequency) will primarily be used here in preference to a wavelength scale because the relations are more linear in frequency. It can be seen above that the ripples in the AR band are more compressed toward the short wavelengths, but they are found to be evenly spaced on a wavenumber scale.

Bandwidth Effects Background

The materials used in this section are confined to indices of 2.35 and 1.46 for a D of 0.89 and an L of 1.46. As an illustration, the percent reflection versus wavenumber of typical BBAR coating design is shown in first figure. This has an R_{ave} of 0.87195% from 10,000 to 35,000 cm^{-1} (1,000 to 285.7 nm). This, incidentally, might be scaled to cover 400–1400 nm or 440–1540 nm.

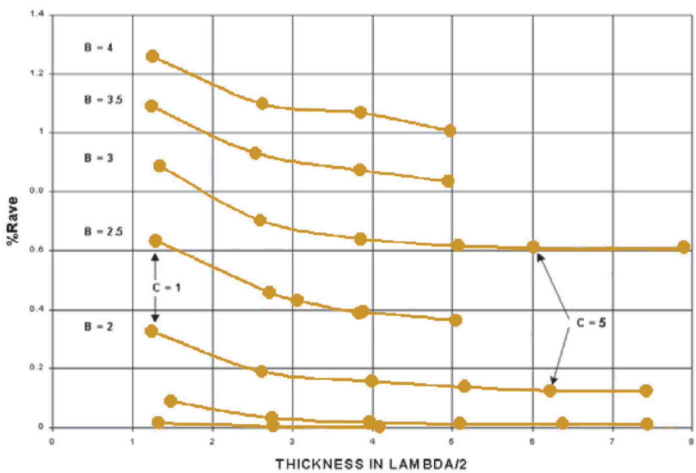


The bottom figure shows the same design over a broader spectral range from 1–40,001 cm^{-1}. It can be seen that there are high reflection bands on either side of the AR band. One might say that the reflection, which would have been in the AR band without the coating, has been redirected to the regions outside of the AR band.



BBAR Effects Background (cont.)

A fairly exhaustive set of designs were studied in search of "true" minima in R_{ave} over a broad range of B and overall thickness T. The quantization as a function of thickness seems to occur at about a half wavelength at the longest wavelength multiplied by 1.25. Henceforth, a thickness cycle C will be referred to as a half wavelength at the longest wavelength multiplied by 1.25. The figure shows the minima in R_{ave} found in the range of B from 1.25 to 4 and C from 1 to 7.



R_{ave} is a strong function of B and C up to C = 3.0, but is a much weaker function of C (T) after that. **Design of experiments** (DOE) statistical techniques were used to find a best fit of the data and equation for R_{ave} as a function of B and C in the range C = 0 to 3. It is not recommended to use C > 3.0 because that is beyond the point of diminishing returns for most cases. When L = 1.46 and H = 2.35, the equation that is recommended to predict $R_{ave}(B,C)$ in the region is:

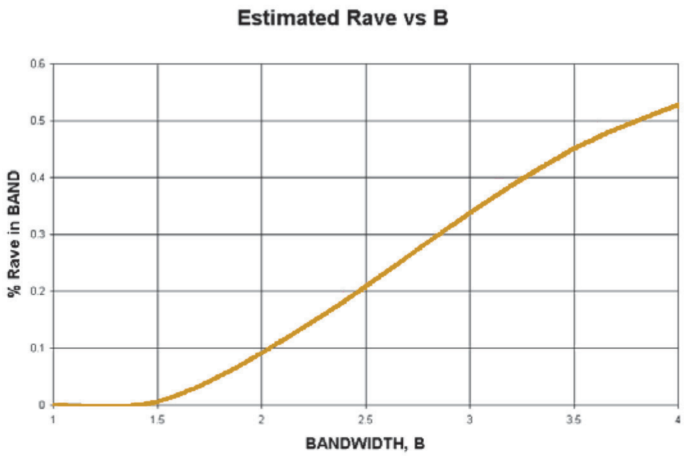
$$R_{ave} = 0.36729 - 0.68978\text{B} + 0.49717\text{BB} - 0.06116\text{BBB} - 0.10757\text{BC} + 0.01875\text{BCC}$$

Estimating the R_{ave} of a BBAR

The estimated best R_{ave} for a BBAR depends on the index of the last layer L, the difference in the index of the high and low layers in the stack D, the bandwidth B, and the cycles of thickness C. The bandwidth has been defined as the longest wavelength in the AR band divided by the shortest. C is defined as one half wavelength of optical thickness at the longest wavelength in the AR band (T) times a factor of 1.25. When the results of the foregoing pages are combined, the following formula results:

$$R_{ave} = (13.48/D)\times(1-L)^{3.5}\times[0.36729 - 0.68978B + 0.49717BB - 0.06116BBB - 0.10757BC + 0.01875BCC]$$

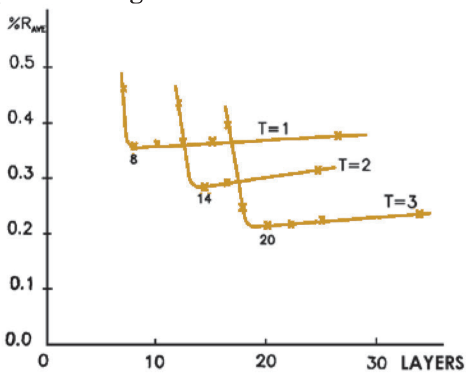
This does not lend itself to easy intuitive observations, but it can be easily evaluated in a computer spreadsheet or on a calculator. For example, when L = 1.38, D = 0.89 (2.35−1.46), and C = 3.0, the R_{ave} versus B is as plotted below.



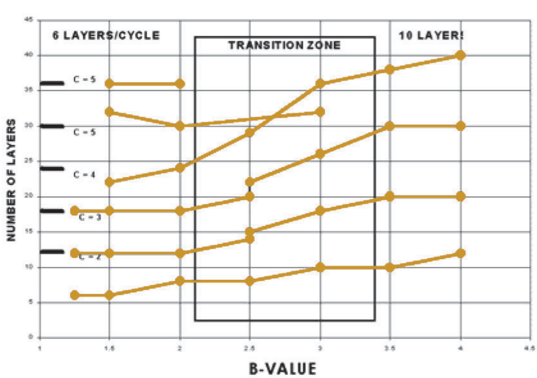
R_{ave} can be seen to be fairly linear with bandwidth over this range. In smaller bandwidths such as 1.0 to 2.5, the final R_{ave} result of a fabricated coating is likely to depend almost entirely on the control capability of the deposition process that produces the coating.

Estimating the Minimum Number of Layers in a BBAR

Earlier data showed that the minimum number of layers for an optimal AR was $6T + 2$. The figure below was part of the origin of that formula. It was found that decreasing the number of layers (and reoptimizing) slightly improved the results with each reduction. This is probably because each interface with a layer produces a reflection that must be dealt with; and therefore the fewer reflections, the better the opportunity to reduce reflections. At some number of layers, the best R_{ave} was found; fewer than that number could not produce as good a result.



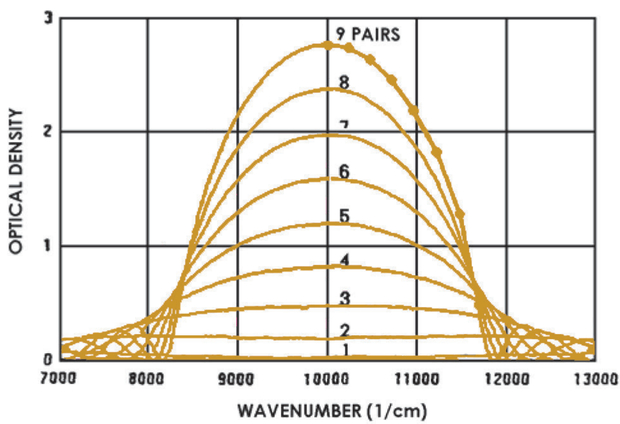
Recent studies have shown that the optimal number depends upon the B-value as seen below. Below $B = 2.0$, $6\times C$ seems to be the right number; above $B = 3.0$, $10\times C$ appears to be correct. For $B = 2.5-3.0$, this is $\sim 6T+2$.



Bandpass and Blocker Coatings

It was pointed out earlier that bandpass, LWP, and SWP filters can be made by properly positioning a QWOT stack or stacks to block or reflect the unwanted wavelengths. It is helpful when working with any of these designs to be able to estimate how many layers will be required to attain the desired reflection/blocking and how wide the blocked band will be. The optical density increases almost linearly with the number of layers in a QWOT stack. As discussed earlier, the width of the blocking band increases with the ratio of the indices of the high- and low-index materials in the stack. The relative width of the blocking band is less with higher orders of the reflection band QWOT wavelength. We can use all of these facts to estimate how many pairs of a given material combination will be required to achieve a given result.

The figure is an example with different numbers of layer pairs of index 2.3 and index 1.46 plotted in linear wavenumbers. This also illustrates how linearly the peak OD increases with additional layer pairs after the first few. Also note that the shape of the OD curve is approximately $\cos^{0.5}(\Pi\delta g/2\Delta g)$, where δg is the distance from the QWOT wavelength in g-units. Points calculated using this formula are plotted to the right.



Bandpass and Blocker Coatings (cont.)

In the previous figure, based on the integrated area for each layer pair (LP) under the curve from $-\Delta g$ to $+\Delta g$, the effective width is approximately $(\prod/2)\Delta g$. This times the ΔOD added per pair at the peak will give an estimate of the OD times bandwidth contribution of each pair. This can be thought of as the area that can be covered by a gallon of paint, when some area of wall needs to be painted.

The optical density at the maximum point of the QWOT stack is given in the equation below, where p is the number of LP in the stack, and n_H and n_L are the high and low indices of the stack. This will depend on whether the stack starts with a high- or low-index layer, but as long as p is more than a few pairs, it gives a good approximation of the average OD_P at the peak:

$$OD_p = 2 \log \frac{1}{2} \left[(n_H/n_L)^p + (n_L/n_H)^p \right] .$$

The change in the OD with the addition of each new LP is derived from the above equation and given below:

$$\Delta OD = 2 \log (n_H/n_L).$$

The Δg width was defined earlier as

$$\Delta g = (2/\Pi) \arcsin \left[(n_H - n_L)/(n_H + n_L) \right].$$

When all of the above is combined, the approximate **OD bandwidth product** (ODBWP) for each additional LP, *like a can of paint*, is

$$\text{ODBWP} = 2 \log (n_H/n_L) \arcsin \left[(n_H - n_L)/(n_H + n_L) \right].$$

Therefore, given the breadth of a spectral band to be covered at a given OD, then number of LP needed can be calculated by dividing by the ODBWP.

Mirror Estimating Example Using ODBWP

The problem is to estimate how many layers might be required to design a 99% reflector from 400 to 700 nm using indices of $H = 2.35$ and $L = 1.46$. Converting to wavenumbers gives 14,286 to 25,000 cm^{-1} with a central value of 19,643 cm^{-1} (509 nm). The total Δg for this works out to be 0.5454, and 99%R converts to an OD = 2.0. Therefore the ODBWP needed for the coating is 1.0908.

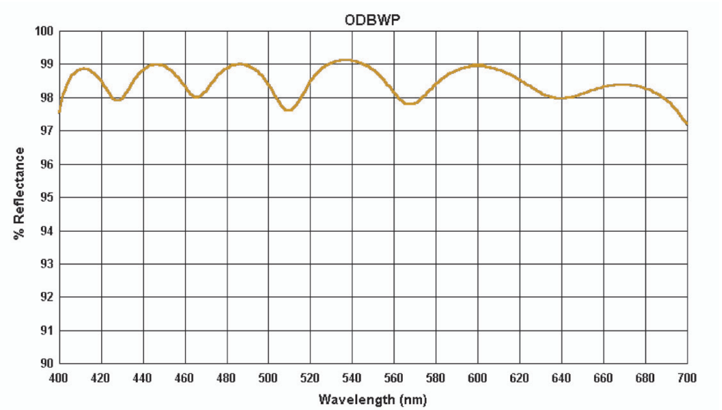
The effective bandwidth of a LP with these indices is

$$(\Pi/2)\ \Delta g = \arcsin\left[(n_H - n_L)/(n_H + n_L)\right] = 0.2358 \text{ radian}.$$

The reflectance (OD) contribution per LP is

$$\Delta OD = 2 \log (n_H/n_L) = 0.41343.$$

The ODBWP per LP is the product of these, or 0.09749; this is the can of paint of the analogy! Dividing this into 1.0908 gives the estimate that 11.19 LP are needed, or about 23 layers. Optimizing a stack of 23 layers with these indices with design targets of 99%R over 400 to 700 nm yields a result as seen below. The average %R is 98.44. This is close to the goal, but more layers would be needed to exceed 99% throughout the range. However, this has given a reasonable starting estimate, and it illustrates the process.



Estimating Edge Steepness in Bandpass Filters

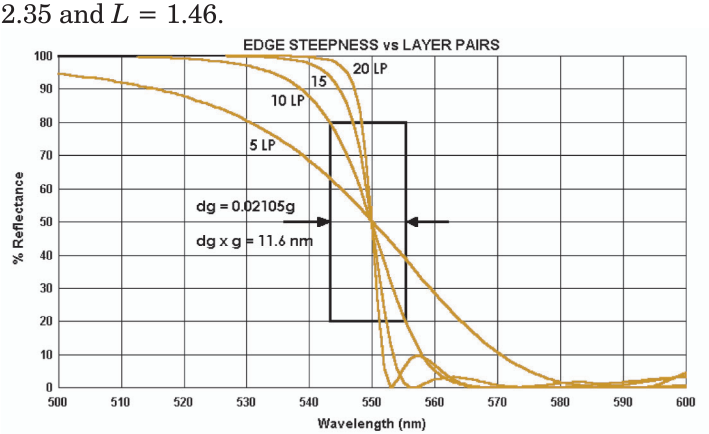
It is often of interest to know how many layers are needed to achieve a certain **edge slope** between the pass and block bands of an edge or bandpass filter (LWP or SWP). The steepness of the side of an edge filter is in inverse proportion to the number of LP. The spectral distance from the high to the low transmittance region is usually the important factor for the designer. This might be specified from 80% to 20% T (about .1 to .7 OD) or some other choice of limits. The spectral distance in *g*-units will be called *dg* and the peak density at the QWOT wavelength OD_P, the effect of steepness may be approximated by the equation below. The Δg and the OD_P are known from previous pages.

$$dg \approx \frac{1}{2}\left(\Delta g / OD_P{}^{1.74}\right)$$

or

$$dg \approx \frac{1}{2}\left(\begin{array}{l} \left\{ \arcsin\left[\left(n_H - n_L\right) / \left(n_H = n_L\right) \right] \right\} / \\ \left\{ \pi \log \frac{1}{2} \left[\left(nH / nL\right)^P + \left(nL / nH\right)^P \right] \right\} \end{array} \right)$$

This can best be calculated in a spreadsheet and visualized in the figure below. The calculated *dg* is 0.02105. When this is multiplied by the edge wavelength, it gives a spectral width of 11.6 nm for the range in which the reflectance drops from 80% to 20% for the 10-layer pair case, with H = 2.35 and L = 1.46.



Estimating Bandwidths of Narrow Bandpass Filters

It is practical to estimate the pass band width and blocking band width of a NBP filter as a function of **refractive indices**, **spacers**, **number of layer pairs**, and **number of cavities** before designing the filter. This can be a practical design guide as to which parameters should be used to gain a desired result. The three-cavity filter seen earlier is generally well suited to the typical 100-GHz DWDM requirements. A systematic investigation of the likely design space using design of experiments methodology (DOE) was performed. Four variables were considered: indices of the high- and low-index materials (2), the number of layer pairs in each mirror, and the number of half waves in the spacer layers. However, the indices were combined to form what was deemed to be two more meaningful variables: the difference in index ($n_H - n_L$), and the average index (($n_H + n_L$)/2). The extremes of the sampling ranges in the DOE were:

#Layer-Pairs	5 to 13
#Spacer-HWs	1 to 5
Index-Difference	.31 to .87
Average-Index	1.615 to 1.895

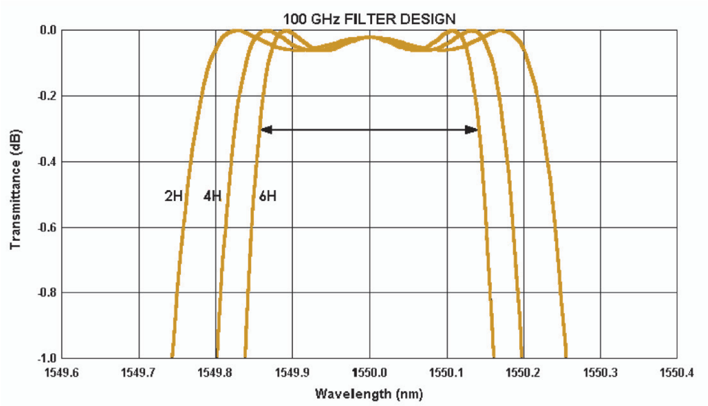
The most likely cases in DWDM are well within these ranges. Experimental values of 25 cases were used in the DOE. The dB bandwidths (BW) of both the pass band and blocking band were found to be a strong functions of the index difference from n_H to n_L, but the average index of n_H and n_L has little effect. It is therefore advisable to obtain the gross features of the design (rough BW and blocking) by index difference (which is usually fixed by other considerations) and the number of layer pairs in the mirrors. The fine details of the BW can then be adjusted with the number of half waves in the spacers. The average index is also usually fixed by other considerations. Therefore, the design is first set as close as possible to the required results with the number of layer pairs and then refined by the number of half waves in the spacers.

Estimating Bandwidths of NBP Filters (cont.)

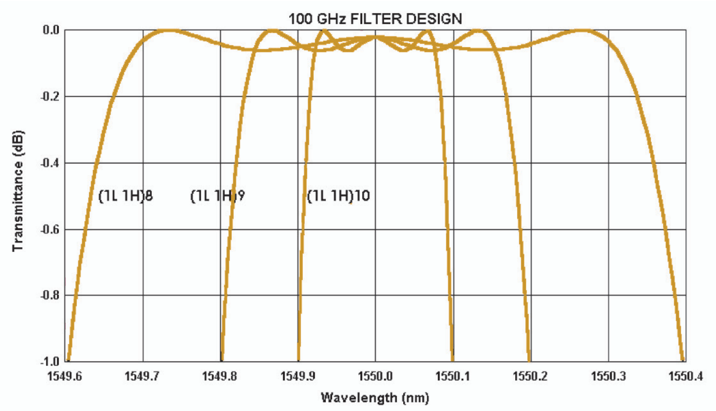
The nominal design of the NBP filter under discussion is:

(1H 1L)9 4H (1L 1H)9 1L (1H 1L)9 4H (1L 1H)9 1L (1H 1L)9 4H (1L 1H)8 1L .52072H .86628L

The three 4H spacers show this to be a three-cavity design; they are two HWOTs or four QWOTs. The effect of changing only the spacers to 2H and 6H is seen below.



The reflectors of the nominal design are (1H 1L)9 or 9 LP. It can be seen below that changing all these mirrors to 8 LP or 10 LP makes a great change in the BW of the filter. The spacers have a somewhat smaller effect.



Estimating Bandwidths of NBP Filters (cont.)

The table below shows the equation and constants that can be used to predict the bandwidth of this class of filter at the 0.3 dB level and also the 20.0 dB level. This is easily entered in a spreadsheet program for routine calculations in the design process. It could also be useful to find the difference between n_H and n_L from and actual design run. Since the index of n_L is likely to be close to 1.45–1.46, the index n_H can be determined to the accuracy that n_L is known.

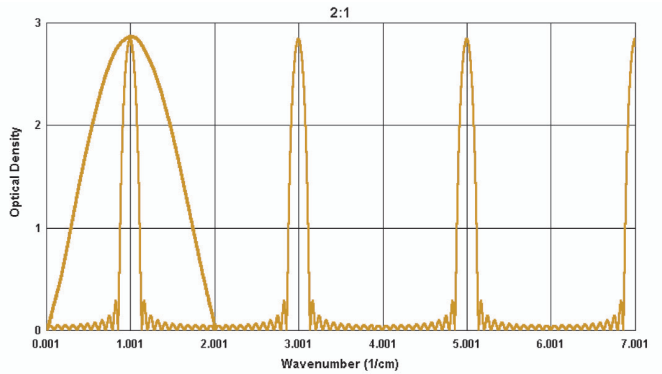
Bandwidth in Nanometers
= Const
+A * #Layer Pairs
+B * #Spacer Half-Waves
+C * Index Difference
+D * Average Index
+AC * #Layer Pairs * Index Difference
+AA * (#Layer Pairs)^2
+CC * (Index Difference)^2

	0.3 dB	**20 dB**
Const	40.42101	99.05472
A	-4.43899	-11.1974
B	-0.1527	-0.39881
C	-65.9173	-150.719
D	2.477614	5.069167
AC	2.696339	6.397634
AA	0.12923	0.339522
CC	28.63107	63.3527

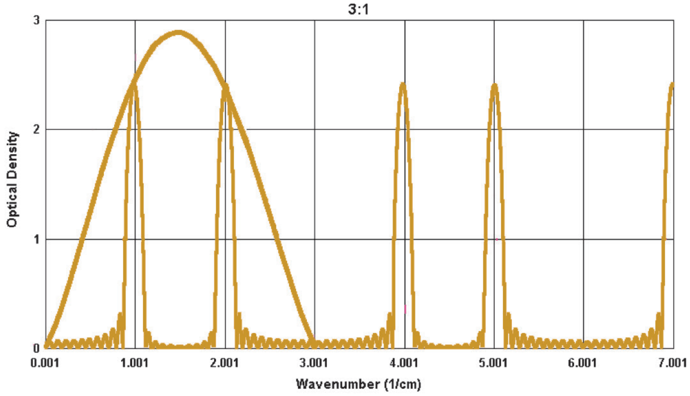
Blocking Bands at Higher Harmonics of a QWOT Stack

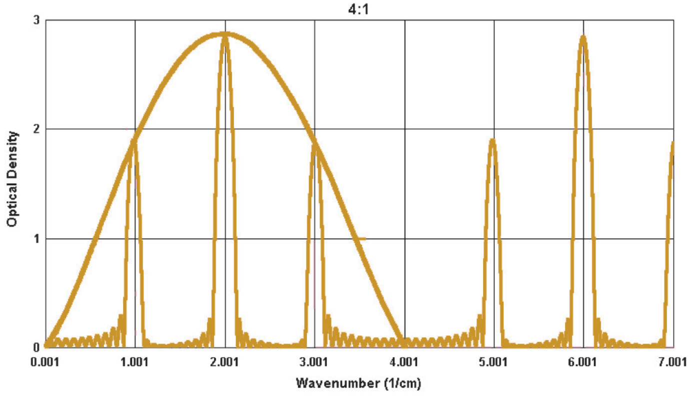
The reflectance or block band of a QWOT stack repeats at each odd multiple of the design wavenumber when the thicknesses for each of the high- and low-index materials are equal. This is seen in the figure below. The width of each of these block bands is the same in **Δg**. This then leads to the fact that the third harmonic frequency will have 1/3 the relative width at that *wavelength* as at the fundamental QWOT wavelength. Similarly, the fifth harmonic will be 1/5 as wide. This then allows the creation of narrower bands when needed, but it is at the expense of 3 or 5 times as thick a stack for a given block band wavelength. Using high and low materials whose indices are closer to each other can accomplish the same thing with thinner stacks if the appropriate materials are usable. The free (unblocked) spectral width between block bands also needs to be considered.

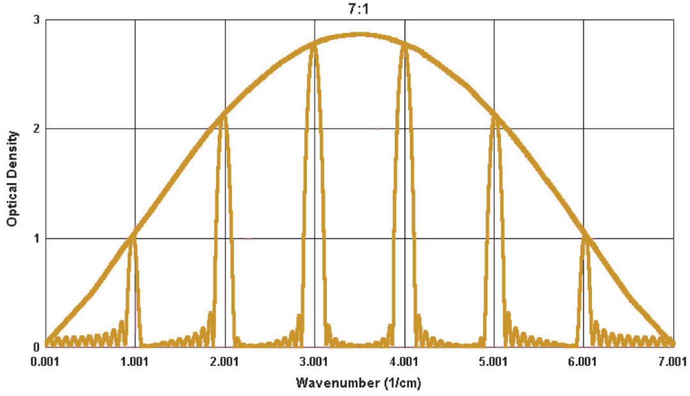
Different bands can be suppressed by using other than the unity ratio (2:1) between the thicknesses of the high- and low-index layers. For example, a 3:1 ratio between the **overall thickness** of the pair to the thinnest layer will add the second and fourth harmonics but suppress the third, sixth, etc., as seen in the figures on the next page. Other examples shown are a 4:1 ratio, which will add the second but not the fourth, etc.; and a 7:1 ratio, which will suppress the seventh harmonic and multiples of it.



Blocking Bands at Higher Harmonics of a QWOT Stack (cont.)

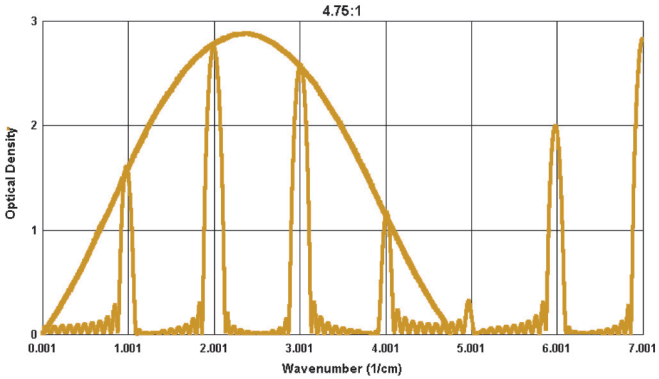






Blocking Bands at Higher Harmonics of a QWOT Stack (cont.)

It is interesting to note that any ratio can be used and will suppress a band at that frequency ratio, even if there is no band there! The figure illustrates a ratio of 4.75:1.



If the ratio is called A:1, it can be seen that the A[th] harmonic of g_0 has a zero value and that the peaks of the harmonics have an envelope that is approximately a sine function of g from 0 to ∏. An empirical fit to the data yields the approximate formula:

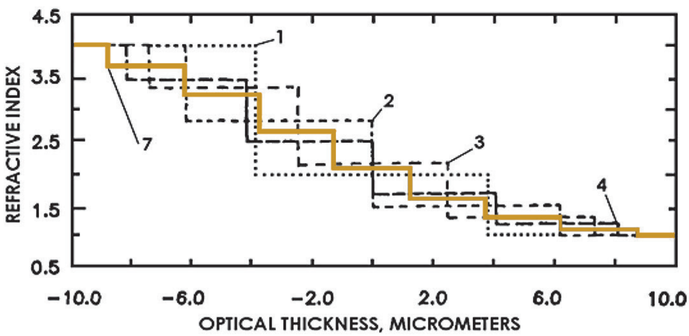
$$OD_N \approx OD_E \sin^{1.2}(\Pi N g / A); \quad N = 1, 2, \ldots$$

Here OD_N is the OD of the peak of the Nth harmonic block band, and OD_E is the OD of the peak achieved by an equal thickness pair stack. This curve is drawn on each of the foregoing figures for the first cycle and it would repeat at intervals of A. The formula allows the estimation of the OD of any given harmonic blocking band.

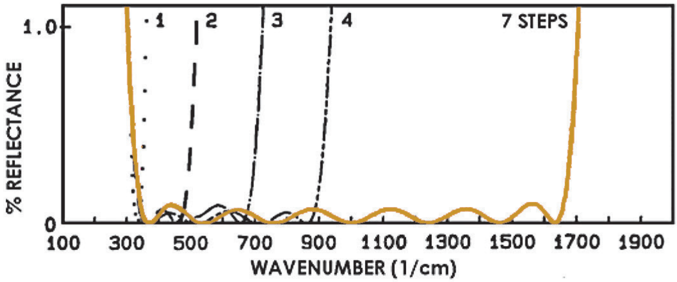
The higher harmonics in these figures result from the sharp "square wave" nature of the transitions from one layer index to the next. If the changes in index were smooth sine waves of the same period as the square waves, there would be no higher harmonics, only a single peak as at 1.001 wavenumber. This, then, would be a classic "rugate" filter used to block only the one line and pass "all" other wavelengths/frequencies.

"Step-Down" Index of Refraction AR Coatings

The merits of **step-down layers**—where the space between the substrate and the ambient medium is divided into approximately equal steps of decreasing index of refraction from the substrate to the medium—was discovered many years ago. The figure illustrates such a step-down layer from germanium to air (index 4.0 to 1.0) with 1, 2, 3, 4, and 7 steps.



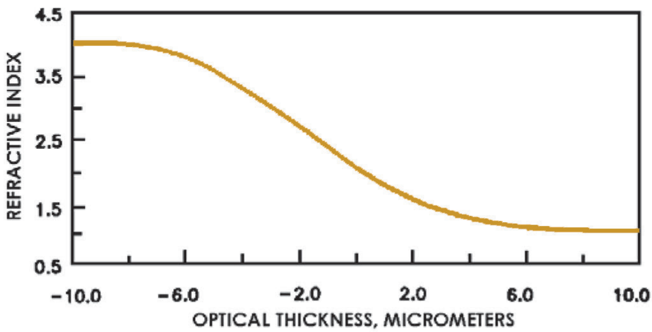
Note that the overall thicknesses have been adjusted to give the same long-wavelength or low-frequency limit at 300 1/cm (for 1% reflectance). Note also that the origin of zero thickness is near the center of this figure and in later plots. This causes the index profiles versus thickness to be more clearly seen as approximations of the "ideal" to be shown later. The next figure shows the performance of each profile. It can be observed that the AR band gets wider with an increasing number of steps as the short- wave limit (high-frequency) moves to the right.



"Step-Down" Index of Refraction AR Coatings (cont.)

In the limit of an infinite number of step-down layers, an index profile like the figure below will result. The spectral curve for such a **rugate** design will be very low reflection from the low wavenumber (long wavelength) limit to infinite frequency (zero wavelength). This, then, is the "ideal" index versus thickness profile, and the index is inhomogeneous as a function of thickness. This inhomogeneity is a property of a rugate coating.

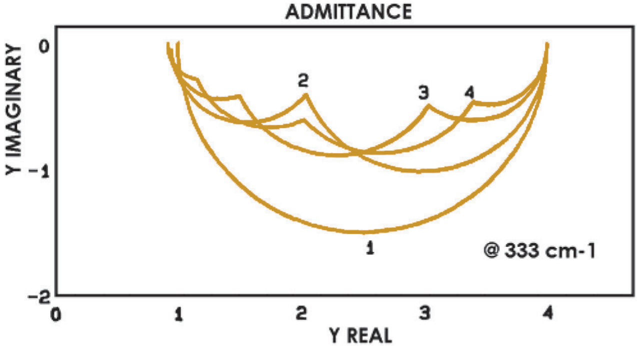
The performance of any of these "ideal" ARs is ultimately limited by the lowest real index of refraction available as has been mentioned earlier (p. 54). The current limits on the indices of real homogeneous materials is approximately the 1.38 of MgF_2. However, a new possibility has appeared, at least as a laboratory curiosity. The photolithography techniques have etched and produced "moth eye" surfaces which have a microscopic texture like "pyramids" that taper from contiguous solid bases to "sharp" points. This then approximates the change in effective index from the substrate to air/vacuum as seen in the figure.



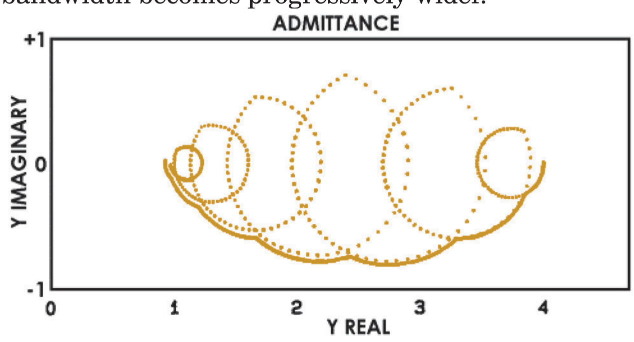
The index of 1.0 is approximated to the degree that the pyramids come to sharp points. It will also be noted that the ideal pyramids cannot be flat sided, but must curve more like a bullet shape in order to obtain the correct density as a function of thickness from the substrate to air or vacuum. All this does not lead directly to practical production coatings, but it can add insight.

"Step-Down" Index of Refraction AR Coatings (cont.)

The next figure shows the admittance diagrams resulting for the 1, 2, 3, and 4-layer versions at the long wavelength end of the AR band of the step-down. Note that the SLAR is just a semi-circle, and the other loci are made up of segments of a circle.

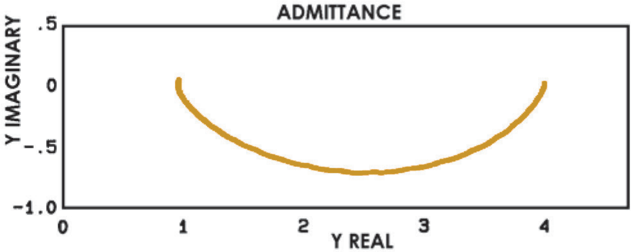


The following figure shows the admittance diagram of the seven layer at the longest *and* shortest wavelengths in the AR band. At the long-wave limit of the AR band, the admittance of the "ideal" AR appears to be of an approximately catenary form, which seems to osculate the admittance locus of the short-wave or high-frequency end of the AR band. The admittance locus of the short-wavelength end has a coiled spring appearance. As the number of layers or steps is increased, the loci become more smooth and the AR bandwidth becomes progressively wider.

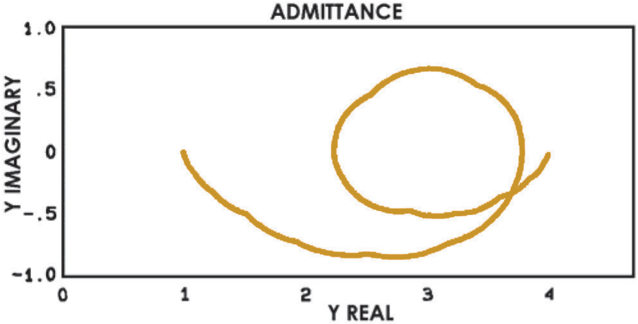


Too Much Overall Thickness in a Design

The next figure shows the admittance for a 24-layer step-down AR at the long-wave end of the band. It approximates the ideal admittance locus for a step-down AR with infinite bandwidth (beyond the long-wave limit).



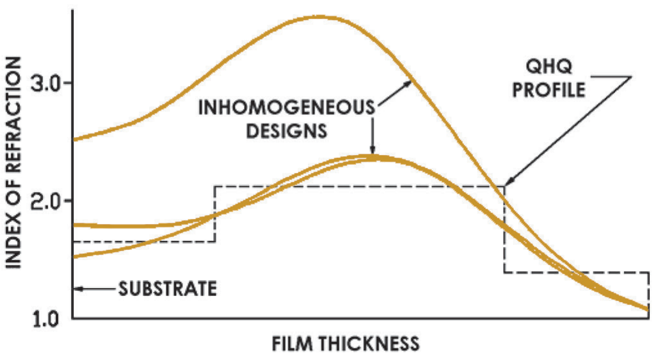
It has been shown that too little overall thickness will result in an inferior BBAR design. It will now be shown what can happen with too much thickness. The next figure shows a profile generated by these methods, which was intentionally started from a base profile that was approximately twice as thick as it needed to be, from the long wavelength limit as discussed above.



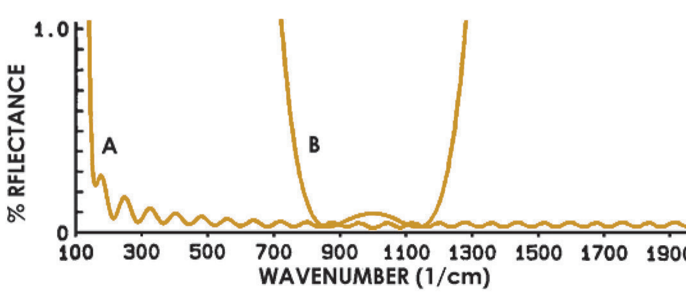
The difference between these two admittances is a loop that has "grown" on the "too-thick" profile's admittance locus. The optimization process had to deal with the excess material thickness in a way that would not disturb the AR performance. This appears to have been done by making a HWOT loop at the longest wavelength. This is wasted material, a benign growth.

Inhomogeneous Index of Refraction Designs

The classical QHQ has a QWOT of index 1.65, HWOT of 2.2, and a QWOT of 1.38 to provide a "step-up" before the step-down. This index profile is shown as a broken line in the next figure. A stack of very thin layers that totaled this same overall thickness was optimized for the index of each layer with respect to as broad an AR goal as possible. The resulting index profiles might be taken as approximations of smooth periodic variations of index with thickness as in the step-down case discussed. Three cases for different substrate indices are shown in the figure. They have similar shapes and fit the QHQ profile.

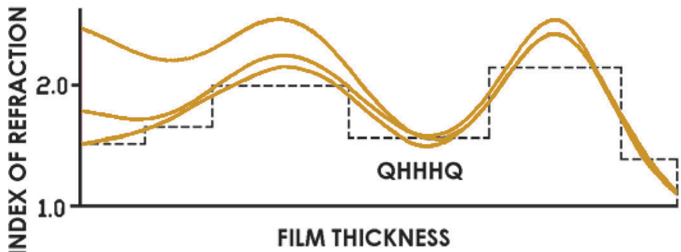


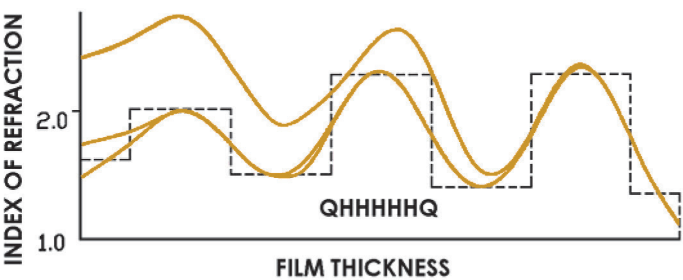
The B-curve in the figure below shows the limited (~1.5) bandwidth of the classic QHQ, homogeneous design. The A-curve shows the almost infinite bandwidth possible with the proper rugate or inhomogeneous index profile, if any indices were available down to 1.0.



Inhomogeneous Index of Refraction Designs (cont.)

It is possible to insert additional half waves of alternating high- and low-index layers between the quarter-wave start and end layers of the classical QHQ in order to improve the bandwidth and minimize the reflectance of broadband ARs. The indices of each layer can be optimized while holding the optical thicknesses constant. The index profiles of the QH^3Q and QH^5Q solutions are shown as broken lines in the next two figures. The inhomogeneous equivalent designs are shown in solid lines on the figures.





These three inhomogeneous designs have 1, 2, and 3 maxima in index profiles, and can be related to the parameter T (or C) of the estimating section as having 1, 2, and 3 units of overall thickness. They are all guides to the ideal AR coating design if any indices were available down to 1.0. In such a case, the simplest ($C = 1$) would be as good as the thicker coatings. This is because they are all semi-infinite in bandwidth. However, in the finite world of real $L > 1.0$, there is some advantage of the thicker coatings up to the point of diminishing returns.

Fourier Concepts

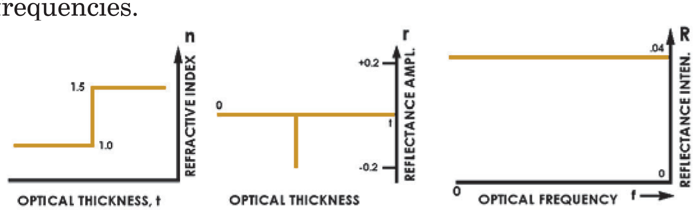
The state of the art at the present time in coating design is based on **analysis** and **adjustment**. It is possible to fully analyze what a specific configuration of layers and indices will give in terms of reflectance, transmittance, absorptance, polarization, etc. Small changes can then be made in the parameters to see if the results are in the direction of better or worse with respect to what is desired. A design can be optimized with respect to defined goals or targets by multiple iterations of these steps. This process is the backbone of the thin-film design process at this time. However, what is really desired is to be able to **synthesize** a design. This would be to directly put together a design that meets the requirements without iteration or optimization. The **Fourier technique** has the promise of being just such a synthesis process. However, as will be illustrated, there is still some work to be done before a true synthesis version is achieved. Because of the approxi-mations that must be used at present, a preliminary synthesis by the existing techniques must then be optimized for a final solution. These **approximations** and **limitations** will be described subsequently, and hopefully they may be overcome in the future. The real potential of the **synthesis technique** seems to be for cases where intuition and experience do not indicate how best to design for some complex reflectance profile. It would be highly desirable in these cases to have a tool to take the required reflectance profile and directly synthesize a solution.

The published works about Fourier techniques are heavy in the mathematical description. The mathematics do not paint a lucid picture of the principles for many people. Graphical illustrations are used here to lend some intuition and understanding of the techniques. Although mathematics may be the engine which propels the vehicle, one need not be an engine designer or automobile mechanic in order to drive a car to where one wants to go. This section is a "Driver's Ed" version of what makes the "car go."

Fourier Background

We reviewed the concepts of the Fresnel reflection amplitude coefficient early in this work. We will later show empirically that this is the "coin of the realm" in which we should deal when working in the Fourier domain. It is not magic, but "it *is* all done with mirrors." **Reflectance amplitude** is the key. This is the reflection caused by a discontinuity of index of refraction or admittance at an interface between two homogeneous media. In the case of inhomogeneous media with no discontinuities, the concepts still apply, but then we must deal with the **rate of change** of index or admittance. If there is no change, there is no reflection. For the rest of this discussion, we will assume homogeneous media and no absorptance or dispersion, for simplicity's sake.

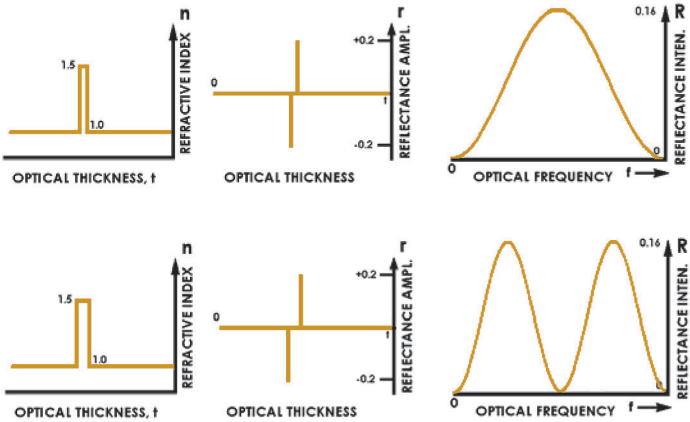
The left section figure shows the index of refraction versus position in space or optical thickness t in the direction of the propagation of the light. The center section of the figure shows the reflectance amplitude, $r = -0.2$ (in math: a Kronecker delta) versus the same position (t) that this discontinuity produces. The right section shows the reflectance intensity [$R = rr^* = 0.04$ (4%)] versus spectral frequency (f = 1/wavelength) that this produces. Note that the reflectance is the same for all frequencies since there is no dispersion. This is similar to a pure sharp electrical pulse, like lightning, that produces "white" noise at all frequencies.



What is seen here from left to right is the index of refraction profile versus thickness [$n(t)$], the reflection amplitude versus thickness $r(t)$ that is generated by that, and the Fourier transform of that to $R(f)$, that is $rr^*(f)$.

Fourier Background (cont.)

A thin slab of material of index 1.5 surrounded by an index of 1.0 will have interference between the reflections from the first and second interface surfaces. The next figure shows this index profile and r vs. t, plus the resulting first cycle of the R vs. f spectrum. This is the familiar result of a higher-index single-layer coating where the r at the first and last interfaces are of equal and opposite amplitudes, much like the soap bubble that was discussed. The r and R go to a maximum at a frequency for which the thickness of the slab is one QWOT and back to a minimum when the frequency is twice that (or t = HWOT). The pattern repeats at all integer multiples of this latter frequency. Note that the first minimum would occur at half the frequency if we doubled the thickness of the slab as illustrated in the bottom figure. Similarly, it would occur at $2f$ if t were cut in half, etc.

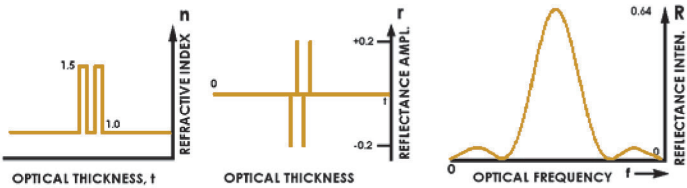


The right subfigures in this series are all the Fourier transforms (squared) of the center subfigures. That is to say, the reflectance amplitude versus optical thickness is Fourier transformed to the reflectance amplitude (and squared to give the R more commonly seen) versus optical frequency. A key point is that the Fourier transform is *reversible*. If the right data is transformed, it will give back the center data.

Fourier Examples

The foregoing implies that it should be possible to define what is wanted in a right curve and transform it to the reflectance vs. thickness that would produce that result (center curve). It is not yet quite that easy, but the state of the art is getting close. A few additional examples will be provided to give more of an intuition for the results.

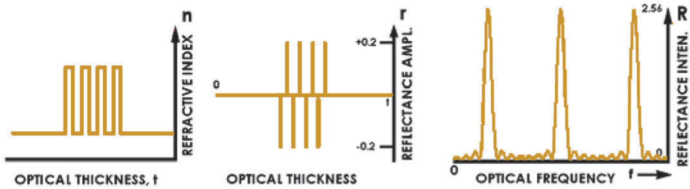
If two slabs are spaced apart by an optical thickness equal to their own, we get the results shown in the next figure.



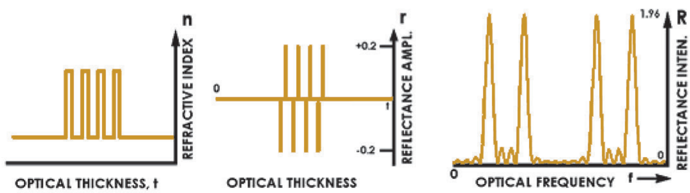
The interactions of the various reflections and phases add to produce the results shown. Note that the scale of R has quadrupled because the reflection amplitudes have added to give $2r$, which converts to $4R$. It may be further noted that there are three of the lowest-frequency components developed by the interferences between each of the three adjacent pairs that have the same frequency spectrum as the first figure, with only two interfaces but differing phase relations to each other. There will also be two interferences between the first and third interfaces and the second and fourth, which will have minima twice as often as the "fundamental" period. There is one more interference between the first and fourth interface, which will have a minimum three times as often as the fundamental. Keeping track of all these reflectance amplitudes, frequencies, and phases can quickly lead to confusion, as can be seen from this most simple three-layer example. However, the Fourier transform algorithms will take care of all of that. The point to remember is that the reflections from each surface interact with the reflections from each other surface through their amplitudes and relative phases.

Fourier Examples (cont.)

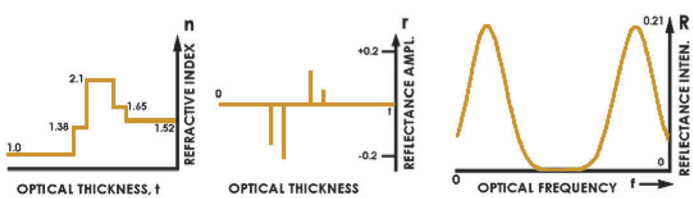
The next figure shows the effect of eight equally spaced reflections as would be found in a 7 QWOT stack. The pattern starts to look consistent with earlier figures of QWOT stacks.



The next figure shows a stack where the ratio of the thickness of a pair to the thickness of the first layer is 3:1; note the similarity to earlier 3:1 stacks. The figure above has peaks at the first, third, fifth, etc., multiple of the fundamental frequency. The 3:1 figure below has the first, second, fourth, fifth, etc., with the third, sixth, etc., multiples missing.

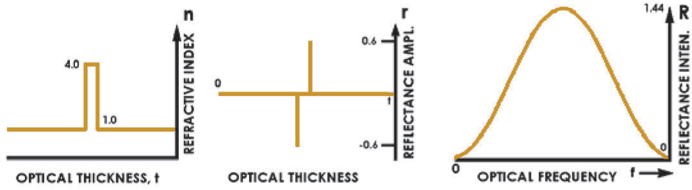


The following figure shows how the common three-layer broad-band AR would look in this scheme where the AR band actually lies between two higher reflection peaks. The regions outside the AR band are seldom examined, but can yield useful information.

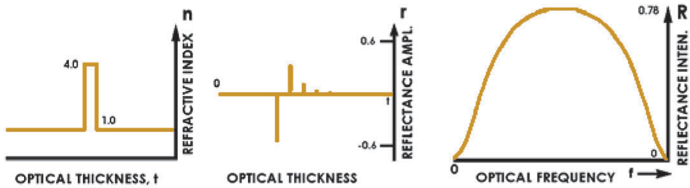


Fourier Limitations

Close examination of the two previous figures relating to 8 reflections stacks reveals a problem. The peaks in R are greater than the physical limit of 1.0. The problem is that the center subfigures shown have not taken the **multiple internal reflections** (MIR) into account. This can be seen more clearly in the comparison of next figure below without proper account of the MIR and its correction in figure below it. This simulates a thin slab of a very high index material such as germanium of index 4.0. The Fresnel reflection at each interface would be of magnitude 0.6.



The next figure shows the proper influence of the multiple reflections and has the correct shape and magnitude.

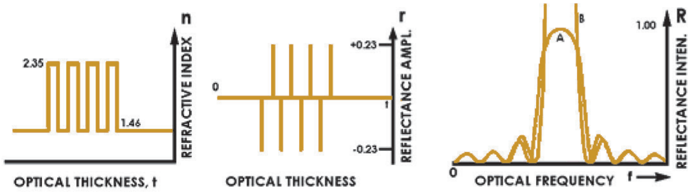


The reflection from the first interface is just as before. The first reflection from the second interface encounters the first surface on its way back, and only 0.4 of it is transmitted to interfere with the first surface reflection. The flux continues to reflect back and forth inside the slab, letting 0.4 of its remaining strength transmit through whichever interface it is reflecting from at that point. This results in a rapidly decaying series of reflectance pulses coming back in the original reflection direction. When the interface reflectances are low, this problem is not so obvious; but for high reflectances the problem of neglecting multiple reflections becomes severe.

Fourier Limitations (cont.)

Even in the cases of the 8 reflections shown on p. (81), the "excess R" problem of simple Fourier is apparent.

Most of the material published to date has been restricted to small reflections in non-absorbing and non-dispersive media. It is apparent from the above examples why the small reflections limitation is imposed. The technology to date does not incorporate the effects of multiple reflections; other than that, it works very nicely. As we have seen above, the shape of the results are distorted as well as the magnitudes. This is illustrated in the next figure where the true result *A* is compared with the distorted result *B* for a 7-layer QWOT stack.



The **Q-function** is the spectral description of the amplitude of Q versus frequency, which when transformed will yield the proper index of refraction versus thickness function to produce the spectrum. Various forms of the Q-function have been tried in an attempt to correct the distortion. The Q functions tried have been various combinations of *R* and *T* (transmittance intensity). As a result of our work, we concluded that the proper function is

$$Q(f) = [1 - T(f)]^{1/2} = [R(f)]^{1/2} = r(f) .$$

However, this does require that the multiple reflections are properly accounted for. It has not yet been worked out how this can be properly done, but it is hoped that someone with the proper mathematical background and insight will be able to do this soon. The approach here has been empirical and has not been proven with mathematical rigor, but the evidence is convincing.

Designing a New Coating

It is important at the outset to review the requirements and goals for a new coating design to be sure that nothing is overlooked. Then the materials might be selected that are expected to perform in the spectral range of interest and under the environmental rigors expected/required. The usable materials may also be limited by the processes and equipment available where the coating is to be produced. Where possible, the indices (both n and k) that the processes produce should be used, at least in the final design stages.

Simple AR coatings, such as V-coats and 3- or 4-layer BBARs, do not require much design work. The above choices of materials and later thickness adjustments to "tune-in" the best reflectance versus wavelength are all that is usually required. The choice between a 3-material BBAR with 3 layers versus a 2-material design with 4 layers is more of an equipment and production issue than a design issue.

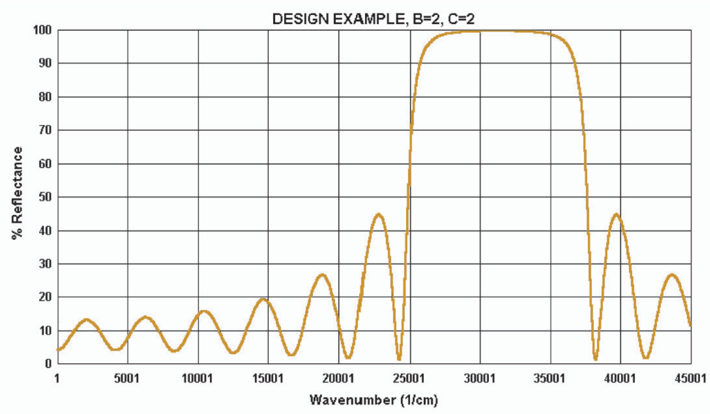
A **test run** is usually done with most coatings, and the spectral (and possibly physical/environmental) properties are measured. The spectral curve is often different from the design. Enough points measured on the resulting curve are entered back into the design program as targets, and the thicknesses reoptimized to best fit the results. If the fit is not satisfactory, the indices of the materials can be added as variables. Given the results of these optimizations, the actual thicknesses can be determined and the monitoring parameters adjusted to produce the thicknesses required to meet the requirements. Iterations of this test and measure process may be required, particularly with the more complex coatings. The greatest difficulty in this procedure often comes from processes and/or measurements that give variable results from run to run. Stability is key to results.

Designing BBAR Coatings

It was discussed under **Estimating** that after the choice of high, low, and last layer indices, the only variables left are the overall thickness of the coating, the number of layers, and the thicknesses of the individual layers. Once the decision is made as to the choice between $C = 1, 2,$ or 3 (where more than three is not recommended) for overall thickness, then the minimum number of layers needed is determined from the bandwidth requirement B.

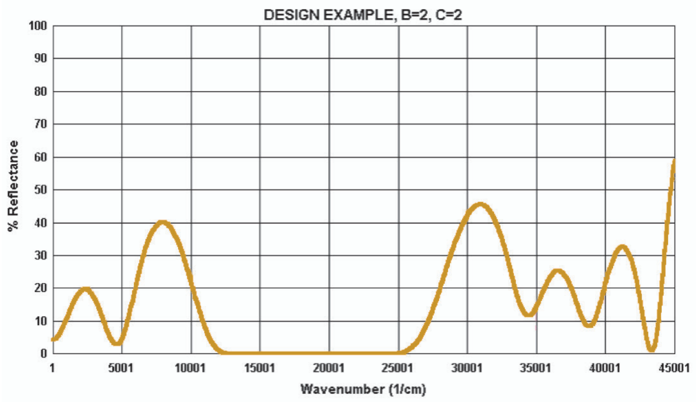
It was pointed out that a BBAR is no more nor less than the low-reflection region between higher-reflection regions (block bands). The recommended starting design would be (1H 1M)m 1H 1L, where 2m is greater than the number of layers expected in the final design. This gives some extra starting layers to be later eliminated. This stack might be QWOTs at 0.8 of the shortest wavelength in the AR band.

As an example, the design goal might be a BBAR over 400 to 800 nm (25,000 cm^{-1} to 12,500 cm^{-1}, $B = 2$), choosing $C = 2$ where the estimated R_{ave} would be 0.106% when $L = 1.38$ and 2.35 and 1.46 are used for the other layers. The expected minimum number of layers would be 12, so m would be 6. The starting design would be (1H 1M)6 1H 1L at 320 nm (14 layers). The starting spectrum would look like this figure.

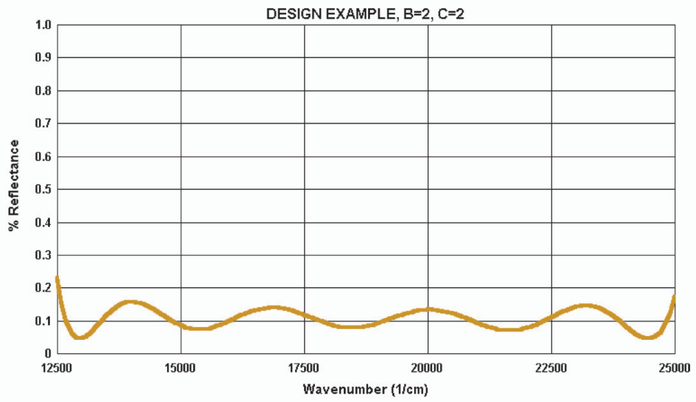


Designing BBAR Coatings (cont.)

After design optimization against 20–30 targets of 0%R from 400 to 800 nm, the optimum AR design is reached whose spectrum is plotted below. The reflection in the AR band is plotted next and has an R_{ave} of 0.107!

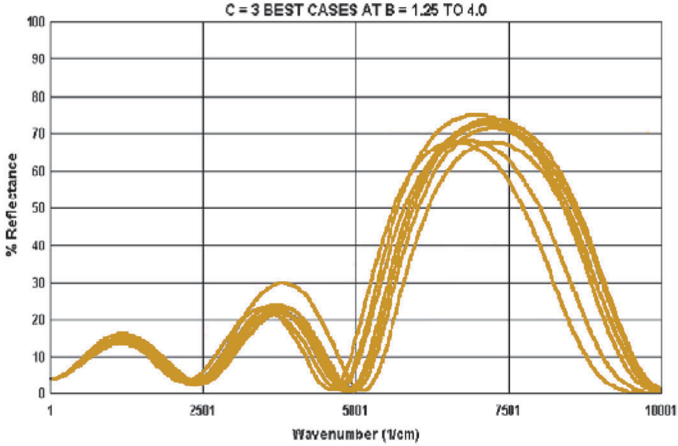


This design still has 14 layers, which might be reduced to 12 by further design effort, but is not expected to make a significant improvement since the result is already very close to the predicted minimum of 0.106%R. An attempt to do that might start with reoptimizing after eliminating the thinnest layer. If that did not give a better result, the next thinnest layer might be tried instead.



Tails in BBAR Coatings

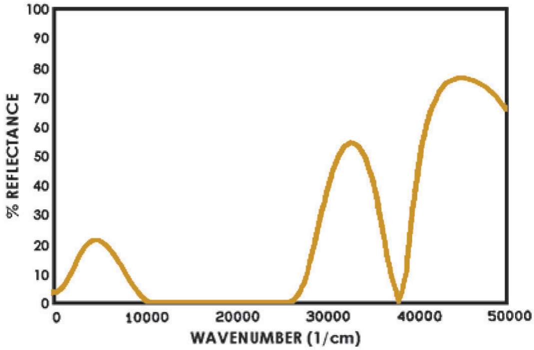
Note in the first figure on the previous page that there are two peaks to the left of the AR band, the long-wavelength or low-frequency side. These peaks have been found to be a reliable indicator of the overall thickness and quality of the AR design. In this case, $C = 2$ as planned. At some intermediate overall thickness, the shape of those peaks will be highly distorted. From previous work, the following figure shows many $C = 3$ cases where B ranged from 1.25 to 4. It can be seen that the reflectance spectra are all very similar. It has been observed that less-than-optimal designs can have significant distortions from this shape. For example, the peaks can tend to merge or distort if the overall thickness is smaller or larger than the ideal T for a given C; and the shape can be distorted if an optimum has not been reached in the design. It can be seen that one of the traces in the figure below is slightly out of line with the others, and this could be an indication of a potential for further improvement in that particular design.

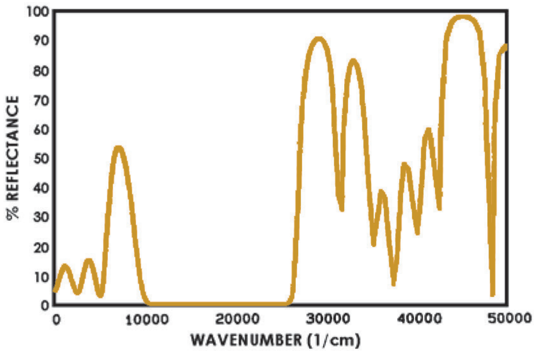


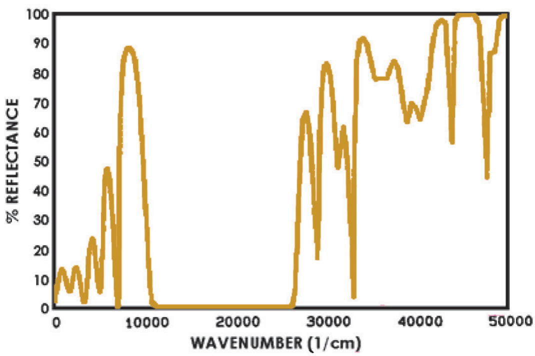
When the best AR designs from the Berlin design contest mentioned earlier were evaluated in this way, similar effects are seen. This is an interesting confirmation of the concept from totally independent sources.

Tails in BBAR Coatings (cont.)

The figures on this page are all for the best designs of various overall thicknesses from the Berlin contest, with *C* equal to 1, 3, and 5.







Designing Edge Filters, High Reflectors, Polarizing and Non-Polarizing Beamsplitters

Edge filters:

The design of edge filters (LWP or SWP) is very similar to the approach for BBARs. The requirements are usually for a certain edge steepness, a blocking band, and a pass band. A QWOT stack is chosen with enough layer pairs to provide the edge steepness and blocking band density required. The pass band (or AR) is then optimized by adjusting the thicknesses of a few matching layers on each side of the stack. One non-QWOT layer on each side is usually not enough, but more than 5 is usually unnecessary unless there is a very broad pass band.

High reflectors:

The design of high reflectors as discussed under ODBWP (p. 62) is primarily an issue of having enough layer pairs for the reflectance needed. When it comes to broad-band high reflectors that are wider than the $2\Delta g$ breadth of band provided by the material of choice, more attention is needed. When two or more stacks are positioned spectrally side by side, there are usually some frustrating holes in the high-reflection band due to interactions between the stacks. There essentially needs to be a few matching layers between the stacks. Another approach is to have the thickness of each layer decrease (or increase) in a smooth progression from that needed to be a QWOT at the longest wavelength to that needed at the shortest. One choice for a 20-layer stack, where B is 2, might be to decrease each layer in turn by dividing its thickness by the 20^{th} root of 2.

Polarizing and Non-Polarizing Beamsplitters:

The design of polarizing beamsplitters has been discussed (p. 40). Again it is a case of choosing a QWOT stack and optimizing the matching layers for the low reflection (AR) of the *p*-polarization. The design of non-polarizing beamsplitters has been covered in the section by that title (pp. 43–46). That is an issue mostly of the narrow range of angles and reflectances that can be made non-polarizing.

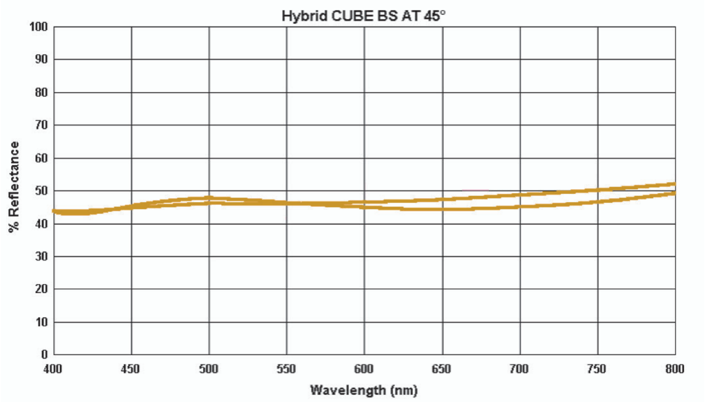
Designing Beamsplitters in General

Beamsplitters are almost always at some angle to the incident flux. This angle causes all of the polarizing effects discussed earlier. These must be kept in mind for any beamsplitter application to avoid surprises.

Another class of beamsplitters of some interest would be represented by a **metal-dielectric** or **hybrid** design with a silver layer surrounded by a few dielectric layers which comes as close as practical to 50/50% at 45° with very little polarization effect. Such a design at 550 nm is

.16644H .55746L .69196H 10A 20.875M 10A .87435H .36514L .2491H ,

where *H* and *L* are TiO_2 and SiO_2, *M* is silver, and *A* is 10 nm of Al_2O_3 as a “glue” layer. This is immersed in glass of index 1.52. The figure shows the reflectance in *s* and *p* at 45°, and the transmittance is similar. The total losses can be less than 10%A. It is about a 45/45% beamsplitter.



All-dielectric beamsplitters can be designed to have 50/50 average for random polarization over such a region with no losses, but the *s* and *p* are widely separated by many percent. Beamsplitters can be designed at any average reflection from 0 to 100% if polarization is not an issue.

Designing to a Spectral Shape & Computer Optimization

There are an infinite variety of spectral shapes that can be designed to transmit or reflect a spectral shape that will provide such things as color correction for a light source, gain flattening for a laser pump, filters for tristimulus color measurement, etc. A few things that might be kept in mind for such design opportunities are: 1) the finer the detail in the spectral shape, the thicker the coating must be (from **Fourier Concepts**), 2) the number of layer pairs will have to be adequate to "paint" the reflectance area required, 3) the optimization targets need to be more numerous and closely spaced than the fine detail of the spectral shape. The rest is usually up to the skillful use of an optimization program, which often means "asking it the right question" (targets).

The use of **optimization** has been mentioned many times. One can generally think of an "optimizer" as a **black box** which takes for its input a starting design, a set of parameters which may be varied, and a set of performance goals, weightings, and constraints. The desired output is a design that is the best fit to the weighted performance goals that can be achieved with the variables and within the constraints given. Optimizers usually deal with a **merit function** that is actually a *demerit* function that is reduced to a minimum by the process. The "merit" function might be computed as the sum of the squares of the difference between the targets and the actual spectrum. The program finds the effect of each variable on the merit function and predicts where the best design would be in variable space. This process iterates for a given number of times or until the goals are met. Such "black boxes" are provided as software packages for personal computers by various organizations. The starting design, such as might have been obtained through the methods discussed above, would be represented by the index of refraction and the thickness of a given number of layers plus the substrate and media indices. Some of the other factors and consequences will be discussed next.

Performance Goals and Weightings

If the goal is to design a four-layer AR for the visible spectrum, one might specify the performance goals as 0.0% reflectance at 50 nm intervals from 400 to 700 nm. This should lead to a good result. The **weighting** on each goal or target value is often determined by the reciprocal of the tolerance on that target. For example, the above targets of 0.0% for wavelengths of 500, 550, and 600 nm might have tolerances of 0.5%, while 450 and 650 nm have 1.0%, and 400 and 700 nm have 2.0%. This would give an approximation of weighting for the visual or photopic response of the eye putting the most "pressure" on wavelengths centered near 555 nm. The same design might also result from leaving out the 400- and 700-nm targets entirely. When computers were slower and more expensive, keeping the number of targets smaller helped speed up and reduce the costs of the optimization process.

The situation changes when a broader-band AR is to be designed. A given design might be optimized from 420 to 1100 nm. If only seven targets were used, a problem would be encountered. The "optimizer" would tend to move the reflection toward 0.0% as much as it could at the targets (only), but the spaces in between the targets would be free to move to much higher values of reflectance. It is advisable to have at least twice as many targets across the band of interest as there are cycles in the ripple pattern, and four times is a better choice. The ripple patterns are more evenly spaced on a wavenumber (cm^{-1}) scale; therefore, it is better to have the targets equally spaced in frequency also.

The **art of designing**, after the indices and number of layers have been determined (which could be aided by the concepts discussed previously), involves the choices of the magnitude and position of the targets and their weightings in order to achieve the desired results. It is sometimes found that the original choices need to be modified to apply more or less pressure on some target points to achieve the best balance.

Constraints

There are sometimes occasions when it is desirable to constrain certain aspects of a design such as minimum and maximum layer thicknesses. Some software packages allow great flexibility in the definition of **constraints**. This is done by bringing design variables and results into a spreadsheet where they can be manipulated and fed back to the optimization part of the program. This approach was recently used in an investigation of which classes (if any) of AR designs might be improved by design techniques which considered the effects of **probable production errors**. The influence was also reported of different error distribution assumptions, such as random errors uniformly distributed within a tolerance range, worst-case error distributions, and various sensitivities to errors that might realistically represent those in actual practice. No class of coatings was discovered to gain a significant benefit from the optimization with respect to the expected **random errors** of production. It was not surprising to find that normal designs result in each layer thickness lying at a minimum of the (de)merit function, wherein a movement from the nominal design thickness in any direction will increase the (de)merit. Since this is also the point of the zero first derivative of the (de) merit with respect to thickness, the sensitivity to small errors is at a minimum.

Global vs. Local Minima

We have seen that there are two solutions to the typical V-coating problem. The optimizer will usually converge on the solution closest to the starting design, or the **local minimum**. The other solution in this case might actually be better than the first, depending on the targets and weightings, but the typical optimizer might not find it. The best of all possible solutions is the **global minimum**. One can find discussions of global optimizers in the literature, but they of course must systematically examine the whole universe of parameter space in enough detail to not miss the global minimum. The analogy would be that a search for the lowest point starting at Salt Lake City would find it to be Salt Lake. A search starting at Barstow, California, should find Death Valley, which is lower. However, if one started at Jerusalem, they might find the true global minimum at the Dead Sea. Therefore, it is well to be alert to the possibility of local minima when using automatic optimizers. This is where some knowledge and understanding can be helpful in evaluating the results of an optimization. It is like the fact that we can know that the Dead Sea is the lowest point on the globe from the collective experience of others, even if we have not been there yet ourselves.

Some Optimizing Concepts

There are a variety of concepts for **optimizers** that have been used for decades, and new ones are being developed all of the time. Some of the most commonly used concepts will be described, but the "simplex" or parabolic approximation and other methods which have appeared over the years in some applications such as lens design software will not be discussed.

Optimization is like letting a ball slide into a bathtub and roll until it rests on/in the drain hole. Its progress will not generally be in a straight line to the drain.

Damped Least Squares Optimization

There are many descriptions in the literature that describe the concept of least squares and **damped least squares** (DLS) optimization. The derivative of the merit function with respect to each variable is calculated. These are used to compute the solution including each variable that would minimize the merit function. The solution is not likely to be the actual minimum because the merit function is not apt to be linear in all of the variables. However, this should move the design in the direction of the minimum (local). There is a risk, particularly as the actual minimum is approached, that the predicted change will overshoot the actual minimum. Therefore, damping is added so that the step taken is shortened to avoid **overshooting**, if that has been detected. Conversely, the step may be lengthened if the movement continues in the same direction without passing over a minimum. This all done automatically in the software.

Needle Optimization

The key idea of this method is to introduce a needle-like variation in the refractive index somewhere in the coating design in such a way that it will optimally decrease the value of the merit function. Once this variation has been inserted into the existing design, the resulting assembly of layers is refined to further decrease the value of the merit function by adjusting the thickness of all the layers. This process may be successively iterated until the introduction of a thin layer of material no longer effects a decrease in the merit function. The influence of a very thin layer on the merit function is evaluated through the whole thickness of the design ("scanned" through it). The point of greatest beneficial effect is then chosen to insert the thin layer and then reoptimize the design.

Flip-Flop Optimization

Southwell introduced a simple but effective optimization scheme, which he called **flip-flop optimization**. It stems from the concepts that we discussed earlier concerning index of refraction approximations and also inhomogeneous index functions. There is generally an advantage to using the highest and lowest indices practical in a design. Since we have seen that we can approximate any index in between the highest and lowest, there is usually little justification for using more than two indices and the attendant complications.

Southwell's proposed synthesis (design) algorithm is as follows:

(1) Select a total physical thickness for the coating. Divide this thickness into thin layers of equal thickness.

(2) Assign some initial index, either high or low, to each layer. Usually the convergence of iterative solutions depends on starting values, so this step may be important. Here are four suggestions:

- Start with all high-index layers.
- Start with all low-index layers.
- Start with alternating high- and low-index layers.
- Start from some known approximate solution.

The first three require no knowledge of thin-film theory, and the fourth attempts to utilize such experience.

(3) Evaluate a merit function based on the desired spectral response. One example is the **least-squares sum**: square the difference between calculated reflectivity and the desired reflectivity at various wavelengths across the band of interest and add them together. Use the **characteristic matrix theory** to evaluate the calculated response.

(4) Change the state of each layer (from low to high index or from high to low) one at a time and reevaluate the merit function. If the performance is better in the flipped state, retain the change; otherwise restore it.

(5) If, after testing all the layers (a single pass), the merit function has improved, go to (4) for another pass; otherwise end.

Equation Summary

Index of refraction n:

$$n \equiv c/v, \quad v = c/n \quad c = 2.99792458 \times 10^8 \, \text{m/s},$$

Wavelength λ and **frequency** ν :

$$\lambda = v/\nu, \quad \text{in vacuum: } \lambda = c/\nu$$

Optical path difference:

$$\text{OPD} = nd,$$

Snell's law of refraction:

$$n_1 \sin \theta_1 = n_2 \sin \theta_{2.}$$

Law of reflection:

$$\theta_1 = -\theta_2$$

Critical angle:

$$\sin \theta_C = n_2/n_1 .$$

Fresnel reflection equation:

$$r = (n_1 - n_2)/(n_1 + n_2).$$

When **absorption** is included:

$$N_1 = n_1 - ik_1,$$

Effective indices:

$$N_S = n \times \cos\theta \text{ and } N_P = n / \cos\theta ;$$

$$r_S = (N_{1S} - N_{2S})/(N_{1S} + N_{2S})$$

$$r_P = (N_{1P} - N_{2P})/(N_{1P} + N_{2P}).$$

Equation Summary

Reflectance intensity:

$$R = rr^*$$

Fresnel reflection:

$r = (r_{1\,+}\, r_2 e^{-i\varphi})/(1 + r_1 r_2 e^{-i\varphi})$ where $e^{-i\varphi} = \cos\varphi - i\sin\varphi$

Relative volts/meter:

$$E = 27.46 / [\mathrm{Re}(Y)]^{0.5}$$

Total flux:

$$R + T + A = 1$$

g-value:

$$g = \lambda_0/\lambda \text{ or } g = \sigma/\sigma_0.$$

Half width of the high reflectance zone:

$$\Delta g \approx (2/\Pi)\arcsin\left[(\mathrm{n_H} - \mathrm{n_L})/(\mathrm{n_H} + \mathrm{n_L})\right].$$

Optical density:

$$OD = \log_{10}(1/T)$$

Decibels (dB):

$$\mathrm{dB} = -10\log_{10}(1/T) = -10\,OD.$$

Factor to make thickness QWOT matched at angle θ:

$$\text{Factor} = \cos\left\{\arcsin\left[(n/n')\sin\theta\right]\right\}/\left\{1 - \left[(n/n')\sin\theta\right]^2\right\}$$

Estimated AR reflectance:

$$R_{\mathrm{ave}} = (13.48/\mathrm{D})\times(1-\mathrm{L})^{3.5}\times[0.36729 - 0.68978\mathrm{B} + 0.49717\mathrm{BB} - 0.06116\mathrm{BBB} - 0.10757\mathrm{BC} + 0.01875\mathrm{BCC}]$$

where B = bandwidth, L = index of refraction of the last layer, T = overall optical thickness of the coating, and D = the difference between the highest and lowest indices used (except for the last layer).

Equation Summary

Average OD_p at the peak:

$$OD_p = 2 \log \frac{1}{2} \left[(n_H/n_L)^p + (n_L/n_H)^p \right] .$$

Change in the OD with the addition of each new LP:

$$\Delta OD = 2 \log (n_H/n_L)$$

Δg width:

$$\Delta g = (2/\Pi) \arcsin \left[(n_H - n_L)/(n_H + n_L) \right].$$

OD bandwidth product:

$$\mathrm{ODBWP} = 2 \log (n_H/n_L) \arcsin \left[(n_H - n_L)/(n_H + n_L) \right].$$

Edge slope:

$$dg \approx \frac{1}{2} (\Delta g/OD_P{}^{1.74})$$

or

$$dg \approx \frac{1}{2} \left(\begin{array}{l} \left\{ \arcsin \left[(n_H - n_L)/(n_H = n_L) \right] \right\} / \\ \left\{ \pi \log \frac{1}{2} \left[(nH/nL)^P + (nL/nH)^P \right] \right\} \end{array} \right)$$

Peaks of the harmonics:

$$OD_N \approx OD_E \sin^{1.2} (\Pi N g / A); \quad N = 1, 2, \ldots$$

Q-function:

$$Q(f) = [1 - T(f)]^{1/2} = [R(f)]^{1/2} = r(f) .$$

Equation Summary

Estimating bandwidths of NBP filters:

Bandwidth in Nanometers
= Const
+A * #Layer Pairs
+B * #Spacer Half-Waves

+C * Index Difference
+D * Average Index
+AC * #Layer Pairs * Index Difference
+AA * (#Layer Pairs)^2
+CC * (Index Difference)^2

	0.3 dB	20 dB
Const	40.42101	99.05472
A	-4.43899	-11.1974
B	-0.1527	-0.39881
C	-65.9173	-150.719
D	2.477614	5.069167
AC	2.696339	6.397634
AA	0.12923	0.339522
CC	28.63107	63.3527

Bibliography

Baumeister, P.W., *Optical Coating Technology*, SPIE Press, Bellingham, WA, 2004.

Baumeister, P.W., *Military Standardization Handbook, Optical Design,* MIL-HDBK- 141, Defense Supply Agency, Washington, D.C., 1962.

Furman, Sh.A. and A.V. Tikhonravov, *Optics of Multilayer Systems,* Éditions Frontières, Gif-sur-Yvette Cedex, France, 1992.

Liddell, H.M., *Computer-aided Techniques for the Design of Multilayer Filters* Adam Hilger Ltd, Bristol, 1981.

Macleod, H.A., *Thin Film Optical Filters,* 3rd Ed. Institute of Physics Publishing, London, 2001.

Palik, E.D., *Handbook of Optical Constants of Solids*, Academic Press, Orlando, FL, 1985.

Palik, E.D., *Handbook of Optical Constants of Solids II*, Academic Press, Orlando, FL, 1991.

Schmidt, S.R. and R.G. Launsby, *Understanding Industrial Designed Experiments*, Air Academy Press, Colorado Springs, CO, 1994.

Thelen, A., *Design of Optical Interference Coatings,* McGraw-Hill, New York, 1988.

Willey, R.R., *Practical Design and Production of Optical Thin Films*, Marcel Dekker, New York, 2002.

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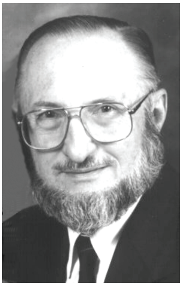

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