

Integrated Optomechanical Analysis

SECOND EDITION

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**Keith B. Doyle
Victor L. Genberg
Gregory J. Michels**

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CONTENTS

Introduction / xv

<Chapter 1>

Introduction to Mechanical Analysis Using Finite Elements / 1

- 1.1 Integrated Optomechanical Analysis Issues / 1
 - 1.1.1 Integration issues / 1
 - 1.1.2 Example: orbiting telescope / 1
 - 1.1.3 Example: lens barrel / 3
- 1.2 Elasticity Review / 4
 - 1.2.1 Three-dimensional elasticity / 4
 - 1.2.2 Two-dimensional plane stress / 6
 - 1.2.3 Two-dimensional plane strain / 8
 - 1.2.4 Principal stress and equivalent stress / 9
- 1.3 Material Properties / 10
 - 1.3.1 Overview / 10
 - 1.3.2 Figures of Merit / 11
 - 1.3.3 Discussion of materials / 14
 - 1.3.4 Common telescope materials / 16
- 1.4 Basics of Finite Element Analysis / 16
 - 1.4.1 Finite element theory / 16
 - 1.4.2 Element performance / 18
 - 1.4.3 Structural analysis equations / 21
 - 1.4.4 Thermal analysis with finite elements / 22
 - 1.4.5 Thermal analysis equations / 23
- 1.5 Symmetry in FE Models / 24
 - 1.5.1 General loads / 24
 - 1.5.2 Symmetric loads / 24
 - 1.5.3 Modeling techniques / 27
 - 1.5.4 Axisymmetry / 28
 - 1.5.5 Symmetry: pros and cons / 28
- 1.6 Model Checkout / 28
- 1.7 Summary / 30
- References / 30
- Appendix
 - A.1 RMS / 31
 - A.2 Peak-to-Valley / 31
 - A.3 Orthogonality / 31
 - A.4 RSS / 32
 - A.5 Coordinate transformation for vectors / 33
 - A.6 Coordinate transformation for stresses or materials / 33
 - A.7 Factor of safety, margin of safety, model uncertainty / 34

<Chapter 2>**Introduction to Optics for Mechanical Engineers / 37**

- 2.1 Electromagnetic Basics / 37
- 2.2 Polarization / 38
- 2.3 Rays, Wavefronts, and Wavefront Error / 40
- 2.4 Pointing Error / 41
- 2.5 Optical Aberrations / 42
- 2.6 Image Quality and Optical Performance / 44
 - 2.6.1 Diffraction / 45
 - 2.6.2 Measures of image blur / 45
 - 2.6.2.1 Spot diagram / 46
 - 2.6.2.2 Point spread function and Strehl ratio / 46
 - 2.6.2.3 Encircled energy function / 47
 - 2.6.3 Optical resolution / 47
 - 2.6.4 Modulation transfer function / 48
- 2.7 Image Formation / 50
 - 2.7.1 Spatial domain / 51
 - 2.7.2 Frequency domain / 51
- 2.8 Imaging System Fundamentals / 54
- 2.9 Conic Surfaces / 55
- 2.10 Optical Design Forms / 56
- 2.11 Interferometry and Optical Testing / 57
- 2.12 Mechanical Obscurations / 57
 - 2.12.1 Obscuration periphery, area, and encircled energy / 58
 - 2.12.2 Diffraction effects for various spider configurations / 59
 - 2.12.3 Diffraction spikes / 59
- 2.13 Optical-System Error Budgets / 60
- References / 61

<Chapter 3>**Zernike and Other Useful Polynomials / 63**

- 3.1 Zernike Polynomials / 63
 - 3.1.1 Mathematical description / 63
 - 3.1.2 Individual Zernike terms / 64
 - 3.1.3 Standard Zernike polynomials / 66
 - 3.1.4 Fringe Zernike polynomials / 68
 - 3.1.5 Magnitude and phase / 69
 - 3.1.6 Orthogonality of Zernike polynomials / 69
 - 3.1.6.1 Noncircular apertures / 70
 - 3.1.6.2 Discrete data / 71
 - 3.1.7 Computing the Zernike polynomial coefficients / 72
- 3.2 Annular Zernike Polynomials / 74
- 3.3 X-Y Polynomials / 74
- 3.4 Legendre Polynomials / 75
- 3.5 Legendre–Fourier Polynomials / 76
- 3.6 Aspheric Polynomials / 77
- References / 78

<Chapter 4>**Optical Surface Errors / 81**

- 4.1 Optical-Surface Rigid-Body Errors / 81
 - 4.1.1 Computing rigid-body motions / 82
 - 4.1.2 Representing rigid-body motions in the optical model / 83
- 4.2 Optical-Surface Shape Changes / 84
 - 4.2.1 Sag displacements / 85
 - 4.2.2 Surface normal deformations / 86
- 4.3 Relating Surface Errors to Wavefront Error / 87
 - 4.3.1 Refractive surfaces / 87
 - 4.3.2 Reflective surfaces / 88
- 4.4 Optical Surface Deformations and Zernike Polynomials / 89
 - 4.4.1 Optical-surface error analysis example / 89
- 4.5 Representing Elastic Shape Changes in the Optical Model / 91
 - 4.5.1 Polynomial surface definition / 91
 - 4.5.2 Interferogram files / 92
 - 4.5.3 Uniform arrays of data / 93
 - 4.5.3.1 Grid Sag surface / 94
 - 4.5.3.2 Interpolation / 94
- 4.6 Predicting Wavefront Error Using Sensitivity Coefficients and Matrices / 95
 - 4.6.1 Rigid-body and radius-of-curvature sensitivity coefficients / 96
 - 4.6.1.1 Sensitivity coefficients example / 96
 - 4.6.1.2 Computing radius of curvature changes / 97
 - 4.6.2 Use of Zernike sensitivity coefficients / 98
- 4.7 Finite-Element-Derived Spot Diagrams / 99
- References / 99

<Chapter 5>**Optomechanical Displacement Analysis Methods / 101**

- 5.1 Displacement FEA Models of Optical Components / 101
 - 5.1.1 Definitions / 101
 - 5.1.2 Single-point models / 102
 - 5.1.3 Models of solid optics / 104
 - 5.1.3.1 Two-dimensional models of solid optics / 104
 - 5.1.3.2 Three-dimensional element models of solid optics / 105
 - 5.1.4 Lightweight mirror models / 108
 - 5.1.4.1 Two-dimensional equivalent-stiffness models of lightweight mirrors / 108
 - 5.1.4.2 Three-dimensional equivalent-stiffness models / 114
 - 5.1.4.3 Three-dimensional plate/shell model / 116
 - 5.1.4.4 Example: gravity deformation prediction comparison of a lightweight mirror / 117
 - 5.1.4.4.1 Two-dimensional effective property calculations / 118
 - 5.1.4.4.2 Three-dimensional effective property calculations / 119

5.1.4.4.3	Three-dimensional plate/shell model effective property calculations / 120
5.1.4.4.4	Comparison of results / 121
5.1.4.5	Example: Lightweight mirror with significant quilting / 122
5.1.5	Generation of powered optic models / 126
5.1.5.1	On-axis slumping / 126
5.1.5.2	Off-axis slumping / 127
5.1.5.3	Calculation of local segment sag / 131
5.1.6	Symmetry in optic models / 131
5.1.6.1	Creating symmetric models / 131
5.1.6.2	Example creation of a symmetric model / 132
5.1.6.3	Example of symmetry verification check / 134
5.2	Analysis of Surface Effects / 137
5.2.1	Composite-plate model / 138
5.2.2	Homogeneous-plate model / 139
5.2.3	Three-dimensional model / 141
5.2.4	Example: coating-cure shrinkage / 141
5.2.4.1	Composite-plate model / 142
5.2.4.2	Homogeneous-plate model / 142
5.2.4.3	Three-dimensional model / 143
5.2.5	Example: Twyman effect / 143
	References / 145

◀Chapter 6▶

Modeling of Optical Mounts / 147

6.1	Displacement Models of Adhesive Bonds / 147
6.1.1	Elastic behavior of adhesives / 147
6.1.2	Detailed 3D solid model / 151
6.1.2.1	Congruent mesh models / 152
6.1.2.2	Glued contact models / 152
6.1.3	Equivalent-stiffness bond models / 153
6.1.3.1	Effective properties for hockey-puck-type bonds / 154
6.1.3.2	Example: modeling of a hockey-puck-type bond / 159
6.1.3.3	Effective properties for ring bonds / 161
6.2	Displacement Models of Flexures and Mounts / 162
6.2.1	Classification of structures and mounts / 162
6.2.1.1	Classification of structures / 162
6.2.1.2	Classification of mounts / 163
6.2.1.3	Mounts in 3D space / 164
6.2.2	Modeling of kinematic mounts / 165
6.2.3	Modeling of flexure mounts / 167
6.2.3.1	Arrangement of strut supports / 167
6.2.3.2	Optimum radial location of mounts / 169
6.2.3.3	Modeling of beam flexures / 172
6.2.3.4	Example: modeling of bipod flexures / 174
6.2.3.5	Design issues with bipod flexures / 176

- 6.2.3.6 Modeling of blade flexures / 180
- 6.3 Modeling of Test Supports / 181
 - 6.3.1 Modeling of air bags / 182
 - 6.3.2 Example: test support deformation analysis of a nonaxisymmetric optic / 186
 - 6.3.3 Modeling of V-block test supports / 189
 - 6.3.4 Modeling of sling and roller-chain test supports / 189
 - 6.3.5 Example: Comparison of three test supports / 190
- 6.4 Tolerance Analysis of Mounts / 191
 - 6.4.1 Monte Carlo analysis / 191
 - 6.4.2 Example: flatness/coplanarity tolerance of a mirror mount / 192
- 6.5 Analysis of Assembly Processes / 195
 - 6.5.1 Theory / 195
 - 6.5.2 Example: assembly analysis of mirror mounting / 197
- References / 198

◀Chapter 7▶

Structural Dynamics and Optics / 199

- 7.1 Natural Frequencies and Mode Shapes / 199
 - 7.1.1 Multi-degree-of-freedom systems / 200
- 7.2 Damping / 201
- 7.3 Frequency Response Analysis / 202
 - 7.3.1 Force excitation / 202
 - 7.3.2 Absolute motion due to base excitation / 205
 - 7.3.2.1 Absolute motion due to base excitation example / 206
 - 7.3.3 Relative motion due to base excitation / 207
 - 7.3.4 Frequency response example / 208
- 7.4 Random Vibration / 209
 - 7.4.1 Random vibration in the time domain / 209
 - 7.4.2 Random vibration in the frequency domain / 210
 - 7.4.3 Random-vibration SDOF response / 211
 - 7.4.3.1 Random force excitation example / 211
 - 7.4.3.2 Base excitation: absolute motion example / 212
 - 7.4.3.3 Base excitation: relative motion example / 212
 - 7.4.4 Random vibration design levels / 213
- 7.5 Vibro-Acoustic Analyses / 214
 - 7.5.1 Patch method / 214
- 7.6 Shock Analyses / 216
 - 7.6.1 Shock response spectrum analyses / 217
 - 7.6.2 Shock analysis in the time domain / 218
 - 7.6.3 Attenuation of shock loads / 218
- 7.7 Line-of-Sight Jitter / 218
 - 7.7.1 LOS jitter analyses using FEA / 219
 - 7.7.2 LOS jitter in object and image space / 221
 - 7.7.3 Optical-element rigid-body motions / 221
 - 7.7.4 Cassegrain telescope LOS jitter example / 222

7.7.5	LOS rigid-body checks / 222
7.7.5.1	LOS rigid-body checks example / 223
7.7.6	Radial LOS error / 224
7.7.7	Identifying the critical structural modes / 225
7.7.8	Effects of LOS jitter on image quality / 227
7.7.8.1	Constant-velocity image motion / 228
7.7.8.2	High-frequency sinusoidal image motion / 229
7.7.8.3	Low-frequency sinusoidal image motion / 230
7.7.8.4	Random image motion / 230
7.7.9	Impact of sensor integration time / 231
7.8	Active LOS Stabilization / 233
7.8.1	Image motion stabilization / 234
7.8.2	Rigid-body stabilization / 234
7.9	Structural-Controls Modeling / 235
7.10	Vibration Isolation / 236
7.10.1	Multi-axis vibration isolation / 237
7.10.2	Vibration isolation system example / 238
7.10.3	Hexapod vibration isolation systems / 240
7.10.4	Vibration isolation roll-off characteristics / 240
7.11	Optical Surface Errors Due to Dynamic Loads / 241
7.11.1	Dynamic response and phase considerations / 241
7.11.2	Method to compute optical surface dynamic response / 242
7.11.3	Dynamic surface response and modal techniques / 243
7.11.4	System wavefront error due to dynamic loads / 244
	References / 245

◀Chapter 8▶

Mechanical Stress and Optics / 249

8.1	Stress Analysis Using FEA / 249
8.1.1	Coarse FEA models and stress concentration factors / 250
8.1.2	FEA post-processing / 250
8.2	Ductile Materials / 251
8.2.1	Microyield / 251
8.2.2	Ultimate strength / 252
8.3	Analysis of Brittle Materials / 252
8.3.1	Fracture toughness / 253
8.3.2	FEA methods to compute the stress intensity / 254
8.4	Design Strength of Optical Glass / 254
8.4.1	Surface flaws / 255
8.4.2	Controlled grinding and polishing / 255
8.4.3	Inert strength / 256
8.4.3.1	Residual stress and inert strength / 256
8.4.3.2	Inert strength based on material testing and Weibull statistics / 256
8.4.4	Environmentally enhanced fracture / 258
8.4.4.1	Crack growth studies / 258
8.4.4.2	Static and dynamic fatigue testing / 259

- 8.4.4.3 Lifetime and time-to-failure analyses / 260
- 8.4.4.4 Lifetime prediction and probability of failure / 262
- 8.4.4.5 Effects of residual stress on time-to-failure / 263
- 8.4.4.6 BK7 design strength example / 264
- 8.4.5 Proof testing / 264
- 8.4.6 Cyclic fatigue / 265
- 8.5 Stress Birefringence / 265
 - 8.5.1 Mechanical stress and the index ellipsoid / 266
 - 8.5.2 Stress birefringence for isotropic materials / 267
 - 8.5.3 Stress-optical coefficients / 270
 - 8.5.4 Computing stress birefringence for nonuniform stress distributions / 271
 - 8.5.5 Stress birefringence example / 274
 - 8.5.6 Stress birefringence and optical modeling / 276
- References / 277

◀Chapter 9▶

Optothermal Analysis Methods / 279

- 9.1 Thermal Design and Analysis / 279
- 9.2 Thermo-Elastic Analysis / 280
 - 9.2.1 Thermal strain and the coefficient of thermal expansion / 280
 - 9.2.2 CTE inhomogeneity / 281
- 9.3 Index of Refraction Changes with Temperature / 283
- 9.4 Effects of Temperature on Simple Lens Elements / 285
 - 9.4.1 Focus shift of a doublet lens example / 286
 - 9.4.2 Radial gradients / 287
- 9.5 Thermal Response Using Optical Design Software / 288
 - 9.5.1 Representing OPD maps in the optical model / 289
- 9.6 Thermo-Optic Analysis of Complex Temperature Fields / 290
 - 9.6.1 Thermo-optic finite element models / 290
 - 9.6.1.1 Multiple reflecting surfaces / 291
 - 9.6.2 Thermo-optic errors using integration techniques / 291
 - 9.6.3 User-defined surfaces / 293
- 9.7 Bulk Volumetric Absorption / 293
- 9.8 Mapping of Temperature Fields from the Thermal Model to the Structural Model / 294
 - 9.8.1 Nearest-node methods / 295
 - 9.8.2 Conduction analysis / 295
 - 9.8.3 Shape function interpolation / 296
- 9.9 Analogous Techniques / 297
 - 9.9.1 Moisture absorption / 298
 - 9.9.2 Adhesive curing / 298
- References / 298

◀Chapter 10▶

Analysis of Adaptive Optics / 301

- 10.1 Introduction / 301

10.2 Method of Simulation / 302
10.2.1 Determination of actuator inputs / 303
10.2.2 Characterization metrics of adaptive optics / 304
10.2.2.1 Example: adaptive control simulation of a mirror segment / 305
10.3 Use of Augment Actuators / 307
10.3.1 Example of augment actuators / 308
10.4 Slope Control of Adaptive Optics / 309
10.5 Actuator Failure / 309
10.6 Actuator Stroke Limits / 311
10.7 Actuator Resolution and Tolerancing / 312
10.7.1 Example of actuator resolution analysis / 313
10.8 Design Optimization of Adaptively Controlled Optics / 314
10.8.1 Adaptive control simulation in design optimization / 314
10.8.1.1 Example: Structural design optimization of an adaptively controlled optic / 315
10.8.2 Actuator placement optimization / 317
10.8.2.1 Example: Actuator layout optimization of a grazing incidence optic / 318
10.9 Stressed-Optic Polishing / 319
10.9.1 Adaptive control simulation in stressed-optic polishing / 319
10.9.2 Example: Stressed-optic polishing of hexagonal array segments / 320
10.10 Analogies Solved via Adaptive Tools / 322
10.10.1 Correlation of CTE variation / 323
10.10.2 Mount distortion / 324
References / 324

◀Chapter 11▶

Optimization of Optomechanical Systems / 327

11.1 Optimization Approaches / 328
11.2 Optimization Theory / 329
11.3 Structural Optimization of Optical Performance / 333
11.3.1 Use of design response equations in the FE model / 333
11.3.2 Use of external design responses in FEA / 335
11.4 Integrated Thermal-Structural-Optical Optimization / 336
References / 337

◀Chapter 12▶

Superelements in Optics / 339

12.1 Overview / 339
12.2 Superelement Theory / 339
12.2.1 Static analysis / 340
12.2.2 Dynamic analysis / 341
12.2.2.1 Guyan reduction / 341
12.2.2.2 Component mode synthesis / 341
12.2.3 Types of superelements / 342

- 12.2.3.1 Conventional superelement / 342
- 12.2.3.2 External superelement / 343
- 12.3 Application to Optical Structures / 343
 - 12.3.1 Kinematic mounts / 343
 - 12.3.2 Segmented mirrors / 343
- 12.4 Advantages of Superelements / 344
- 12.5 Telescope Example / 344
- References / 345

◀Chapter 13▶

Integrated Optomechanical Analysis of a Telescope / 347

- 13.1 Overview / 347
- 13.2 Optical Model Description / 348
- 13.3 Structural Model Description / 349
- 13.4 Optimizing the PM with Optical Metrics / 351
- 13.5 Line-of-Sight Calculations / 352
- 13.6 On-Orbit Image Motion Random Response / 352
- 13.7 On-Orbit Surface Distortion in Random Response / 355
- 13.8 Detailed Primary Mirror Model / 356
- 13.9 RTV vs Epoxy Bond / 359
- 13.10 Gravity Static Performance / 360
- 13.11 Thermo-Elastic Performance / 362
- 13.12 Polynomial Fitting / 364
- 13.13 Assembly Analysis / 365
- 13.14 Other Analyses / 366
- 13.15 Superelements / 367
- References / 369

◀Chapter 14▶

Integrated Optomechanical Analyses of a Lens Assembly / 371

- 14.1 Double Gauss Lens Assembly / 371
 - 14.1.1 Thermal analysis / 372
 - 14.1.2 Thermo-elastic analysis / 373
 - 14.1.3 Stress birefringence analysis / 374
 - 14.1.4 Thermo-optic analysis / 374
 - 14.1.5 Optical analysis / 375
- 14.2 Seven-Element Lens Assembly / 378

Index / 381

Introduction

Optomechanical engineering is the application of mechanical engineering principles to design, fabricate, assemble, test, and deploy an optical system that meets performance requirements in the service environment. The challenge of optomechanical engineering lies in preserving the position, shape, and optical properties of the optical elements with specified tolerances typically measured in microns, microradians, and fractions of a wavelength.

Optomechanical analyses are an integral part of the optomechanical engineering discipline to simulate the mechanical behavior and performance of the optical system. These analyses include a broad range of thermal, structural, and mechanical analyses that support the design of optical mounts, metering structures, mechanisms, test fixtures, and more. This includes predicting the performance, dimensional stability, and structural integrity of optomechanical designs subject to internal mechanical loads and often harsh environmental disturbance, including inertial, pressure, thermal, and dynamic disturbance. Designs must provide for positive margin against failure modes that include yielding, buckling, ultimate failure, fatigue, and fracture.

Analysis starts with first-order estimates using analytical solutions based on classic elasticity and heat transfer theory. These closed-form solutions provide rapid estimates of structural and thermal behavior and an understanding of the governing parameters controlling the response. Finite element analysis (FEA) methods are widely used to provide more-accurate and higher-fidelity mechanical response predictions. Models of varying complexity may be developed by discretizing the structure into one-, two-, or three-dimensional elements to meet both efficiency and accuracy requirements. Thermal analysis models use both finite element methods and finite difference techniques to predict the thermal behavior of optical systems. Models are developed to predict thermal response quantities such as temperature distributions and heat fluxes that account for conduction, convection, and radiation modes of heat transfer.

Integrated optomechanical analysis involves the coupling of the structural, thermal, and optical simulation tools in a multi-disciplinary process commonly referred to as structural-thermal-optical performance or STOP analyses. The benefit of performing integrated analyses is the ability to provide insight into the interdisciplinary design relationships of thermal and structural designs and their impact through a deterministic assessment of optical performance. Engineering decisions during both the conceptual and execution stages of a program can then be based on high-fidelity performance simulations that are combined with program performance and reliability requirements, risk tolerance, schedule, and cost objectives to optimize the overall system design.

Integrated optomechanical analyses benefit optical system concept development by providing a rigorous and quantitative evaluation to explore the mission and design-trade spaces. The benefits of a wide variety of optical design configurations can be evaluated to account for factors such as the mechanical

design, pointing control and stability, thermal management, and materials selection for architecture down select.

During the execution stages of a program, integrated optomechanical analyses capture complex environmental conditions and concurrent disturbances. These analyses can be performed to compute performance as a function of time such as during operational scenarios that provide insights beyond which can be captured by a roll-up of static error-budget contributions. The simulations can be used in conjunction with numerical algorithms to optimize the design, serve as a predictive test bed for system-performance predictions, or provide for diagnostic evaluations of systems underperforming in the field.

The development and use of integrated optomechanical analyses has significantly increased over the past decade to support the ever-increasing challenges in optical system design, leveraging advances in computational resources. Government organizations have employed integrated tools in support of large-scale programs and advanced technologies, including space- and ground-based telescopes and high-powered beam systems. In addition, commercial organizations have sought to improve their effectiveness and efficiency in the design of optical systems through the application and development of custom-integrated optomechanical software tools. A variety of commercial software has been developed to provide an integrated analysis capability to the broader community.

Several approaches have been taken to integrate or couple the thermal, structural, and optical modeling tools. The “bucket brigade” approach relies on scripts to format and pass data between software tools. The “wrapper” approach uses custom-developed software to automate the data-sharing process. Fully integrated software tools offer the ability to model each discipline in a single, stand-alone modeling environment. Each of these approaches has its advantages and disadvantages, and one may be more appropriate over another for a given application or organization.

An essential piece of successful optomechanical analyses is the verification and validation of the models. Verification may be considered as the assessment of the numerical correctness of the model, i.e., ensuring that the models and the software do not have errors. Analytical solutions, stick models, check-out runs, and crawl-walk-run strategies are all verification methods to help ensure that a model is sound.

Validation may be considered as the assessment of how well the model represents the physical behavior of the hardware. Model validation via testing is performed at various stages of a design cycle. Early testing at the component and subassembly level can be used to validate basic physics and model uncertainties. System-level validation supports requirements verification and provides confidence in analyses that are used to extrapolate performance outside of a limited test domain.

This book serves as a compilation of many of the analyses and integrated methods that the authors have employed and developed in their collective experience supporting the development of optical systems. There are 14 chapters

that address key aspects of optomechanical analysis, including the detailed use of FEA methods and techniques to integrate and couple the thermal, structural, and optical analysis tools. There are additional disciplines involved in optical system engineering that may also be incorporated in a broader integrated analysis process that includes controls, radiometry, stray light, and aerodynamics, whose discussions are beyond the scope of this text.

Chapter 1 starts with an introduction to mechanical analysis using finite element methods and considerations in the integration of thermal, structural, and optical analyses. Included is a review of mechanical engineering basics, an overview of materials commonly used in optical systems, and finite element theory. A section on FEA modeling checks is presented that underscores the importance of verifying models and analyses.

Chapter 2 presents the fundamentals of optics, common optical performance metrics, and image formation. Included are discussions on polarized light, diffraction, conic surfaces, the impact of mechanical obscurations on optical performance, and optical system error budgets. This chapter serves as the basis of how mechanical perturbations, including optical surface errors and index of refraction changes due to temperature and stress, affect the performance of optical systems.

Chapter 3 provides an overview of Zernike polynomials and their utility in representing discrete data such as finite element results and as a means of data transfer from the thermal and structural tools into optical design software. Other relevant polynomial forms are also discussed.

Chapter 4 presents optical-surface-error analyses and methods to predict optical performance that account for FEA-derived optical surface errors. Two methods using optical sensitivity coefficients are discussed to predict wavefront error as a function of both rigid-body errors and higher-order elastic surface deformations. Use of optical sensitivity coefficients are beneficial early in the design stages for “closed-loop” analyses that allow mechanical engineers to predict optical performance as a function of mechanical design variables and account for the effects of environmental disturbances. The integration of FEA-derived optical surface errors within commercially available optical design software enables the development of a “perturbed” optical model, from which the full range of optical simulations and performance evaluations may be exercised to assess thermal and structural effects.

Chapters 5 and 6 discuss finite element model construction and analysis methods for predicting displacements of optical elements and support structures. Specific topics include modeling methods for individual optical components, various techniques to model lightweight mirrors, methods to create powered optical surfaces, use of symmetry for efficient modeling practices, and methods to analyze the effects of a variety of surface coating effects. Chapter 6 introduces kinematic mounting principles and focuses on the modeling of optical mounts, adhesive bonds, flexures, test supports, and the use of Monte Carlo methods to evaluate the effects of optical mount misalignments.

For many of the topics discussed in Chapters 5 and 6, analysis and modeling approaches range from first-order to detailed, high-fidelity simulations. The engineer may adopt an analysis strategy where the model fidelity maps to design maturity and requirements accuracy. Low-fidelity models are performed early in the design stages for the “80% solution.” These models are easily modified as the design evolves to support design trades and sensitivity studies. High-fidelity models that are more time consuming to build, modify, run, and post-process can be developed when the design has matured to provide high accuracy.

Chapter 7 provides an overview of structural dynamics, including normal modes, damping, harmonic, random, vibro-acoustic, and shock analyses. Analysis techniques are presented to predict pointing errors and LOS jitter using FEA and optical sensitivity coefficients, including the subsequent impact on optical system performance. Strategies and techniques to reduce the LOS jitter, including the identification of critical modes in the mechanical structure, the use of passive and active stabilization techniques, and the impact of sensor integration time, are included in the discussion. For large-aperture optical systems, methods are presented to predict optical surface distortions and wavefront error due to dynamic excitation of the optical surfaces.

Chapter 8 focuses on mechanical stress. Stress needs to be managed for several reasons in an optical system including structural integrity where excessive stress can lead to permanent misalignments or structural failure of optical elements, mounts, and support structures. An introduction to stress analysis using FEA is presented along with methods to predict the design strength of optical glass. The latter half of Chapter 8 describes the phenomenon of stress birefringence and presents analysis techniques to account for the effects of mechanical stress on optical performance. First-order estimates are provided using the photo-elastic equations along with more involved methods to compute optical performance metrics such as retardance and polarization errors due to complex mechanical stress states.

Chapter 9 presents optothermal analysis methods, including thermo-elastic and thermo-optic modeling techniques. This class of analyses helps drive thermal management strategies used to preserve optical-element surface errors and index-of-refraction changes in the presence of temperature changes. Methods to compute externally derived OPD maps using interferogram files and phase surfaces along with techniques to map temperatures between thermal and structural models that have varying mesh densities are presented. This latter process is a critical step in the STOP modeling effort and is often a technical challenge for program teams. Additional topics include a discussion on bulk volumetric absorption and the use of thermal analysis software to perform analogous analyses, including moisture effects and adhesive curing.

Chapter 10 provides an introduction to the analysis of adaptive optics. Adaptive optic concepts and definitions, including correctability and influence functions, are discussed along with the mathematics to compute actuator motion to minimize optical surface deformations. Practical details on adaptive optics are discussed, including predicting residual surface errors due to actuator failure,

stroke limits, resolution, and tolerancing are also presented. Examples are provided on the design of adaptive optics and actuator placement using design optimization methods. Additional topics in the chapter include stress-optic polishing and the use of adaptive tools to solve an analogous class of problems. This latter topic utilizes the same mathematical process for determining actuator inputs to predict the combination of a set of predefined disturbances to best match any arbitrary surface error. Examples are presented that solve for the combination of mount distortions and CTE variations to match interferometric test data.

Chapter 11 discusses structural optimization theory and applications. Numerical optimization consists of powerful techniques that enable a more-efficient evaluation of a broad design space beyond which may be evaluated via parametric design trades. The chapter discusses the use of optical performance metrics in structural optimization simulations and also provides a general discussion on multidisciplinary optimization.

Chapter 12 presents the use of FEA substructuring techniques for optical systems. The use of substructuring or superelements provides many benefits in detailed FEA simulations to provide for a more rapid turnaround of results for greater insight and impact. Superelement theory is presented along with common types of superelements. Examples of modeling kinematic mounts and segmented optical systems using superelements are presented.

The final two chapters present examples of the optomechanical and integrated analyses discussed in the previous chapters. Chapter 13 addresses a variety of analyses on a reflective telescope, and Chapter 14 details the integrated optomechanical analysis of two lens assemblies.

Keith B. Doyle
Victor L. Genberg
Gregory J. Michels
October 2012

◀Chapter 1▶

Introduction to Mechanical Analysis Using Finite Elements

1.1 Integrated Optomechanical Analysis Issues

1.1.1 Integration issues

The optical performance of telescopes, lens barrels, and other optical systems are heavily influenced by mechanical effects. Fig. 1.1 depicts the interaction between thermal, structural, and optical analysis. Each analysis type has its own specialized software to solve its own field-specific problems. To predict interdisciplinary behavior, data must be passed between analysis types. In this book, emphasis is placed on the interaction of the three analysis disciplines.

1.1.2 Example: orbiting telescope

A simple finite element structural model of an orbiting telescope is shown in Fig. 1.2, and a corresponding optical model is shown in Fig. 1.3. Because of dynamic disturbances, the optics may move relative to each other and elastically deform. From the finite element model, the motions of each node point are predicted. To determine the effect on optical performance, it is necessary to pass the data to the optical analysis program in an importable form. This usually requires a special post-processing program as described in later chapters. Typically, the structural data must be converted to the optical coordinate system, optical units, and sign convention, then fit with Zernike polynomials or interpolated to interferogram arrays (Chapter 3).

To create a valid and accurate structural model, the analyst must be aware of modeling techniques for mirrors, mounts, and adhesive bonds (Chapters 5 & 6). Incorporating image-motion equations inside the finite element model (Chapter 4) allows for image-motion output directly from a vibration analysis. The vibrations may be due to transient, harmonic or random loads. To determine if a mirror will fracture, the analyst must understand detailed stress modeling and the type of failure analysis required (Chapter 8). During its fabrication processes (grinding, polishing, and coating), a mirror may be tested under various support conditions that require their own analysis (Chapter 6). Analysis of the assembly process (Chapter 6) will predict locked-in strains and create an optical back out that can be factored into the overall system performance. Performance of the flexible primary mirror can be improved by adding actuators and sensors to create an adaptive mirror (Chapter 10). Using optimum design techniques (Chapter 11), the design can be made more efficient and robust. The specific details of the analyses on this telescope are demonstrated in Chapter 13.

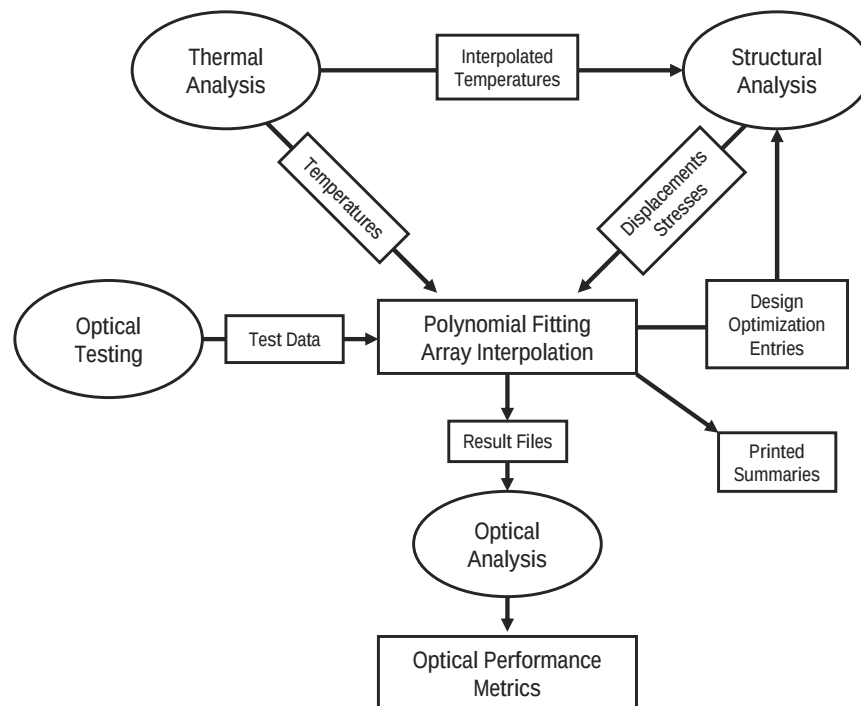


Figure 1.1 Optomechanical analysis interaction.

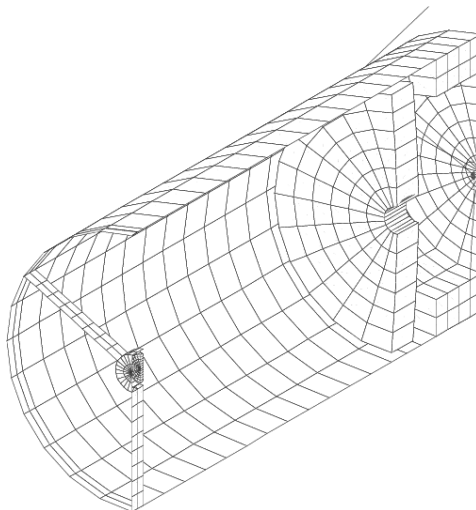


Figure 1.2 Telescope structural analysis model.

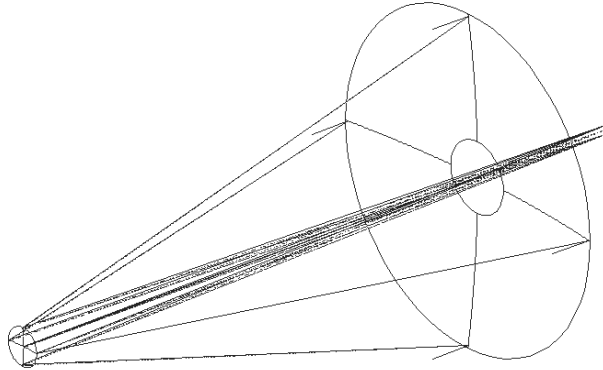


Figure 1.3 Telescope optical analysis model.

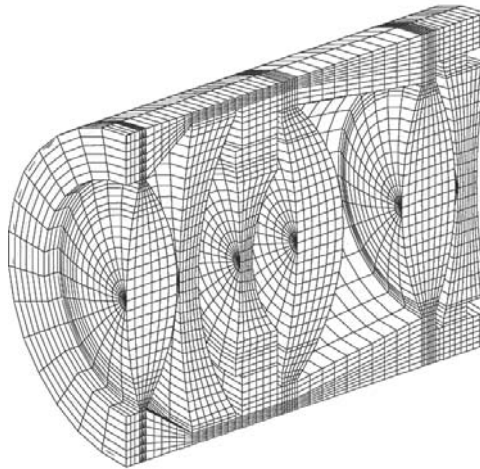


Figure 1.4 Lens barrel structural model.

1.1.3 Example: lens barrel

The lens barrel in Fig. 1.4 is representative of components used in a variety of applications from optical lithography to projection systems. Often the optical beam causes thermal loading on the lenses. Analyzing for the steady-state or transient temperature distribution is the first analysis required (Chapter 9). The resulting temperature profiles may cause an optical index change throughout each lens, which affects the optical performance (Chapter 9). As part of the structural analysis, temperatures must be applied that will require interpolation if the structural model is different from the thermal model. The thermoelastic stresses cause distortion (Chapter 4), and may cause stress birefringence effects (Chapter 8). Each of these effects requires special software to analyze the FEA results and

present the data in a format suitable for optical programs. If the structure and loading have symmetry, techniques can be used to reduce the computation required. The example lens barrel in Chapter 14 demonstrates many of the techniques discussed throughout the text.

1.2 Elasticity Review

1.2.1 Three-dimensional elasticity

TERMINOLOGY:

E = Young's modulus = slope of stress-strain curve

ν = Poisson's ratio = contraction in y, z due to elongation in x

α = Coefficient of thermal expansion (CTE)

σ = Stress = force/unit area

u, v, w = Displacements in x, y, z directions

e = Total strain = $\delta u / \delta x$ = stretch/unit length = $\epsilon + e_T$

ϵ = Mechanical strain = due to applied stress

e_T = Thermal strain = due to temperature change $\Delta T = \alpha \Delta T$

Stress components are shown in Fig. 1.5. The strain-component notation is analogous to the stress notation. Pictorially represented in Fig. 1.6, the strain-displacement relations are

$$\begin{aligned} \epsilon_x &= du/dx & \gamma_{xy} &= [du/dy + dv/dx] \\ \epsilon_y &= dv/dy & \gamma_{yz} &= [dv/dz + dw/dy] \\ \epsilon_z &= dw/dz & \gamma_{zx} &= [dw/dx + du/dz] \end{aligned} \quad (1.1)$$

Shear strain may be defined as above or as half of that value. The engineer must be aware of which definition is used in the analysis software.

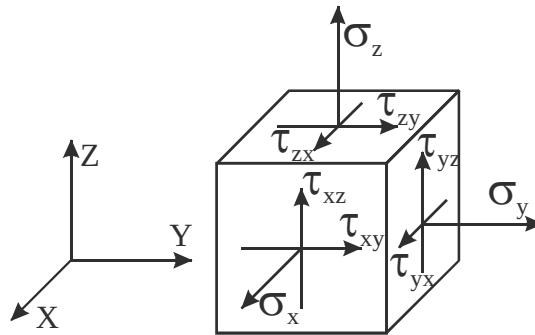


Figure 1.5 Stress components.

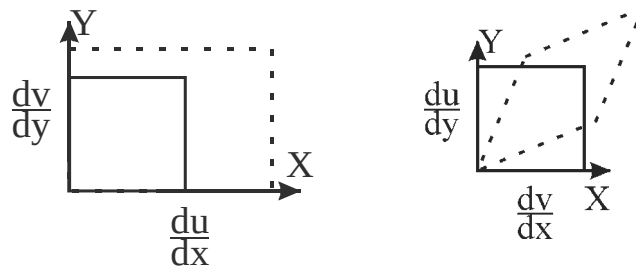


Figure 1.6 Strain-displacement relations.

For isotropic materials, the full 3D stress–strain relations may be represented as

$$\begin{Bmatrix} e_x \\ e_y \\ e_z \\ e_{xy} \\ e_{yz} \\ e_{zx} \end{Bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ -\nu & 1 & -\nu & 0 & 0 & 0 \\ -\nu & -\nu & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2(1+\nu) & 0 & 0 \\ 0 & 0 & 0 & 0 & 2(1+\nu) & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(1+\nu) \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} + \alpha \Delta T \begin{Bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{Bmatrix}, \quad (1.2)$$

or in inverted form:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix} \begin{Bmatrix} e_x \\ e_y \\ e_z \\ e_{xy} \\ e_{yz} \\ e_{zx} \end{Bmatrix} - \frac{E\alpha\Delta T}{1-2\nu} \begin{Bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{Bmatrix}. \quad (1.3)$$

The form in Eq. (1.2) is more intuitive since one can see how applied stress causes strain effects. However, the form in Eq. (1.3) is commonly used in FEA programs. The coefficient matrix in Eq. (1.3) is often referred to as the material matrix. If the material is orthotropic, then the stress–strain relations are represented as

$$\begin{Bmatrix} e_x \\ e_y \\ e_z \\ e_{xy} \\ e_{yz} \\ e_{zx} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_x} & -\frac{\nu_{yx}}{E_y} & -\frac{\nu_{zx}}{E_z} & 0 & 0 & 0 \\ -\frac{\nu_{xy}}{E_x} & \frac{1}{E_y} & -\frac{\nu_{zy}}{E_z} & 0 & 0 & 0 \\ -\frac{\nu_{xz}}{E_x} & -\frac{\nu_{yz}}{E_y} & \frac{1}{E_z} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{xy}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{yz}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{zx}} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} + \Delta T \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \\ 0 \\ 0 \\ 0 \end{Bmatrix}, \quad (1.4)$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} = \begin{bmatrix} \frac{1-\nu_{yz}\nu_{zy}}{\Psi} E_x & \frac{\nu_{xy}+\nu_{zy}\nu_{xz}}{\Psi} E_y & \frac{\nu_{xz}+\nu_{xy}\nu_{yz}}{\Psi} E_z & 0 & 0 & 0 \\ \frac{\nu_{yx}+\nu_{zx}\nu_{yz}}{\Psi} E_x & \frac{1-\nu_{xz}\nu_{zx}}{\Psi} E_y & \frac{\nu_{yz}+\nu_{yx}\nu_{xz}}{\Psi} E_z & 0 & 0 & 0 \\ \frac{\nu_{zx}+\nu_{yx}\nu_{zy}}{\Psi} E_x & \frac{\nu_{zy}+\nu_{xy}\nu_{zx}}{\Psi} E_y & \frac{1-\nu_{xy}\nu_{yx}}{\Psi} E_z & 0 & 0 & 0 \\ 0 & 0 & 0 & G_{xy} & 0 & 0 \\ 0 & 0 & 0 & 0 & G_{yz} & 0 \\ 0 & 0 & 0 & 0 & 0 & G_{zx} \end{bmatrix}, \quad (1.5)$$

where $\Psi = 1 - \nu_{xy}\nu_{yx} - \nu_{yz}\nu_{zy} - \nu_{zx}\nu_{xz} - 2\nu_{yx}\nu_{zy}\nu_{xz}$, and ν_{ij} is $-\epsilon_j/\epsilon_i$ for uniaxial stress σ_i .

The above equations may be used to analyze material that is orthotropic in nature, or they may be used to analyze isotropic materials that are fabricated by a method so they act in an orthotropic fashion (see Chapter 5).

1.2.2 Two-dimensional plane stress

Although all structures are truly 3D, it is computationally efficient to approximate thin structures (plates and shells) with 2D plane-stress relations for isotropic materials [Eqs. (1.6) and (1.7)]. If a thin structure lies in the X - Y plane, then the normal (Z) stress components are assumed to be zero:

$$\sigma_z = \tau_{yz} = \tau_{zx} = 0$$

$$\begin{Bmatrix} e_x \\ e_y \\ e_{xy} \end{Bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\nu & 0 \\ -\nu & 1 & 0 \\ 0 & 0 & 2(1+\nu) \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} + \alpha \Delta T \begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix}, \quad (1.6)$$

and

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{Bmatrix} e_x \\ e_y \\ e_{xy} \end{Bmatrix} - \frac{E\alpha\Delta T}{1-\nu} \begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix}. \quad (1.7)$$

Under this assumption, the normal strains are not zero but given as

$$\begin{aligned} e_z &= \frac{-\nu}{E} (\sigma_x + \sigma_y) + \alpha \Delta T, \\ e_{yz} &= e_{zx} = 0. \end{aligned} \quad (1.8)$$

Thus, in-plane stretching causes the material to get thinner.

For orthotropic materials, such as a graphite-epoxy panel, the plane stress relations are given as

$$\begin{Bmatrix} e_x \\ e_y \\ e_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_x} & -\frac{\nu_{yx}}{E_y} & 0 \\ -\frac{\nu_{xy}}{E_x} & \frac{1}{E_y} & 0 \\ 0 & 0 & \frac{1}{G_{xy}} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} + \Delta T \begin{Bmatrix} \alpha_x \\ \alpha_y \\ 0 \end{Bmatrix}, \quad (1.9)$$

and

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{1}{1-\nu_{xy}\nu_{yx}} E_x & \frac{\nu_{xy}}{1-\nu_{xy}\nu_{yx}} E_y & 0 \\ \frac{\nu_{yx}}{1-\nu_{xy}\nu_{yx}} E_x & \frac{1}{1-\nu_{xy}\nu_{yx}} E_y & 0 \\ 0 & 0 & G_{xy} \end{bmatrix} \begin{Bmatrix} e_x \\ e_y \\ e_{xy} \end{Bmatrix} - \Delta T \begin{Bmatrix} \alpha_x \\ \alpha_y \\ 0 \end{Bmatrix}. \quad (1.10)$$

Through the thickness, strains are again nonzero:

$$e_z = -\frac{\nu_{xz}}{E_x}\sigma_x - \frac{\nu_{yz}}{E_y}\sigma_y + \alpha_z\Delta T; \quad (1.11)$$

$$e_{yz} = e_{zx} = 0.$$

1.2.3 Two-dimensional plane strain

An alternate approximation is to assume that the normal strains are zero, $\epsilon_z = \gamma_{yz} = \gamma_{zx} = 0$. This condition can occur for very wide, thin-bond areas, or for long (in Z) uniform structures. The isotropic plane-strain relations are

$$\begin{Bmatrix} e_x \\ e_y \\ e_{xy} \end{Bmatrix} = \frac{1+\nu}{E} \begin{bmatrix} 1-\nu & -\nu & 0 \\ -\nu & 1-\nu & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} + (1+\nu)\alpha\Delta T \begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix}, \quad (1.12)$$

and

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix} \begin{Bmatrix} e_x \\ e_y \\ e_{xy} \end{Bmatrix} - \frac{E\alpha\Delta T}{1-2\nu} \begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix}. \quad (1.13)$$

The normal stress is not zero in this assumption, but given as

$$\sigma_{zz} = \nu(\sigma_{xx} + \sigma_{yy}) - E\alpha\Delta T, \quad (1.14)$$

$$\tau_{yz} = \tau_{zx} = 0.$$

To be complete, the orthotropic plane-strain relations are given as

$$\begin{Bmatrix} e_x \\ e_y \\ e_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{1-\nu_{zx}\nu_{xz}}{E_x} & \frac{\nu_{yx}-\nu_{yz}\nu_{xz}}{E_y} & 0 \\ \frac{\nu_{xy}-\nu_{zx}\nu_{zy}}{E_x} & \frac{1-\nu_{yz}\nu_{zy}}{E_y} & 0 \\ 0 & 0 & \frac{1}{G_{xy}} \end{bmatrix} \begin{Bmatrix} \sigma_x + \Delta T\alpha_x \\ \sigma_y + \Delta T\alpha_y \\ \tau_{xy} \end{Bmatrix}, \quad (1.15)$$

and

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{1-\nu_{yz}\nu_{zy}}{\Psi} E_x & \frac{\nu_{xy}+\nu_{zy}\nu_{xz}}{\Psi} E_y & 0 \\ \frac{\nu_{yx}+\nu_{zx}\nu_{yz}}{\Psi} E_x & \frac{1-\nu_{xz}\nu_{zx}}{\Psi} E_y & 0 \\ 0 & 0 & G_{xy} \end{bmatrix} \begin{Bmatrix} e_x - \Delta T \alpha_x \\ e_y - \Delta T \alpha_y \\ e_{xy} \end{Bmatrix}. \quad (1.16)$$

1.2.4 Principal stress and equivalent stress

Stress failure cannot be determined directly from a general 2D or 3D state of stress. A general state of stress is processed to determine principal stresses or an equivalent stress, which is then used as a failure criterion. For a general 2D state of stress at a point $(\sigma_x, \sigma_y, \sigma_{xy})$, Mohr's circle (Fig. 1.7) is used to find the state of principal stress, which is defined as an orientation with *no* shear stress,

$$(\sigma_1, \sigma_2, 0), \text{ where } C = \frac{\sigma_x + \sigma_y}{2} \text{ and } R = \sqrt{\tau_{xy}^2 + \left(\frac{\sigma_x - \sigma_y}{2} \right)^2}$$

$$\text{and } \sigma_1 = C + R, \sigma_2 = C - R.$$

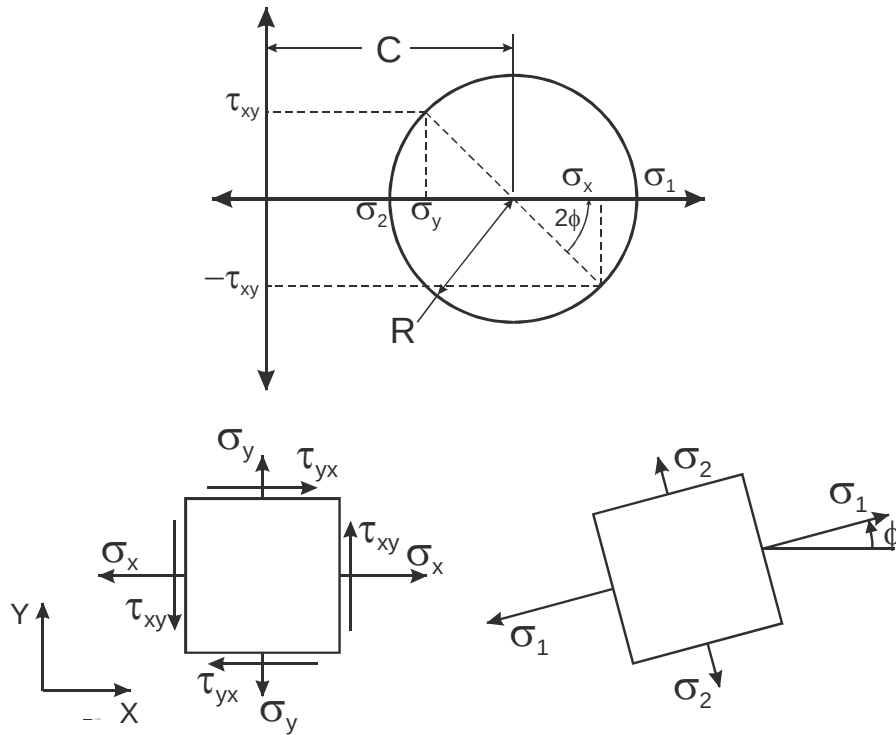


Figure 1.7 Mohr's Circle for 2D stress.

Ductile materials such as aluminum or steel follow the Maximum Distortion Energy Theory, in which yielding occurs when the Von Mises stress (σ_{vm}) from

$$\sigma_{vm} = \frac{1}{\sqrt{2}} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} \quad (1.17)$$

reaches the material yield stress.

Brittle materials such as common glasses follow fracture-mechanics laws in which fracture occurs when the stress-intensity factor (K) reaches the fracture toughness (K_c) of the material. K is computed from maximum principal stress, or maximum shear stress, flaw size, and geometry, and K_c is a material property. See Chapter 8 for more details.

1.3 Material Properties

1.3.1 Overview

Material selection is an integral part of the design process, affecting thermal, structural, and optical performance. Key material properties include stiffness and thermal stability to ensure that optical element alignment and surface figure is preserved over the thermal, inertial, and dynamic operational environments.

In general, optical structures are stiffness limited, rather than stress limited. For example, a metering structure must maintain the optical surface figure and the alignment of the optics subject to gravity loads in operation and during testing while meeting line-of-sight dynamic response requirements. These operational performance criteria are driven by the stiffness of the design provided by the elastic module E and the weight or density ρ of the material rather than meeting stress requirements. Thus, high stiffness and low weight are very desirable properties for operation. Material selection must also account for nonoperational stresses such as those found in launch conditions that may be quite high. In this case, stiffness/strength is an important material characteristic for nonoperational load conditions.

Material selection is also critical in the thermal and thermoelastic behavior of an optical system. Materials with high thermal conductivity K and high thermal diffusivity D minimize the presence of thermal gradients and the time a material takes to reach thermal equilibrium. The thermoelastic response of a structure in the presence of temperature differentials ΔT is dictated by the material's coefficient of thermal expansion (CTE), resulting in thermal strain. Materials with low CTE will minimize thermal strain and distortions in an optical element and in the metering structure. Isothermal temperature changes are usually less critical than thermal gradients, because optical structures can be designed to be athermal (see Chapter 9). Properties of common materials are shown in Table 1.1.

Table 1.1 Properties of common materials in optical structures.

	E	ρ	ν	CTE	K	C_p
	(Gpa)	(kg/m ³)		(ppm/C)	(W/M K)	(W sec/Kg K)
Aluminum	68	2700	0.33	23.6	167	960
Beryllium	287	1850	0.08	11.3	216	1820
Titanium	114	4430	0.31	8.8	7.3	522
Stainless 304	193	8000	0.27	14.7	16.2	477
Stainless 416	200	7800	0.28	9.9	24.9	480
Magnesium	45	1770	0.35	25.2	138	1024
Copper	117	8940	0.34	16.9	391	420
Invar	141	8050	0.36	1.4	10.4	515
SiC (RB 12%)	373	3110		2.68	147	680
SiC (RB 30%)	310	2920	0.14	2.44	158	660
SiC CVD	466	3210	0.21	2.4	146	700
Silicon	131	2330	0.28	2.5	137	710
Carbon/SiC	245	2650		2.5	135	660
AlBeMet	197	2100	0.17	13.9	212	1560
Borosilicate	59	2180	0.20	2.8	1.1	710
Fused Silica	73	2205	0.17	0.58	1.4	741
ULE	67	2205	0.18	0.03	1.3	766
Zerodur	91	2530	0.24	0.05	1.6	821
GY-70/x30	93	1780		0.02		

 E = modulus of elasticity ρ = mass density ν = Poisson's ratio α = CTE = coefficient of thermal expansion K = thermal conductivity C_p = heat capacity $D = K/(\rho C_p)$ = thermal diffusivity

1.3.2 Figures of Merit

Common figures of merit useful in optical structures include the specific stiffness, the steady-state thermal distortion metric, and the transient thermal distortion metric as expressed in Table 1.2. Plots of the following data make comparisons of material easy. In Fig. 1.8, the structure performance metric specific stiffness is plotted versus mass density. In Fig. 1.9, specific stiffness is plotted versus transient thermal stability.

Figures of merit can be useful when starting the design process, but the designer must choose the proper material for the application. For example, when designing a mirror, the specific stiffness E/ρ is a useful criterion because it determines the mirror's natural frequency and self-weight deflection. The natural frequency of a circular plate (ignoring transverse shear) is determined from

Table 1.2 Common figures of merit.

	E/ρ	K/α	D/α
Aluminum	25	7.1	2.7
Beryllium	155	19.1	5.7
Titanium	26	0.8	0.4
Stainless 304	24	1.1	0.3
Stainless 416	26	2.5	0.7
Magnesium	25	5.5	3.0
Copper	13	23.1	6.2
Invar	18	7.4	1.8
SiC (RB 12%)	120	54.9	25.9
SiC (RB 30%)	106	64.8	31.1
SiC CVD	145	60.8	27.1
Silicon	56	54.8	33.1
Carbon/SiC	92	54.0	30.9
AlBeMet	94	15.3	4.7
Borosilicate	27	0.4	0.3
Fused Silica	33	2.4	1.5
ULE	30	43.3	25.7
Zerodur	36	32.0	15.4
GY-70/x30	52		

E/ρ = Specific stiffness characterizes the stiffness-to-weight ratio. High E/ρ minimize self-weight deflections and maximize natural frequency.

K/α = Steady-state thermal distortion minimizes the presence of thermal gradients and the resulting distortion.

D/α = Transient thermal distortion, which minimizes the time for gradients to equilibrate and the resulting distortion.

$$f_n = \frac{C}{r^2} \sqrt{\frac{E}{\rho}} \sqrt{\frac{h^2}{12(1-\nu^2)}}, \quad (1.18)$$

where C depends on the support condition. The self-weight deflection of a circular plate, ignoring transverse shear, is

$$d = C \left(\frac{r^4}{h^4} \right) \left(\frac{\rho}{E} \right) (1-\nu^2), \quad (1.19)$$

where C depends on the support condition.

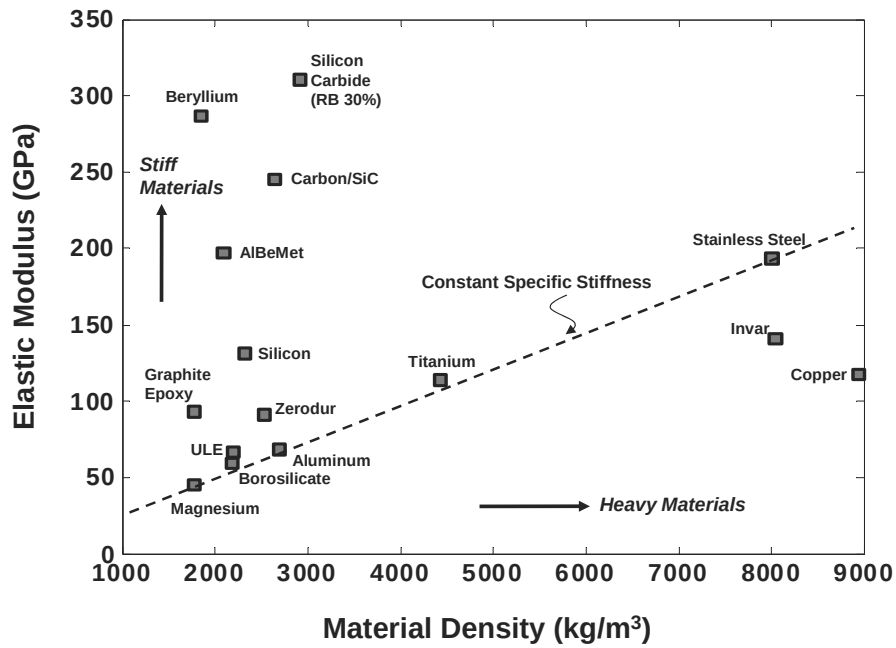


Figure 1.8 Modulus versus density.

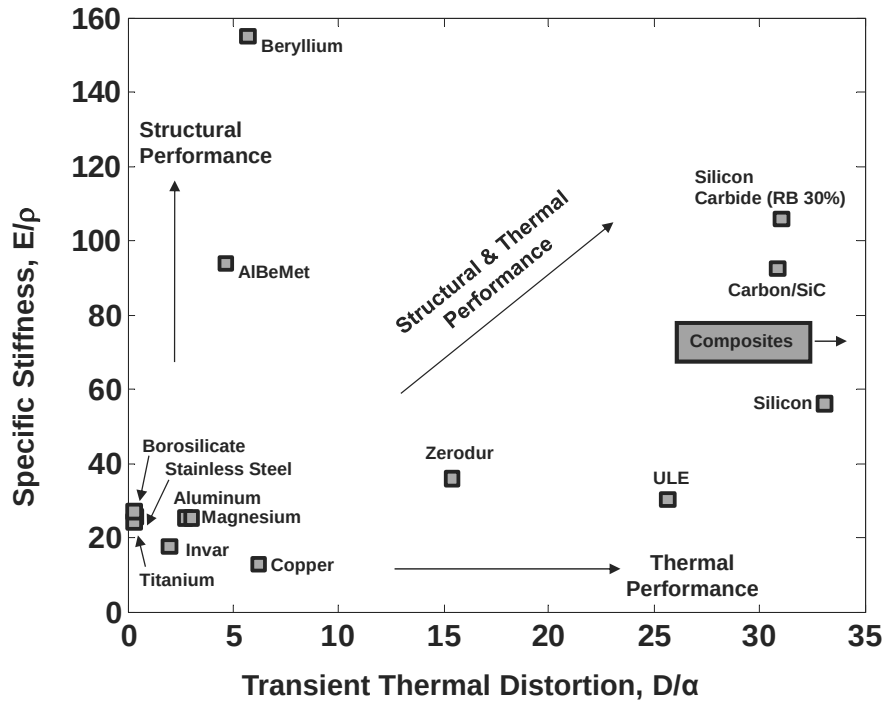


Figure 1.9 Specific stiffness versus transient thermal distortion.

However, when picking a material for the flexures to support a large mirror, specific stiffness is not an important criterion because the flexures represent such a small portion of weight. Instead, E of the flexures determines the natural frequency of the supported mode and the overall pointing error due to gravity loads. Under launch loads, the most important property is the yield stress of the flexure material. The best choice of materials depends on several factors, of which the above figures of merit are but one consideration.

1.3.3 Discussion of materials

- **ULE** and **Zerodur[®]** have excellent thermal characteristics at room temperature. ULE is fused silica doped with titanium, yielding a near-zero CTE. Zerodur[®] is a combination of two-phase materials—one crystalline with a negative CTE, and one amorphous with a positive CTE—yielding a near-zero net CTE. Lightweight mirrors may be created by fusing facesheets and ribs or by water-jet milling a solid blank. Both materials may be polished with a very low micro-roughness.
- **Silicon carbide** offers excellent thermal and structural characteristics with a low CTE, high thermal conductivity, high stiffness, and moderate density, and is an attractive material for mirror substrates and support structures. The material is a ceramic and is produced using several methods, including CVD (chemical vapor deposition) and reaction bonding (sintering). A drawback to silicon carbide is its inherent brittleness; design efforts must ensure appropriate margins of safety to minimize fracture. Silicon carbide and carbon–silicon carbide are developing materials that offer high stiffness and thermal stability.
- **Beryllium** is an attractive material used for mirror substrates and support structures due to its high stiffness, low mass density, and high thermal conductivity. Drawbacks include a relatively high cost and high CTE, making it susceptible to thermal gradients (although at cryo-temperatures the tangent CTE is near zero). Material fabrication and machining processes are complex and require special facilities (the fine particles produced during machining are hazardous to human health).
- **Aluminum alloys** are commonly used for optical mirrors and support structures. Characteristics of aluminum include high thermal conductivity, ease of machining, low cost, moderate stiffness, and high CTE. Thermal gradients must be minimized using aluminum due to its high CTE.
- **Borosilicate** glass has for the majority of applications been replaced by ULE or Zerodur[®] due to their near-zero CTE. However, advantages of this material include low cost and the ability to cast lightweighted mirrors.

- **Steel** has three times the stiffness and weight of aluminum with moderately high CTE and low conductivity. Ground-based telescope structures often employ steel for its low cost, but due to its weight and poor thermal metrics, steel is not commonly used as a support structure for non-terrestrial applications.
- **Copper**'s advantage is its high thermal conductivity; it is commonly used in thermal design applications of optical systems. Copper is heavy with moderate stiffness and has a high CTE.
- **Magnesium** offers similar characteristics to aluminum, but it is lighter, making it an option for relative weight savings. Its conductivity is slightly lower, CTE slightly higher, and a stiffness-to-weight ratio comparable to aluminum. Magnesium is susceptible to corrosion and must be coated for protection.
- **Invar**, an iron and nickel alloy with a low CTE, is commonly used to maintain optical element stability over temperature. Disadvantages of Invar include a relatively low specific stiffness, low conductivity, and high density.
- **Titanium**'s material properties include a CTE that is well matched to optical glass, moderate stiffness and density, low thermal conductivity, and high toughness and yield strength. Titanium is commonly used in high-performance lens assemblies to minimize CTE mismatches. It is also commonly used to thermal isolate components and as a flexure material due to its high strength.
- **Aluminum–beryllium** metal matrix composite combines pure aluminum and pure beryllium. This material offers a high specific stiffness, good thermal characteristics, and the machinability of aluminum. However, limited heritage exists in using this material as a mirror substrate.
- **Composite** materials such as graphite epoxy represent a general class of materials known as carbon fiber reinforced polymers (CFRP). In general, CFRP material properties are characterized by high stiffness, low density, and low CTE. The properties of these materials are direction dependent, and stiffness and CTE may be tailored for specific application by varying the orientation of the laminate plies. A disadvantage of CFRP materials is dimensional instability due to absorption/desorption of moisture.

See Chapter 3 in Yoder¹ for a more complete discussion of materials with tables of mechanical, thermal and optical properties of more materials, especially optical glasses and plastics.

1.3.4 Common telescope materials

One common approach to telescope material selection uses a single material for the mirrors and the metering structure. This design approach performs well in an isothermal environment, resulting in only a scale change, but it may suffer from thermal gradients depending on the material's CTE. Materials used for this single-material approach include aluminum, beryllium, and silicon carbide.

A second approach utilizes low-CTE but different materials for the mirrors and metering structure that minimize response due to isothermal temperature changes as well as thermal gradients. These designs usually use Zerodur® or ULE as the mirror material, and either Invar or CFRP composites for the metering structure material.

1.4 Basics of Finite Element Analysis

1.4.1 Finite element theory

FINITE DIFFERENCE (APPROXIMATE THE MATH):

- <1> Write equilibrium as the governing differential equation.
- <2> Write derivatives as differences on a uniform grid.
- <3> Solve the resulting matrix equation for behavior at the grid points.
- <4> Odd-shaped boundaries are difficult to handle.

FINITE ELEMENTS (APPROXIMATE THE PHYSICS):

- <1> Subdivide the body into simple elements of arbitrary size and shape.
- <2> Assume simple polynomial behavior in each element.
- <3> Write equilibrium at the nodes and solve for nodal values.
- <4> Odd geometry is easily handled.

Structural behavior in a continuous body is defined by differential equations, which are usually impossible to solve for real problems with complex geometry. Two common methods of approximation are finite difference and finite element. This text concentrates on the finite element analysis (FEA) technique that is widely used in the analysis of optical structures.

In FEA, the displacement is assumed to have a simple polynomial behavior over an element. For the 1D truss element in Fig. 1.10, the assumed linear displacement is given by

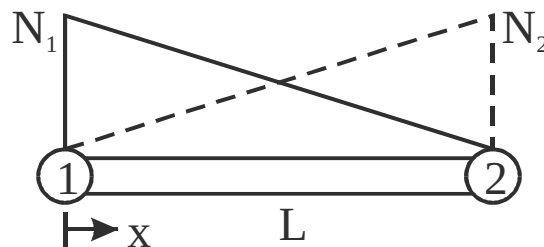


Figure 1.10 Linear shape functions.

$$u(x) = N_1 U_1 + N_2 U_2 = \sum N_j U_j = [N]\{U\},$$

$$N_1 = 1 - x/L,$$

and

$$N_2 = x/L,$$

where U_j = displacement of node j (variable to be solved for), and N_j = shape function for node j ($N_j = 1$ at j , $N_j = 0$ at all other nodes). Thus, a continuous function, $u(x)$, can be written in terms of discrete values, U_j . Using this relationship, stress and strain can also be written as a function of nodal variables U , as follows:

$$\varepsilon = du/dx = (d/dx) \sum N_j U_j = \sum dN_j/dx U_j = [B] \{U\}, \quad (1.21)$$

$$\sigma = E \varepsilon = [G] [B] \{U\}.$$

Potential energy Π is written as an integral over the element volume of the strain energy minus the work done, W_p , by the vector of applied nodal forces P :

$$\Pi = 0.5 \int \varepsilon^T \sigma dV - W_p = 0.5 \int U^T B^T G B U dV - U^T P. \quad (1.22)$$

Minimize Π with respect to the variables U = nodal displacements:

$$d\Pi / dU = 0 = \int B^T G B dV U - P = kU - P. \quad (1.23)$$

Thus, the element stiffness matrix $[k]$ is:

$$k = \int B^T G B dV, \quad (1.24)$$

which, for the 1D truss element is

$$k = \frac{AE}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}. \quad (1.25)$$

Generally, each element's stiffness matrix must be transformed into the global coordinate system used for nodal displacements via a coordinate transformation matrix, T :

$$k_g = T^T k T. \quad (1.26)$$

The element matrices are then assembled into system matrix $\mathbf{K}$, resulting in the system level equilibrium equations:

$$[K] \{U\} = \{P\}. \quad (1.27)$$

After proper boundary conditions and loads are applied to the model, the above equations are solved for nodal displacements U . If desired, element stresses are determined from Eq. (1.21).

The same derivation may be applied to 2D plate and 3D solid elements if the shape functions add the appropriate spatial variables y and z . The order of the shape functions can be increased from linear with two nodes per edge, to quadratic with three nodes per edge, and higher. For additional information on finite element theory, see Refs. 2–4.

1.4.2 Element performance

It is useful for the analyst to understand the performance of the element formulations of the finite element software employed for analysis because the element-shape functions in the previous section determine the behavior and accuracy of the model. The best way to quantify such performances is to run an analysis for which the answer is known. For example, consider the simple cantilever beam illustrated in Fig. 1.11, which is subject to a variety of load conditions. Load cases 1 through 3 exercise the membrane (in-plane) behavior, whereas load cases 4 and 5 exercise bending (out-of-plane) behavior. The structure was modeled with a variety of 2D shell elements as shown in Fig. 1.12.

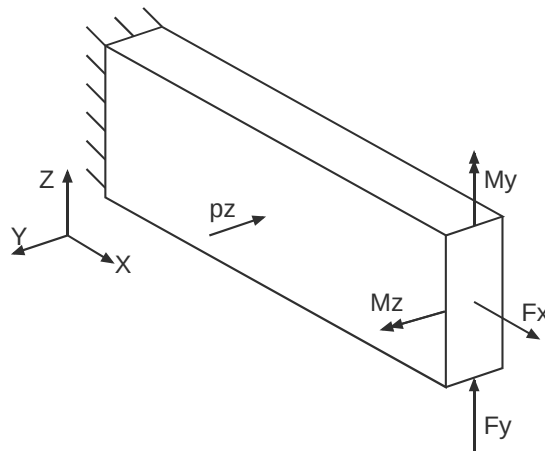


Figure 1.11 Cantilevered beam test case.

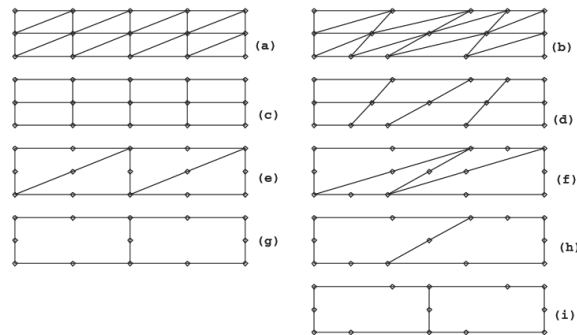


Figure 1.12 2D plate-element models of cantilever beam.

From the results obtained from MSC/Nastran version 2001 listed in Table 1.3, where the results are normalized by dividing by the exact value, the following conclusions can be drawn:

- <1> All elements correctly predict constant stress (case 1).
- <2> Tria3 is very poor for linear membrane stress (cases 2 and 3).
- <3> Other elements do well, even if distorted, for linear membrane stress.
- <4> All elements, including Tria3, do well for plate bending.
- <5> Quadratic elements must have nodes located at the mid-point of the edges or accuracy degrades.

Table 1.3 2D shell results for cantilever beam.

MEMBRANE (IN-PLANE) BEHAVIOR:

- <1> F_x = axial load = uniform, constant stress
- <2> M_z = moment in-plane = axial stress is linear in y
- <3> F_y = shear force in-plane = axial stress linear in x and y

PLATE BENDING (OUT-OF-PLANE) BEHAVIOR:

- <1> M_y = moment out-of-plane = stress constant in x
- <2> p_z = normal pressure = stress linear in x

MODELS:	F_x	M_z	F_y	M_y	p_z
<a> Tria3–uniform	1.00	0.30	0.32	1.00	1.00
 Tria3–distorted	1.00	0.12	0.16	1.00	0.96
<c> Quad4–uniform	1.00	1.00	0.98	1.00	1.00
<d> Quad4–distorted	1.00	0.98	0.96	1.00	1.00
<e> Tria6–uniform	1.00	1.00	0.96	1.00	1.00
<f> Tria6–distorted	1.00	1.00	0.82	1.00	0.84
<g> Quad8–uniform	1.00	1.00	1.00	1.00	1.00
<h> Quad8–distorted	1.00	0.98	0.94	0.91	0.83
<i> Quad8–midside-offset	1.00	0.59	0.59	0.43	0.44

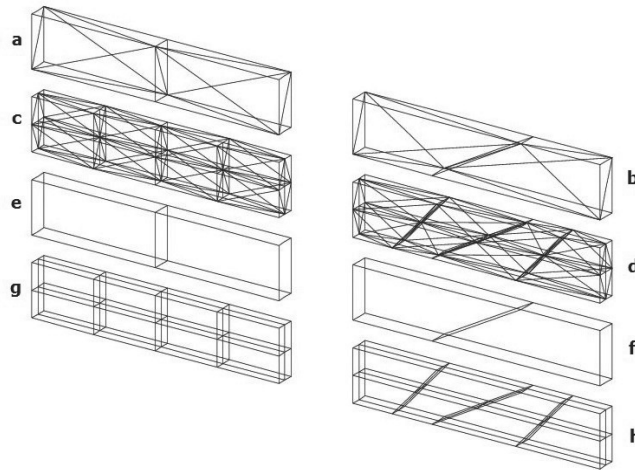


Figure 1.13 3D solid elements.

This is a simple test case, testing only a few of the capabilities of shell elements. Reference 4 lists many other test cases required to fully check element performance characteristics. There are many different shell-element formulations in the literature, so other FEA programs may not yield the same results as Table 1.3. The analyst is encouraged to run similar test cases on the particular program of interest.

FE is an approximate solution. As the FE mesh is made finer, more degrees of freedom (DOF) are added to the model, and the approximation improves. For example, if the cantilevered beam above is increased from 8 to 64 Tria3 elements, the tip displacement error is reduced from 70% to 10%. In FE theory, as the number of elements is increased, with their resulting size (h) reduced, the improvement in accuracy is called h -convergence. If the polynomial order (p) of the elements is increased, the response is called p -convergence. Analysts are encouraged to try increasing the model resolution to see if the results have converged. If a finer mesh significantly changes the response, the original model had not reached convergence, and there is no guarantee that the finer model has either.

The 3D solid elements in Fig. 1.13 can be tested in the same manner, with similar results. As seen in Table 1.4, the higher-order 20-noded hexagonal and 10-noded tetrahedral elements performed well in all cases. The 8-noded hexagonal element performed well but degraded somewhat with distortion. The 4-noded tetrahedral element performed badly except for constraint stress conditions. Based on tests like these, use of the 4-noded tetrahedral element is discouraged.

Table 1.4 3D solid element performance.

MEMBRANE (IN-PLANE) BEHAVIOR:			
<1> F_x = axial load = uniform, constant stress			
<2> M_z = moment in-plane = axial stress is linear in y			
<3> F_y = shear force in-plane = axial stress linear in x and y			
MODELS:	F_x	M_z	F_y
<a> Tet10–uniform	1.00	1.00	0.94
 Tet10–distorted	1.00	1.00	0.84
<c> Tet4–uniform	1.00	0.19	0.17
<d> Tet4–distorted	1.00	0.13	0.14
<e> Hex20–uniform	1.00	1.00	1.00
<f> Hex20–distorted	1.00	0.95	0.89
<g> Hex8–uniform	1.00	1.00	0.98
<h> Hex8–distorted	1.00	0.74	0.74

1.4.3 Structural analysis equations

NOTATION USED:

K = stiffness matrix	M = mass matrix	C = damping matrix
U = displacements	U' = velocity	U'' = acceleration
P = applied load	P_{cr} = buckling load	K_s = stress stiffening
Φ = mode shape	ω = forcing frequency	ω_n = natural frequency

Linear static analysis: small displacement, linear material. Solve by a variety of techniques such as Gauss elimination or Cholesky decomposition:

$$K U = P. \quad (1.28)$$

Nonlinear static analysis: contact, plasticity, large displacements. Solve by some form of a Newton's method or other iterative scheme:

$$K(U) U = P(U). \quad (1.29)$$

Linear buckling analysis: eigenvalue problem. Solve by the Lanczos method:

$$[K + \lambda_{cr} K_s] \Phi = 0,$$

where

$$P_{cr} = \lambda_{cr} P,$$

$$K_s = K_s(P). \quad (1.30)$$

Linear transient analysis: general time varying load. Solve by a numerical integration technique:

$$\mathbf{M} U'' + \mathbf{C} U' + \mathbf{K} U = P(t). \quad (1.31)$$

Nonlinear transient analysis: general time-varying load with contact, nonlinear materials, and large displacements. Typically, solve by an implicit integration for mildly nonlinear, and explicit integration for short duration, highly nonlinear:

$$\mathbf{M} U'' + \mathbf{C} U' + \mathbf{K}(U) U = P(t, U). \quad (1.32)$$

Direct-frequency response analysis: steady-state harmonic condition. Solve like a linear static analysis except with complex mathematics:

$$P = P e^{i\omega t} \Rightarrow U = U e^{i\omega t},$$

$$[-\omega^2 \mathbf{M} + i\omega \mathbf{C} + \mathbf{K}] U = P. \quad (1.33)$$

Real-natural-frequency analysis: no damping, no load. Solve with an eigenvalue technique, such as the Lanczos method:

$$[-\omega_n^2 \mathbf{M} + \mathbf{K}] \Phi = 0. \quad (1.34)$$

Modal-frequency response analysis: approach creates uncoupled equations. The substitution of $U = \sum z_j \Phi_j$ creates diagonal coefficient matrices $k = \Phi^T \mathbf{K} \Phi$ and $m = \Phi^T \mathbf{M} \Phi$, which reduces the direct frequency response equations to:

$$[-\omega^2 m + i\omega c + k] z = \Phi^T P. \quad (1.35)$$

For additional discussion of FEA techniques, see Refs. 2 and 3.

1.4.4 Thermal analysis with finite elements

Heat transfer problems are commonly solved by finite difference or finite element methods. In the finite element approach, the temperature is assumed to vary over an element according to a simple polynomial relationship as shown by the shape functions

$$T(x) = N_1 T_1 + N_2 T_2 = \sum N_j T_j = [N] \{T\},$$

$$N_1 = 1 - x/L,$$

$$N_2 = x/L, \quad (1.36)$$

where N_1 and N_2 are the same as the structural shape functions in Fig. 1.8. The thermal gradient and thermal flux can be written as a function of nodal variables T :

$$\begin{aligned} dT/dx &= (d/dx) \sum N_j T_j = \sum dN_j/dx T_j = [B] \{T\}, \\ q &= -\kappa dT/dx = [G] [B] \{T\}, \end{aligned} \quad (1.37)$$

where κ is the material conductivity. From variational principles, the thermal conductivity matrix can be derived as

$$k = \int B^T G B dV. \quad (1.38)$$

For the simple 1D rod, the thermal conductivity and structural stiffness matrix can be compared:

$$\begin{aligned} \text{Thermal: } k &= \frac{A\kappa}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \\ \text{Structural: } k &= \frac{AE}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}. \end{aligned} \quad (1.39)$$

Thus, if E is replaced by κ , the structural element becomes a thermally conducting element. By analogy, all common structural elements (1D, 2D, and 3D) can become heat-conducting elements with a change of material properties. For an additional discussion of finite elements in heat transfer, see Ref. 2.

1.4.5 Thermal analysis equations

NOTATION USED:

K = conduction matrix	C = capacitance matrix	R = radiation matrix
T = temperatures	Q = applied flux	H = convection matrix

Linear steady-state analysis: linear properties and constant convection coefficients. Solve by a linear solver:

$$\mathbf{K} T + \mathbf{H} T = Q. \quad (1.40)$$

Nonlinear steady-state analysis: radiation, temperature dependent properties. Solve by some form of Newton's method:

$$\mathbf{R} T^4 + \mathbf{K}(T) T + \mathbf{H}(T) T = Q(T). \quad (1.41)$$

Nonlinear transient analysis: general time varying loads and boundary conditions. Solve by numerical integration:

$$\mathbf{C} \dot{T} + \mathbf{R} T^4 + \mathbf{K}(T) T + \mathbf{H}(t, T) T = Q(t, T). \quad (1.42)$$

Some issues involved with heat transfer analysis include

- ◁1▷ surface elements that are required for convection and radiation, and their associated convection coefficients and emissivities;
- ◁2▷ the radiation view factor matrix is very costly to compute;
- ◁3▷ nonlinear control algorithms such as thermostats must be included; and
- ◁4▷ thermal and structural models that may use different meshes such that the nodal temperatures must be interpolated from the thermal model to the structural model.

1.5 Symmetry in FE Models

There are techniques within FEA to take advantage of symmetry within a structure to reduce the model size and computer resources required. Even with the advances in computing hardware technology, the use of symmetry to reduce model size is measurably helpful when computationally intensive analyses are undertaken such as detailed stress analysis or natural frequency extraction.

1.5.1 General loads

In the most general form of the use of symmetry the structure and boundary conditions (BC) must be symmetric with no such restriction on the applied loads. In Fig. 1.11, an example with one plane of symmetry shows that a general load case can be decomposed into a symmetric case and an antisymmetric case. In this example, only half of the structure is modeled. This approach requires some effort by the analyst to calculate the symmetric load (P_s) and antisymmetric load (P_a). Some programs, such as MSC/Nastran have automated this technique in a cyclic symmetry solution.

Definitions assuming mirror plane = yz plane

Mirror symmetry = conventional mirror behavior

$$T_{x'} = -T_x, T_{y'} = T_y, T_{z'} = T_z$$

Antisymmetry = negative mirror

$$T_{x'} = T_x, T_{y'} = -T_y, T_{z'} = -T_z$$

Asymmetry = not symmetric

1.5.2 Symmetric loads

A common special case is a symmetric structure with symmetric loads shown in Fig. 1.14. To solve this problem, the model is one-half of the full structure with

symmetric boundary conditions on the cut. Only loads appearing on the modeled half are used, with loads on the symmetry plane cut in half.

Typical models of mirrors on three-point supports are shown in Fig. 1.16. The one-half models may be used for general loads by using the combining two cases (symmetric and antisymmetric), as shown in Fig. 1.14. There are several common load cases that can be run on the half model using only symmetric boundary conditions, including gravity in x , gravity in z , isothermal temperature change, radial thermal gradient, thermal gradient in x , and thermal gradient in z . The half model with antisymmetric boundary conditions can solve gravity in y and thermal gradient in y .

For natural frequency analysis, a half model may be used. Note that symmetric structures have both symmetric and antisymmetric mode shapes. Thus, when using a half model to calculate dynamic modes or buckling modes, modes must be calculated with symmetric BC, and again with antisymmetric BC, to find all modes. Both BC must be used in dynamic and buckling analyses, because the lowest mode may be either symmetric or antisymmetric.

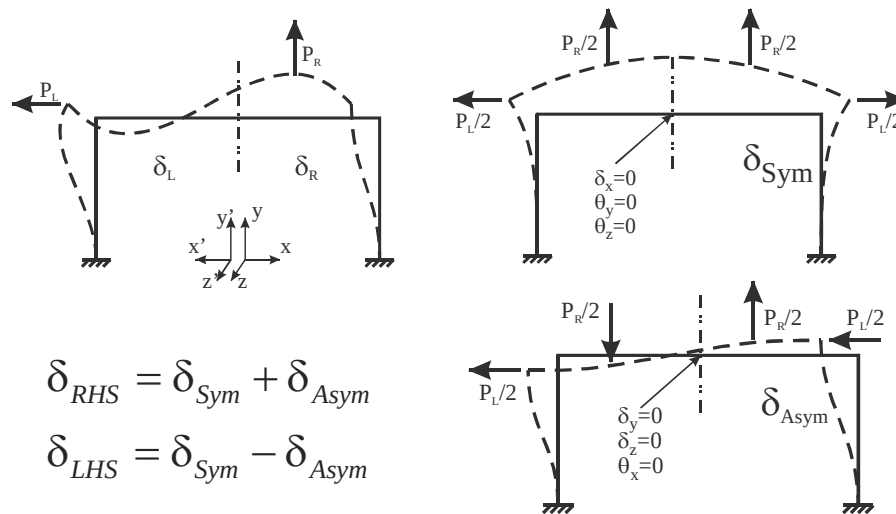


Figure 1.14 Symmetric structure with general load.

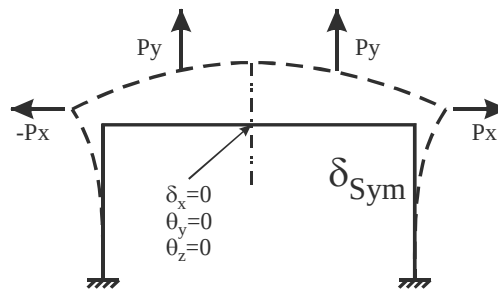


Figure 1.15 Symmetric structure with symmetric load.

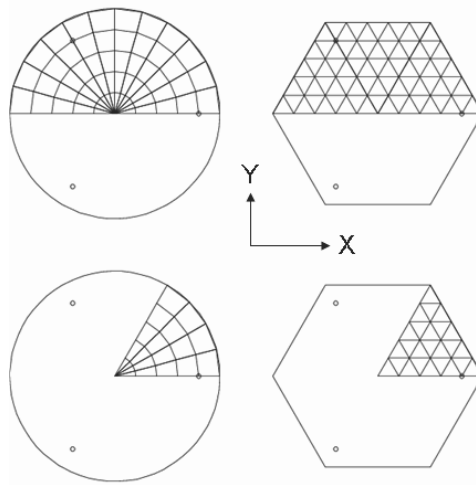


Figure 1.16 Typical symmetric models in optics.

The 1/6 model with symmetric boundary conditions on both cuts can solve load cases for gravity in z , isothermal temperature change, radial thermal gradient, and thermal gradient in z . The 1/6 model with symmetric boundary conditions on one cut and antisymmetric boundary conditions on the other is mathematically impossible. Symmetry planes are infinite planes, so every plane cuts the structure in half. In Fig. 1.17(a), three symmetry planes reduce the modeled structure to a 1/6 model (60 deg). It is possible to create a 1/3 model (120 deg), but it would contain a central plane of symmetry. Thus, half of the 1/3 model would be a reflection of the other half. No new information would be gained over the 1/6 model. A submodel containing both symmetric and antisymmetric BC must have an even number of cutting planes. In Figure 1.17(b), four cutting planes with alternating symmetric and antisymmetric BC is a viable model.

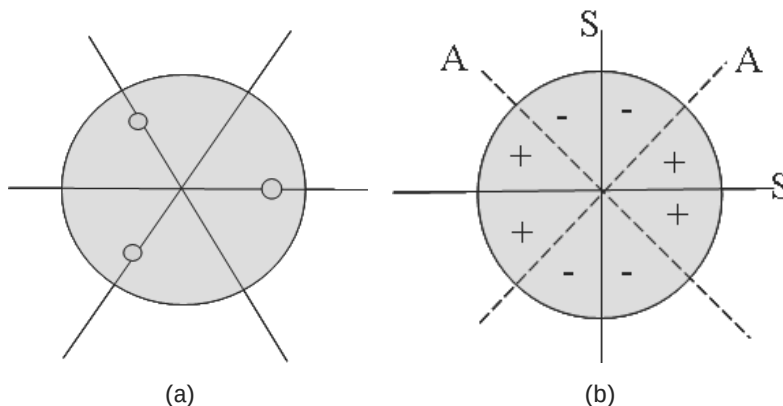


Figure 1.17 Three and four planes of symmetry.

1.5.3 Modeling techniques

NOTATION USED:

d_N = displacement normal to the symmetry plane

d_{T1}, d_{T2} = displacements in the symmetry plane

Θ_N = rotation normal to the symmetry plane

Θ_{T1}, Θ_{T2} = rotations in the symmetry plane

For any general orientation, the model must use a displacement-coordinate system that aligns with the plane of symmetry. The appropriate boundary conditions are

Symmetric BC	Antisymmetric BC	
$d_N = 0$	$\Theta_N = 0$	(1.43)
$\Theta_{T1}, \Theta_{T2} = 0$	$d_{T1}, d_{T2} = 0$	

For a pie-shaped sector of an optic (Fig. 1.17), cylindrical coordinates are naturally defined, so the symmetric BC are

$$\begin{aligned} d_\Theta &= 0, \\ \Theta_R &= \Theta_Z = 0. \end{aligned} \quad (1.44)$$

If a node exists on axis of the cylindrical system, only axial displacement (d_z) is free to move. For axial points, the symmetric BC are

$$d_R = d_\Theta = 0, \Theta_R = \Theta_\Theta = \Theta_Z = 0. \quad (1.45)$$

Solid elements are 3D in geometry, so they cannot lie in a plane of symmetry. However, 1D beams and 2D plates may lie in the plane of symmetry.

FOR 2D PLATES/SHELLS WITH ORIGINAL THICKNESS AT T_0 :

Membrane thickness: $T_m = T_0/2$

Bending Inertia: $I_b = I_0/2 = 4(T_m^3/12)$

or bending ratio: $R_b = 4.0$

Transverse shear factor scales T_m , so use original R_s

Stress recovery: $z = T_0/2$, not $T_m/2$

FOR 1D BEAMS WITH ORIGINAL PROPERTIES A_0, I_0, J_0, K_0, C_0 :

Cross-sectional area: $A = A_0/2$ (cut in half)

Bending inertia: $I = I_0/2$ (cut in half)

Torsional factor: $J = J_0/2$ (cut in half)

Transverse shear factor: $K = K_0$ (use original)

Stress recovery location: $C = C_0$ (use original)

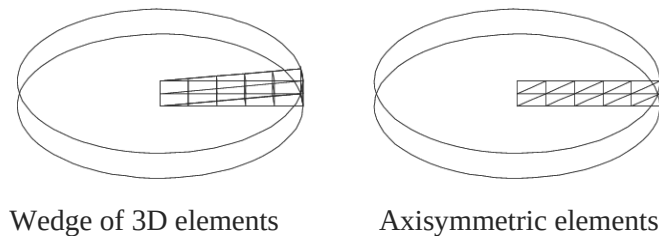


Figure 1.18 Axisymmetric models.

An element lying *in* the plane of symmetry must have its stiffness cut by half so that upon reflection, the other half will be added. This action is not always the same as cutting a thickness by half and then computing the stiffness, because bending properties are a function of thickness cubed.

1.5.4 Axisymmetry

Many optical elements, such as circular lenses, are axisymmetric in geometry and mounted in a ring-type mount. Another example would be a lightweight mirror sitting on an air-bag test support with the core structure modeled with smearable (effective) properties. For these applications, an axisymmetric model is an option if the loads are also axisymmetric, such as axial *g* loads or axisymmetric temperature distributions.

The analyst may choose between using special purpose axisymmetric elements or creating a thin wedge with conventional 3D elements and symmetric BC (Fig. 1.18). The wedge model has the advantage that it can easily be expanded to a full 3D model to study nonsymmetric effects. In most finite element programs, axisymmetric elements have limited capabilities and may not be mixed with other element types.

1.5.5 Symmetry: pros and cons

Even though a structure displays symmetry, the analyst must consider the advantages and disadvantages of using symmetric models before starting an analysis. A few of the considerations are as seen in Table 1.5.

1.6 Model Checkout

The results of any FE analysis should be considered guilty until proven innocent. The analyst must take full advantage of the model pre-processor to conduct as many model checks as possible, including

- <1> duplicate nodes and elements,
- <2> free boundaries,
- <3> surface normals, and
- <4> element geometry quality.

As additional proof of a valid model, checkout runs should be made as follows:

Table 1.5 Considerations for the use of symmetric models.**ADVANTAGES OF USING SYMMETRIC MODELS:**

- <1> Faster modeling.
- <2> Faster run times.
- <3> Smaller input and output files.
- <4> Smaller databases.

DISADVANTAGES OF USING SYMMETRIC MODELS:

- <1> Requires multiple solutions and combinations if it is an asymmetric load.
- <2> Requires multiple BC runs to get all dynamic and buckling modes.
- <3> Requires interpretation of results for imaged segments.
- <4> Cannot get full model plots easily.

RIGID-BODY ERROR CHECK

- <1> Remove all real BC; ground one node in all six DOF.
- <2> Apply six load cases of unit motion in each DOF at the grounded node.
- <3> Motion should be exactly stress free.
- <4> This finds “hidden” reactions to ground and bad MPCs
- <5> Image-motion equations should satisfy this check.

FREE-BODY MODES CHECK

- <1> Remove all boundary conditions.
- <2> Calculate natural frequencies.
- <3> Model should have six, and *only* six, zero modes (rigid body modes).
- <4> Compare values of f_{1-6} to f_7 as an indication of modeling problems:

$$f_{\text{rigid-body}} \ll f_{\text{elastic}}$$

- <5> This finds hidden reactions to ground and bad MPCs.
- <6> This also finds mechanisms that prevent static solutions.

STATIC LOAD CHECK

- <1> Apply a 1-g static load case in each coordinate direction.
- <2> Check that it is reasonable and compare to intuition.
- <3> Compare to known solutions of similar structures.
- <4> Check for expected symmetry in results.
- <5> Check equilibrium ($\Sigma \text{Forces} = \Sigma \text{Reactions}$).

- ◁6> Check error measures and warning messages.
- ◁7> Check mass property summary; compare to known mass and cg.

THERMAL SOAK CHECK

- ◁1> Temporarily set all CTE to a common value (e.g., 10 ppm).
- ◁2> Apply a kinematic boundary condition (e.g., one node for all six DOF).
- ◁3> Apply a significant isothermal load (e.g., 100 °C).
- ◁4> Check to see if all resulting stresses are zero.
- ◁5> This locates rigid links or constraints that prevent thermal growth.
- ◁6> Spherical optics radius of curvature changes by

$$\Delta RoC = RoC * \alpha * \Delta T.$$

In addition, the analyst should make as many checks as possible against hand solutions, classical textbook solutions, and engineering intuition. Whenever possible, compare results to experimental data from prototype structures or previous designs.

1.7 Summary

Modern analysis tools are very valuable, but the burden is still on the *engineer* to:

- ◁1> Understand structural/thermal/optical theory.
- ◁2> Understand FE theory and assumptions.
- ◁3> Understand details of the analysis program.
- ◁4> Understand details of the pre/post-processor program.
- ◁5> Make modeling decisions and assumptions.
- ◁6> Verify the model.
- ◁7> Interpret the results and draw conclusions.
- ◁8> Document the model and assumptions, and report the results.

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3. Cook, R. D., Malkus, D. S., Plesha, M. E., and Witt, R. J., *Concepts and Applications of Finite Element Analysis*, 4th Ed., John Wiley & Sons, New York (2002).
4. Macneal, R. H., *Finite Elements: Their Design and Performance*, Marcel Dekker, New York (1993).

Appendix

A.1 RMS

Root-mean-square (RMS) is calculated from N values of discrete data x_i :

$$\text{RMS} = \sqrt{\frac{1}{N} \sum_{j=1}^N x_j^2}. \quad (1.46)$$

In optics, RMS is used as a measure of surface distortion.

d_j = displacement of node j normal to optical surface

A_j = surface area associated with node j

A_t = total surface area

$w_j = A_j/A_t$ = fraction of area at node j = area weighting

$$\text{RMS} = \sqrt{\frac{1}{A_t} \sum_{j=1}^N A_j d_j^2} = \sqrt{\sum_{j=1}^N w_j d_j^2}. \quad (1.47)$$

A.2 Peak-to-Valley

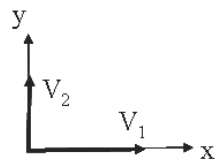
Another measure of surface distortion of an optical surface is the peak-to-valley (P–V) distortion which is the difference between high point on the surface to the low point on the surface:

$$P - V = \max(d_i) - \min(d_i). \quad (1.48)$$

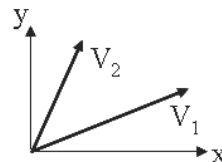
A.3 Orthogonality

Two vectors are said to be orthogonal if their dot product is zero. The scalar product of vectors, where $V_1 = V_{1x}i + V_{1y}j$ is given by

$$V_1 \bullet V_2 = |V_1||V_2|\cos\Theta = (V_{1x} * V_{2x}) + (V_{1y} * V_{2y}). \quad (1.49)$$



Orthogonal $V_1 \bullet V_2 = 0$



Not Orthogonal $V_1 \bullet V_2 > 0$

Functions are said to be orthogonal if they satisfy the relationship:

$$\iint \Phi_1 \Phi_2 dA = 0. \quad (1.50)$$

For example, Zernike polynomials satisfy the above equation when integrated over a full circular optic and represent continuous data.

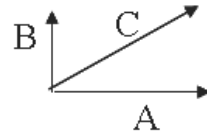
A.4 RSS

Root-sum-square (RSS) is calculated from N values of discrete data x_i :

$$\text{RSS} = \sqrt{\sum_{j=1}^N x_j^2}. \quad (1.51)$$

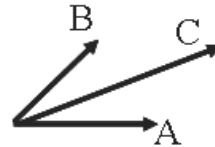
If two orthogonal (uncorrelated) vectors are combined, the resultant is found by RSS:

$$C = \sqrt{A^2 + B^2}$$



If two non-orthogonal (correlated) vectors are combined, the resultant is not found from a simple RSS:

$$C = \sqrt{A^2 + B^2 - 2AB \cos \Theta}$$



In optics, RMS surface errors $E_1, E_2, E_3, \dots$ caused by “independent” sources are often combined with the RSS equation:

$$E_{\text{Total}} = \sqrt{\sum_{j=1}^N E_j^2}. \quad (1.52)$$

These could be RMS surface errors due to:

E_1 = gravity distortion

E_2 = polishing error

E_3 = thermal distortion

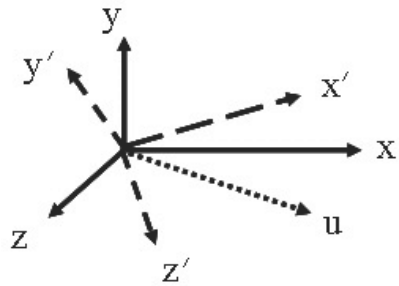
This is usually an approximation, because errors often have some coupling. If surface errors are coupled, the resultant cannot be found from the RMS of the

individual sources. The deformed surface must be combined (accounting for sign) and the RMS of the net surface calculated.

Since Zernike polynomials are orthogonal (uncoupled) in many situations, their individual RMS values may be combined by RSS. Individual cell quilting in lightweight mirrors is independent of global Zernike polynomials, allowing RMS due to quilting to be combined with global behavior by RSS.

A.5 Coordinate transformation for vectors

To transform displacements⁴ (or other vectors) from one coordinate system (x, y, z) to another (x', y', z'), a transformation matrix of cosines $[\Lambda]$ is used



$$[\Lambda] = \begin{array}{c|ccc} & x & y & z \\ \hline x' & l1 & m1 & n1 \\ y' & l2 & m2 & n2 \\ z' & l3 & m3 & n2 \end{array}$$

$$\{u'\} = [\Lambda] \{u\} \text{ and } \{u\} = [\Lambda]^T \{u'\}, \text{ where } [\Lambda]^T = [\Lambda]^{-1}$$

A.6 Coordinate transformation for stresses or materials

The direction cosines defined above are used to transform strain, stress, and material matrices⁴:

To transform strain:

$$\{\epsilon'\} = [T_\epsilon] \{\epsilon\} \text{ and } \{\epsilon\} = [T_\epsilon]^{-1} \{\epsilon'\}.$$

To transform stress:

$$\{\sigma'\} = [T_\epsilon]^{-T} \{\sigma\} \text{ and } \{\sigma\} = [T_\epsilon]^T \{\sigma'\}.$$

To transform material properties:

$[E] = 6 \times 6$ material matrix,

$$[E] = [T_\epsilon]^T [E'] [T_\epsilon] \text{ and } [E'] = [T_\epsilon]^{-T} [E] [T_\epsilon]^{-1}.$$

The transformation matrices are

$$[T_\varepsilon] = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \quad [T_\varepsilon]^{-T} = \begin{bmatrix} T_{11} & 2T_{12} \\ .5T_{21} & T_{22} \end{bmatrix},$$

where the submatrices are

$$[T_{11}] = \begin{bmatrix} l_1^2 & m_1^2 & n_1^2 \\ l_2^2 & m_2^2 & n_2^2 \\ l_3^2 & m_3^2 & n_3^2 \end{bmatrix} \quad [T_{12}] = \begin{bmatrix} l_1 m_1 & m_1 n_1 & n_1 l_1 \\ l_2 m_2 & m_2 n_2 & n_2 l_2 \\ l_3 m_3 & m_3 n_3 & n_3 l_3 \end{bmatrix}$$

$$[T_{21}] = \begin{bmatrix} 2l_1 l_2 & 2m_1 m_2 & 2n_1 n_2 \\ 2l_2 l_3 & 2m_2 m_3 & 2n_2 n_3 \\ 2l_3 l_1 & 2m_3 m_1 & 2n_3 n_1 \end{bmatrix} \quad [T_{22}] = \begin{bmatrix} l_1 m_2 + l_2 m_1 & m_1 n_2 + m_2 n_1 & n_1 l_2 + n_2 l_1 \\ l_2 m_3 + l_3 m_2 & m_2 n_3 + m_3 n_2 & n_2 l_3 + n_3 l_2 \\ l_3 m_1 + l_1 m_3 & m_3 n_1 + m_1 n_3 & n_3 l_1 + n_1 l_3 \end{bmatrix}.$$

A.7 Factor of safety, margin of safety, model uncertainty

The following definitions are commonly used in the aerospace industry:

Factor of safety (FS) is a design requirement enforced upon a design to get design allowable (σ_{Allow}). The FS is based on political and economic decisions, such as the cost of failure. There are design allowable stresses for each possible failure mode. In the following equation, σ_{Fail} may be ultimate, yield, or microyield stress:

$$\sigma_{Allow} = \sigma_{Fail} / FS. \quad (1.53)$$

Typical Factors of Safety

$FS = 2.0$ on σ_{ult} = fracture

$FS = 1.4$ on σ_y = yield

$FS = 1.0$ on $\sigma_{\mu y}$ = microyield

Margin of safety (MS) is a measure of over or under design. If MS is positive, the stresses are below the allowable and thus acceptable. If the MS is negative, the stresses exceed the allowable and the design fails to meet requirements. In the following equation, σ_{Peak} is the maximum stress found from analysis:

$$MS = \frac{\sigma_{Allow}}{\sigma_{Peak}} - 1.0. \quad (1.54)$$

Model uncertainty factor (MUF) is used to account for a variety of “uncertainties” in analysis predictions such as:

- modeling coarseness of early models (too stiff),
- modeling errors (minor) of early models not thoroughly debugged,
- under-prediction of loads from coarse “early” models,
- joints assumed rigid rather than flexible as in detailed models,
- overlooking mass of wiring, insulation, attachments, etc., and
- allowing for some future “minor” design updates.

In preliminary design, predictions for displacement and stress commonly use a MUF of 15%. Thus, multiply FEA-predicted results (σ_{FEA}) by MUF (1.15) to compare to requirements:

$$\sigma_{Peak} = MUF * \sigma_{FEA} < \sigma_{Allow}. \quad (1.55)$$

As the design becomes more mature and analysis models are more detailed and accurate, the MUF factor is reduced (or eliminated).

◀Chapter 2▶

Introduction to Optics for Mechanical Engineers

This chapter presents the fundamentals of optics, image formation, and performance metrics to help mechanical engineers improve their understanding of optical system terminology. In addition, the material is intended to help mechanical engineers better relate their mechanical designs and analyses, including the impact of environmental factors on the performance of optical systems.

2.1 Electromagnetic Basics

Light is a transverse electromagnetic wave where the electric and magnetic fields vibrate or oscillate perpendicular to the direction of propagation. Light propagation in one dimension is illustrated in Fig. 2.1. The mathematical equation describing the electric-field vector E is given by

$$E(z, \tau) = Ae^{i(\omega\tau - kz)}, \quad (2.1)$$

where the electric field is a function of both position z and time τ . The amplitude of the wave is denoted by A , and the phase is given by $\omega\tau - kz$, where $k = 2\pi/\lambda$ is the wave number, and ω is the angular frequency of the light wave. This representation may be extended to 3D waves such as planar, cylindrical, and spherical waves typical of imaging systems.

When an electromagnetic wave enters a medium such as a lens element, the speed of the wave decreases. The ratio of the speed of the wave in a vacuum to the speed in a medium is called the *index of refraction*, n . The index of refraction for common optical glasses in the visible spectrum ranges from 1.5 to 2.0. The index of refraction for common IR materials extends from 1.5 to 4.0.

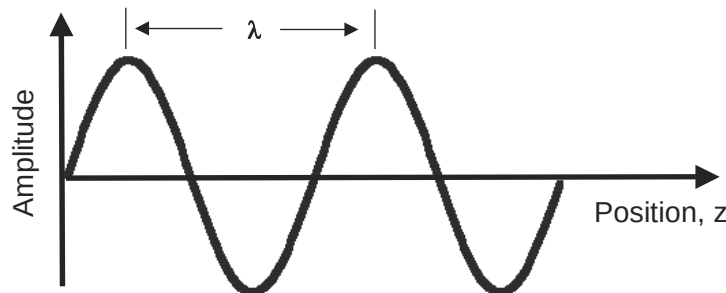


Figure 2.1 1D representation of an electromagnetic wave.

The *wavelength* λ is the distance an electromagnetic wave travels in one cycle. Wavelengths in the visible spectrum range from 0.45–0.70 μm . Electromagnetic radiation may also be described by its *optical* frequency ν , given in number of cycles per second (Hz). For example, the optical frequency for light at a wavelength of 546 nm is 5.5×10^{14} Hz. The relationship between wavelength and frequency is given by

$$\nu = \frac{c}{\lambda}, \quad (2.2)$$

where c is the speed of light.

Two electromagnetic waves are considered in-phase when the peaks and troughs for each wave coincide. Two waves out-of-phase with each other are shown in Fig. 2.2. Since a full cycle represents 360 deg or a wavelength, the phase difference between two waves may be expressed either in deg or in waves. For example, two waves out-of-phase by 90 deg are out-of-phase by a quarter wavelength. Two waves that have a phase difference that is an integer number of waves are considered in-phase because they overlap.

2.2 Polarization

Many optical systems use polarized light or polarizing optics to control and manage the characteristics of light. A well-known example is polarized sunglasses, which are often used to reduce the reflection of light or glare from water. This section defines and describes various states of polarization. The impact of mechanical stress on the state of the polarization is discussed in Chapter 8.

As previously stated, light is a transverse electromagnetic wave where the electric field vibrates perpendicular to the direction of propagation. Light, such as natural light, where the direction of the electric-field vector varies randomly and rapidly (approximately every 10^{-8} sec), is known as unpolarized light and is illustrated in Fig. 2.3. Linearly polarized or plane polarized light describes light whose electric-field vector oscillates in a plane known as the plane of vibration as shown on the right side in Fig. 2.3. Here, the plane of vibration is the xz plane, and the direction of the electric field moves up and down along the x axis.

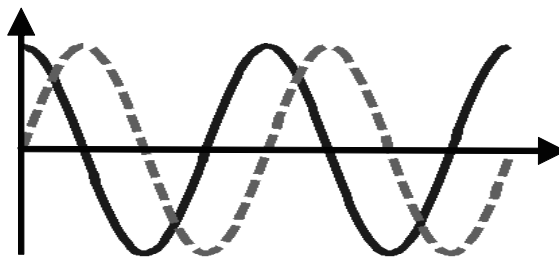


Figure 2.2 Two electromagnetic waves out-of-phase.

In general, since the electric field is a vector quantity, the electric field may be decomposed into components E_x and E_y along an arbitrary set of x and y axes, respectively. The relative magnitude and phase of the components describes the state of polarization.

For linear polarization, the electric-field components are in-phase with each other, as shown in Fig. 2.4. In this example, the amplitudes of E_x and E_y are equal, and their sum results in an electric-field vector vibrating in a plane at 45 deg. Elliptical polarization occurs when E_x and E_y are out-of-phase. A special case of elliptical polarization is circular polarization, which occurs when E_x and E_y are of equal amplitude and out-of-phase by 90 deg, as shown in Fig. 2.5. Here, the tip of the electric-field vector carves out a helix of circular cross-section.

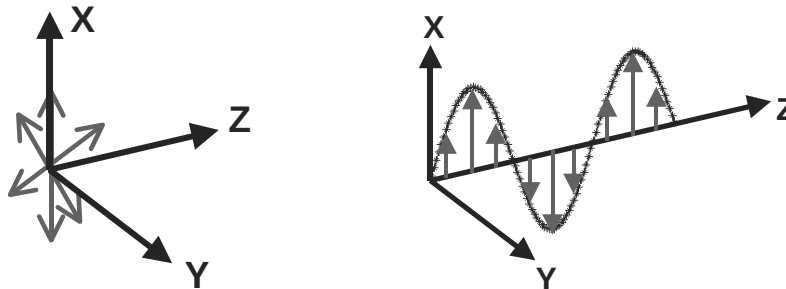


Figure 2.3 Unpolarized light is illustrated on the left and linearly polarized light on the right (arrows represent the direction of the electric field).

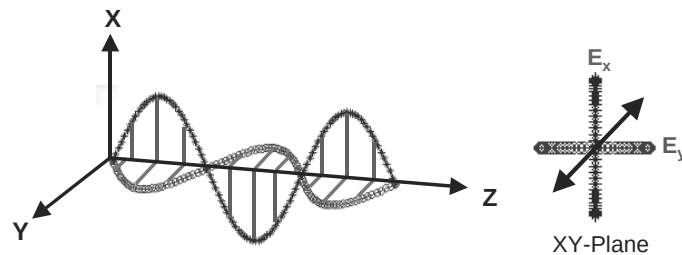


Figure 2.4 Linearly polarized light at 45 deg.

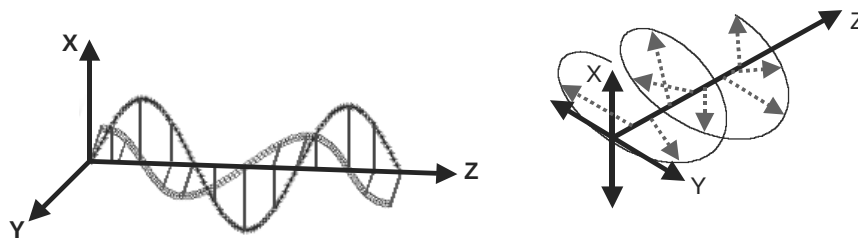


Figure 2.5 Circular polarization.

2.3 Rays, Wavefronts, and Wavefront Error

The propagation of light waves from a point source in an isotropic and homogeneous medium takes a spherical shape, as shown in Fig. 2.6. At any instant in time, each surface joining all points of constant phase is called the wavefront. Neighboring surfaces of constant phase are separated by a wavelength. Rays are fictitious entities normal to each wavefront surface and are useful for understanding and analyzing optical systems. The optical distance traveled by a ray is known as the *optical path length* (OPL). The OPL is computed as the physical distance a ray has traveled, s , multiplied by the index of refraction of the medium in which it travels, as given by

$$OPL = \int n(s) ds \quad (2.3)$$

Across the surface of a given wavefront, the OPL is the same for each point. This is the basis for how images are formed by an optical system. For example, consider a diverging spherical wavefront incident upon a lens element shown in Fig. 2.6. After the wavefront passes through the lens element, the wavefront is converging. The reversal of the wavefront curvature is a consequence of the center rays traveling a greater distance through the lens element and slowing down relative to the edge rays.

For an optical system to form a perfect image point, the exiting wavefront must be spherical, and the rays normal to the wavefront must converge to the wavefronts' center of curvature. The departure of the OPL of the actual wavefront to a spherical reference wavefront measured over the wavefront surface is a measure of *wavefront error*. The difference in OPL is known as the *optical path difference* (OPD). A depiction of an optical system producing wavefront error is shown in Fig. 2.7. Wavefront error is commonly quantified by the peak-to-valley error (P-V) and by the root-mean-square (RMS) error. P-V errors represent the difference between the maximum and minimum OPD over the wavefront, as shown in Fig. 2.8. The RMS is typically a more meaningful measure of wavefront error because it accounts for the deviation over

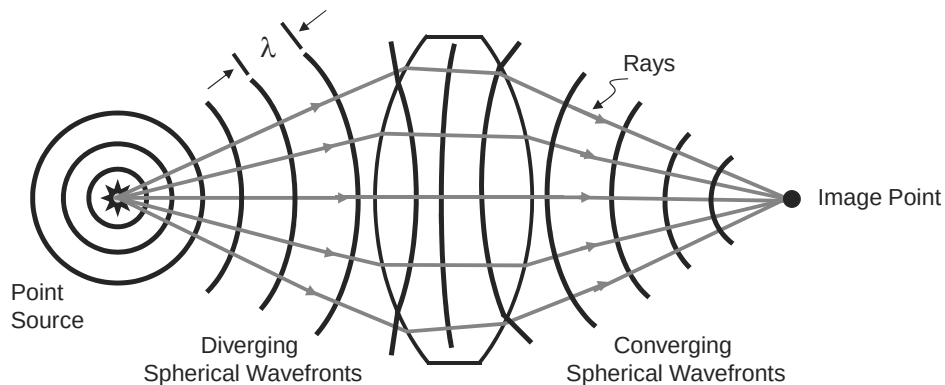


Figure 2.6 Lens element forming an image.

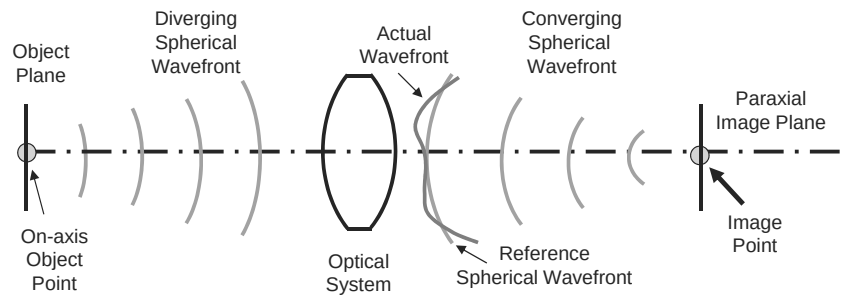


Figure 2.7 Lens element introducing wavefront error.

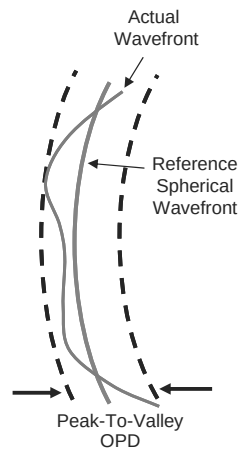


Figure 2.8 Wavefront error is the variation in optical path length between the actual wavefront and a spherical reference surface.

the entire surface of the wavefront. It is a simple extension to see how mechanical loads that deform the surface of an optical element create wavefront error. Furthermore, temperature and mechanical stress acting on an optical element modify the index of refraction of the material and hence introduce wavefront errors by changing the OPL at various points of the wavefront.

2.4 Pointing Error

Pointing error (commonly referred to as line-of-sight or boresight error) is the angular error between the desired pointing and the actual pointing direction of an optical system. Pointing errors are important for numerous types of optical systems including beam delivery, communication, and imaging systems. Pointing errors can be created by fabrication, alignment, and environmental influences that create deviations in the ideal position and shape of the optical elements. An illustration of pointing error due to a lateral displacement (decenter) of an optical element is shown in Fig. 2.9. Modeling methods to compute pointing errors due to mechanical and environmental disturbances are presented in Chapter 7.

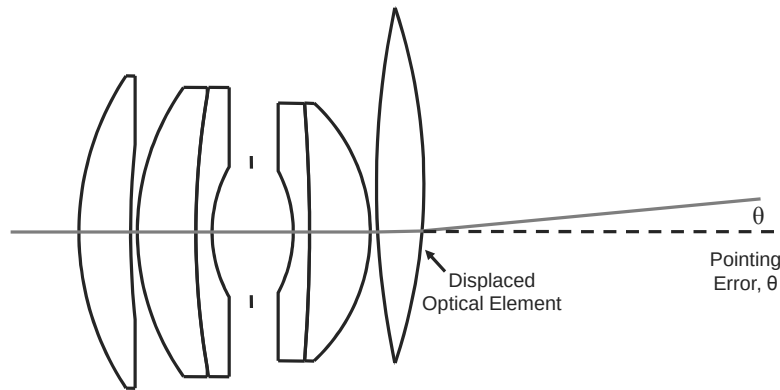


Figure 2.9 Pointing error due to lateral displacement of an optical element.

2.5 Optical Aberrations

Perfect imaging requires point-to-point correspondence between the object points and the image points. This is prevented by the presence of optical aberrations in an optical system prevents this point-to-point correspondence and degrades the performance of optical systems. One form of optical system aberrations are involves chromatic aberrations that, which are caused by refractive index changes with wavelength, where the location of the image point is a function of the wavelength or color of light. Axial and lateral color are examples of chromatic aberrations, as shown in Fig. 2.10, where the three wavelengths come to an image point at different axial (left) and lateral (right) locations.

Geometric aberrations are due to lens constructional parameters. that preclude point-to-point correspondence. Multiple elements and surfaces are often employed to minimize this class of aberration in a lens assembly. Simple geometric aberrations include tilt and defocus. Tilt places the image in the wrong orientation and defocus places the image in the incorrect axial location. The higher-order aberrations create a distorted image and include spherical aberration, coma, astigmatism, distortion, and field curvature. Spherical aberration is the variation of focal length with aperture. For an image of an on-axis object point, rays at the edge of the pupil focus at a different point than rays near the axis as shown in Fig. 2.11.

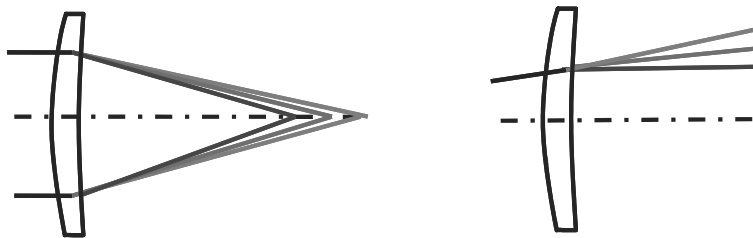


Figure 2.10 Chromatic aberrations: axial (left) and lateral color (right) illustrated using three wavelengths.

Coma is the variation of magnification with aperture, as shown in Fig. 2.12. For an off-axis point, rays traversing the edge of the aperture intersect the image plane at different heights than rays through the center of the aperture. Coma gets its name from its characteristic spot diagram that looks like a comet.

Astigmatism is created in the wavefront when the optical system has different powers in orthogonal planes, as shown in Fig. 2.13.

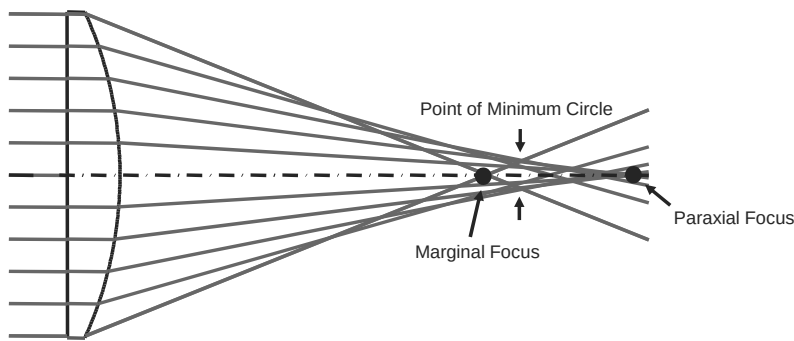


Figure 2.11 Spherical aberration is the variation of focal length with aperture.

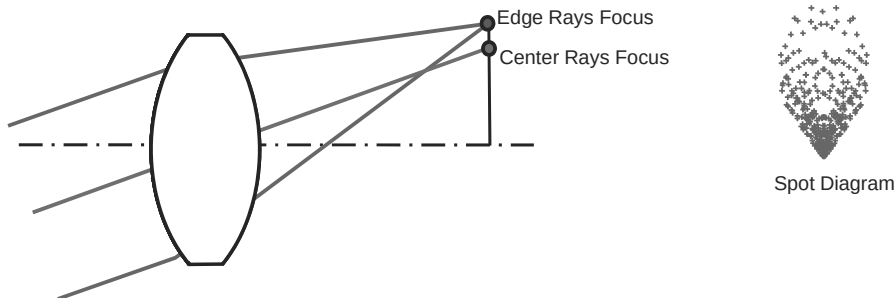


Figure 2.12 Coma is the variation in magnification with field angle.

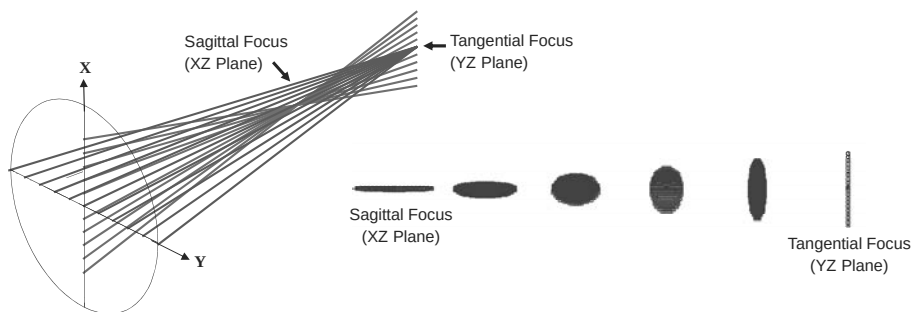


Figure 2.13 Astigmatism produces two foci in orthogonal planes.

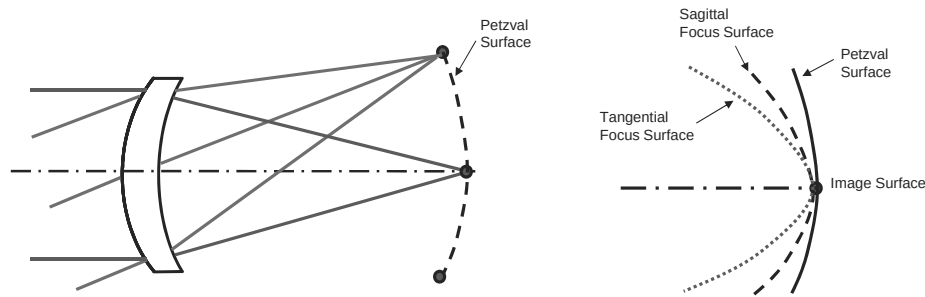


Figure 2.14 Field curvature creates an image on a curved surface.

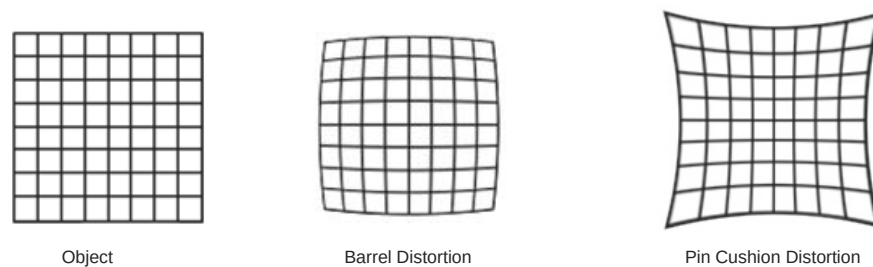


Figure 2.15 Barrel and pin cushion distortion.

Field curvature causes the image of a planar object to lie in a curved image plane. The curved image plane is known as the Petzval surface, as shown in Fig. 2.14. One solution to this form of aberration is to use a curved focal plane.

Distortion is a change in magnification with field of view. Off-axis points are imaged to the incorrect location, and thus images of rectilinear objects are not rectilinear. Barrel distortion occurs when the magnification decreases with distance, and pin cushion distortion occurs when the magnification increases with distance as illustrated in Fig. 2.15.

2.6 Image Quality and Optical Performance

A variety of optical performance metrics are used to measure the quality or the performance of an optical system. The image of an object point is never a perfect point but a smeared or blurred point whose physical extent is commonly referred to as the image blur, blur radius, or blur diameter. The goal for many imaging systems in optimizing optical performance is to minimize the size of the blur. There are many factors that may contribute to the inability of an optical system to produce a perfect point, including the effects of diffraction, chromatic and geometrical aberrations, fabrication errors, alignment errors, and environmental effects. However, if all is perfect, diffraction limits the quality of the image and hence provides the reference for which image quality is measured.

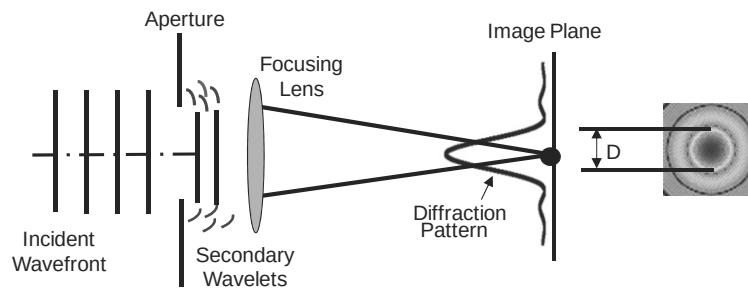


Figure 2.16 Diffraction effects of a circular aperture imaging a planar wavefront.

2.6.1 Diffraction

Diffraction is due to the wave nature of light and occurs at the boundary of obstacles in the light path that alter the amplitude and phase of an incident wavefront. The obstacle may be an aperture of an optical element or a mechanical support structure that causes the light to bend, or be redirected, from the paths predicted by geometrical optics. For example, the image of a point source at infinity for an optical system with a circular lens element is shown in Fig. 2.16. The interaction of the incident plane wavefront with the boundaries of the aperture results in the creation of secondary wavelets that constructively and destructively interfere. The image produced by the focusing lens is not a perfect point but a series of concentric light and dark rings. For an aberration-free system, the central bright spot is known as the *Airy disk* and contains 84% of the incident energy. The diameter of the Airy disk represents the smallest blur diameter that an optical system can produce and is given by

$$D = 2.44\lambda (f/\#), \quad (2.4)$$

where $f/\#$ (f -number) is a measure of the light-collecting properties of an optical system. As a rule of thumb, for visible systems operating at a wavelength near $0.5 \mu\text{m}$, the size of the Airy disk is equal to the f -number in microns.

An optical system is called *diffraction limited* when the effects of diffraction dictate the size of the blur diameter. An acceptable amount of wavefront error may exist in an optical system where the system is still considered diffraction-limited. The allowable wavefront error is given by the Rayleigh criterion, which states that diffraction-limited performance is maintained for up to a quarter-wave of OPD P–V. This corresponds approximately to a RMS wavefront error of $\lambda/14$.

2.6.2 Measures of image blur

Several optical performance metrics are used to measure the size of the image blur or blur diameter. The type of performance metric used depends on the application of the optical system. When the blur diameter approximates the size of the Airy disk, which is typical for high-performance optical systems, then

diffraction-based metrics are employed. If the blur diameter is much larger than the Airy disk, the effects of diffraction may be ignored, and geometric-based metrics may be used.

2.6.2.1 Spot diagram

Spot diagrams are created by tracing a grid of rays from a single object point through an optical system and plotting their intersection with the image plane. Spot diagrams are geometrically based and exclude the effects of diffraction. The distribution of points on the image plane is a measure of the size of the blur diameter. Commonly, a RMS spot diameter is computed that encloses approximately 68% of the energy. Spot diagrams are useful to determine the aberrations present in an optical system since each aberration produces a characteristic pattern. A spot diagram of a singlet lens exhibiting spherical aberration is shown in Fig. 2.17.

2.6.2.2 Point spread function and Strehl ratio

The *point spread function* (PSF) is another measure of the size and shape of the image of a point source. The PSF calculation includes both the effects of diffraction and geometrical aberrations. The PSF for an aberration-free system and for an optical system with coma error is shown in Fig. 2.18 and Fig. 2.19, respectively. Both 3D isometric views and intensity plots of the PSF are shown. The intensity plot uses a logarithmic scale to reveal the ring structure of the PSF more clearly. Notice how the energy in the aberrated case is spread over a much larger diameter than the aberration-free system.

The ratio of the peak intensity of an optical system's PSF to that of a perfect optical system is called the Strehl ratio. This is a useful measure for systems concerned with power delivery. Optical systems are considered diffraction-limited when the Strehl ratio is greater than 0.8. The relationship between Strehl ratio and wavefront error is provided as

$$\text{Strehl Ratio} = 1 - 4\pi^2 WFE_{RMS}^2. \quad (2.5)$$

This relationship is valid for diffraction-limited optical systems where the RMS wavefront error is less than $\lambda/14$.

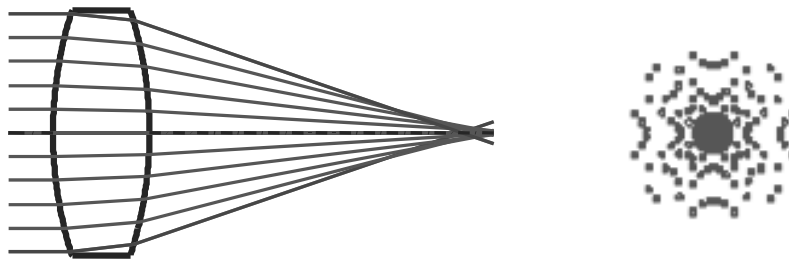


Figure 2.17 Spot diagram formed by a singlet lens exhibiting spherical aberration.



Figure 2.18 PSF for an aberration-free system.

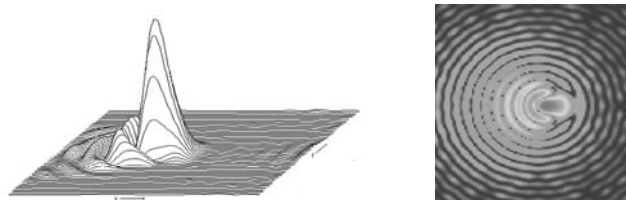


Figure 2.19 PSF for a system with coma error.

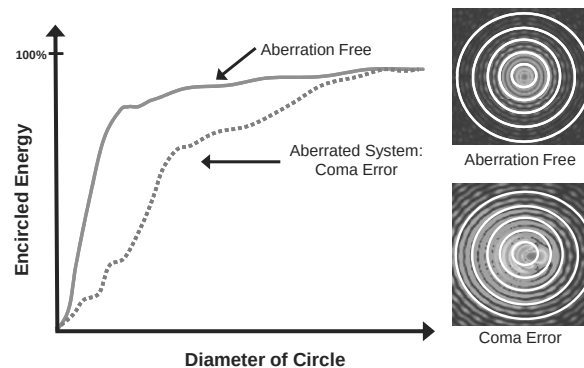


Figure 2.20 Encircled energy function for an unaberrated and aberrated image.

2.6.2.3 Encircled energy function

The encircled energy function is a plot of the energy contained in concentric rings of increasing diameter centered on the image centroid. An example of the encircled energy function is plotted in Fig. 2.20 for the aberration-free PSF and the aberrated PSF that are shown in Figs. 2.18 and 2.19.

2.6.3 Optical resolution

The ability of an optical system to resolve two objects is a common measure of optical performance. The Hubble Space Telescope, for example, can resolve two dimes from approximately 30 miles away. It should be clear that the effects of diffraction limit the resolution of an optical system as depicted in Fig. 2.21. As the diameters of the Airy disk for each image point increase, the intensity

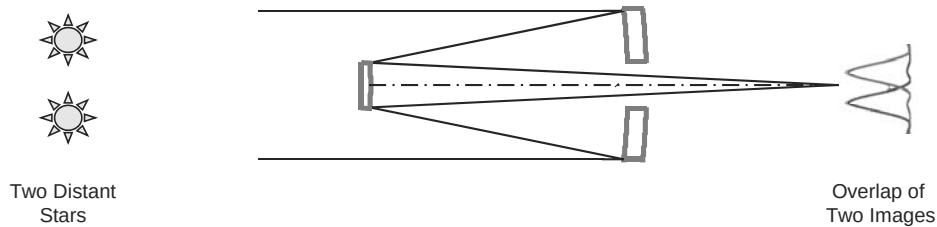


Figure 2.21 A measure of resolution is the ability of an optical system to resolve two point sources.

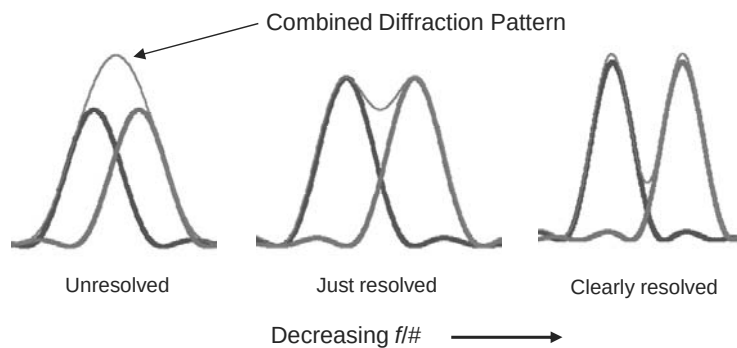


Figure 2.22 Combined intensity pattern for two point sources as a function of f -number.

distributions begin to overlap and resolution decreases. Thus, the same parameters controlling the size of the Airy disk dictate the resolution of an optical system—namely, the system f -number and the wavelength of light. The combined diffraction pattern of the image of two points is shown as a function of f -number in Fig. 2.22. As the f -number of the optical system decreases, the combined intensity distribution begins to show two distinct peaks representing two object points. For systems where the light source can be selected, a corresponding increase in resolution can be achieved by decreasing the wavelength of the source. For example, the optical lithography industry has increased the resolution in their optical instruments by decreasing their illumination wavelength. In addition, they have increased the numerical aperture (analog to $f/\#$ for finite conjugate systems) that also has increased resolution. These steps have allowed smaller feature sizes to be created on integrated circuits.

2.6.4 Modulation transfer function

A second, more comprehensive measure of the resolution of an optical system is given by the *modulation transfer function* (MTF). Here, the MTF considers the response of the optical system to sinusoidal intensity distributions of varying spatial frequency. This is illustrated for three spatial frequencies in Fig. 2.23. As

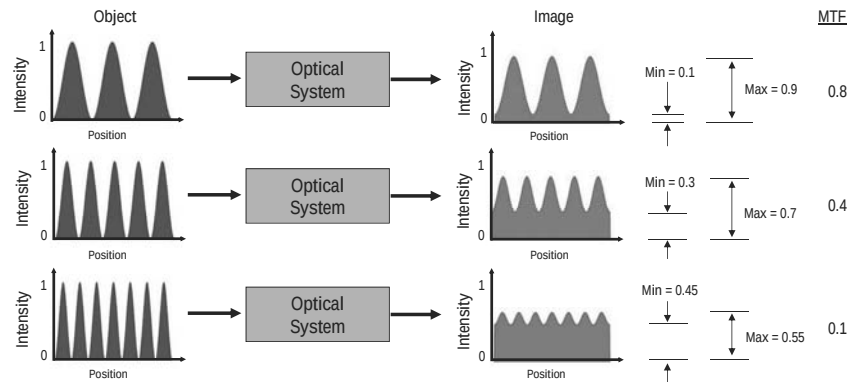


Figure 2.23 Image contrast computed for three spatial frequencies.

the spatial frequency of each object is increased, the more difficult it is for the optical system to distinguish the peaks from the valleys. Resolving ability is quantified by the contrast ratio (also known as modulation), which is given by the following relationship:

$$\text{Image Contrast} = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}, \quad (2.6)$$

where $I_{\max}$ is the maximum intensity, and $I_{\min}$ is the minimum intensity of the image. For the image to be an exact duplicate of the object, the peaks would have a value of one, and valleys would have a value of zero, yielding a contrast ratio of one. As resolving capability diminishes, the contrast ratio decreases, and there is little difference in the magnitude between the peaks and valleys. When the contrast ratio drops to zero, the optical system can no longer resolve the object, and a solid intensity pattern results. For incoherent light (light consisting of different wavelengths that are out of phase such as from the sun or light bulbs), the spatial frequency in which the optical system can no longer resolve is known as the cut-off frequency ρ_c , which is a function of the f -number and is given as

$$\rho_c = \frac{1}{\lambda(f/\#)}. \quad (2.7)$$

The MTF curve is computed by plotting image contrast as a function of spatial frequency, and is shown for a diffraction-limited system and an aberrated system in Fig. 2.24. Notice how the diffraction-limited system is able to resolve higher spatial frequencies as compared to the aberrated system. The MTF is a valuable quantitative description to understand the resolving capability of an optical system by measuring image contrast over a range of spatial frequencies. The mid- and low-end spatial frequencies are often important to image quality, not just the cut-off frequency.

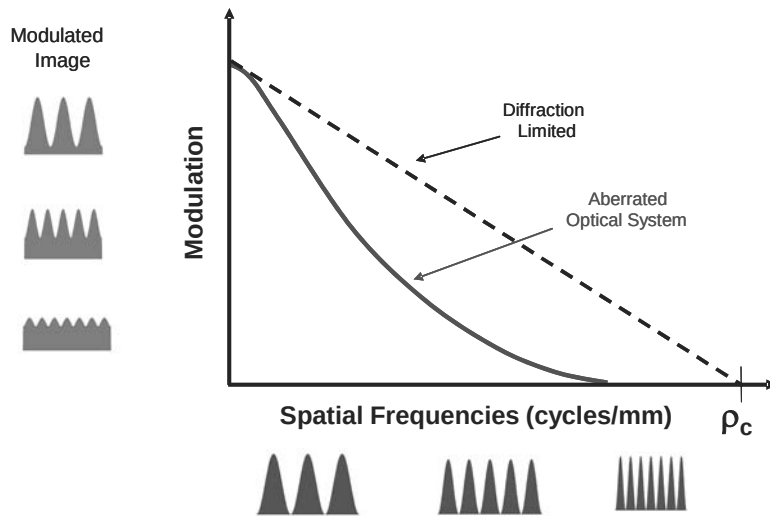


Figure 2.24 Modulation transfer function.

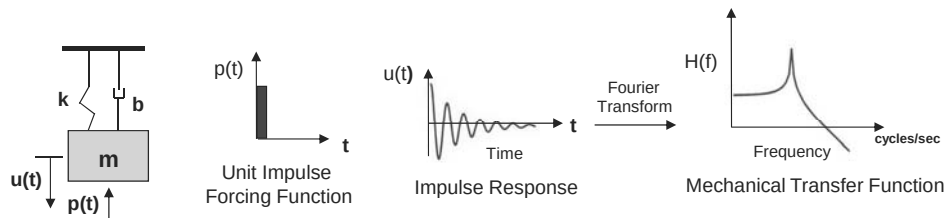


Figure 2.25 Mechanical system impulse response and transfer function.

A system MTF can be computed as the product of the MTFs of each of the components. For example, the MTF of a photograph generated by a digital camera using a telephoto lens can be computed as the product of the MTF of the camera, the telephoto lens, and the detector array.

2.7 Image Formation

Linear-systems theory may be used to describe a broad category of physical systems, including many optical and mechanical systems. The response of these systems may be characterized by their impulse response and transfer functions. (Here, we will consider image formation only for spatially broad, incoherent light sources such as incandescent light bulbs and the sun). Consider a single degree of freedom (DOF) mechanical system, as shown in Fig. 2.25. Subjecting this system to a unit-impulse forcing function (an infinitesimally short-duration impact force) produces a displacement of the mass known as the impulse response. The transfer function of the mechanical system is computed as the Fourier transform of the impulse response. In analogous fashion, the impulse response of the optical

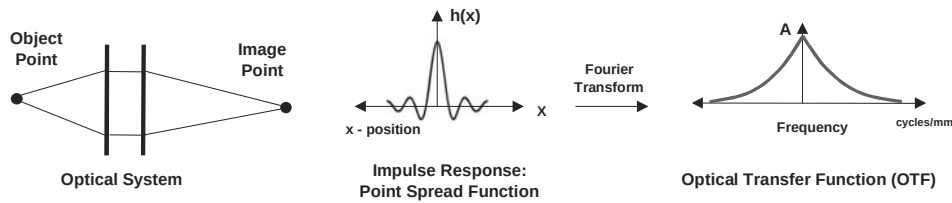


Figure 2.26 Optical system impulse response and transfer function.

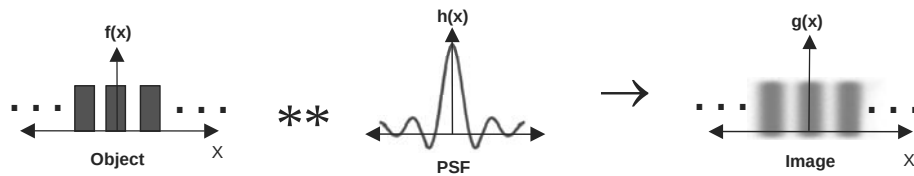


Figure 2.27 Image formation in the spatial domain (** represents the convolution operation).

system is the image of a point source, which is simply the PSF, as shown in Fig. 2.26. Taking the Fourier transform of the PSF yields the *optical transfer function* (OTF). Both the impulse response and the transfer function represent physical characteristics of the mechanical and optical system, and either can be used to compute the response of the system due to arbitrary inputs.

2.7.1 Spatial domain

Computing the response of a physical system using the impulse response requires use of the convolution operation or convolution integral (also known as the Duhamel or superposition integral). This is a common method to compute the response of a mechanical system to an arbitrary time history. This same mathematical operation may be used to compute the image of any object by convolving the object with the PSF. An illustration of incoherent image formation for a periodic rectangle function is shown in Fig. 2.27. Note how the boundaries of the image are blurred as compared to the sharp boundaries of the object. This is a consequence of the smoothing effect of the convolution operation. For an optical system to generate an image that is an exact duplicate of the object, the PSF would have to be a perfect, infinitesimally sized point. As the size of the PSF increases, the smoothing effect increases and the quality of the image decreases. This should help explain why it is so important for high-performance optical systems to minimize the size of the blur diameter.

2.7.2 Frequency domain

A simple way to think of computing the response of a linear system in the frequency domain is to consider the physical system acting as a frequency filter. For mechanical systems, loads that are a function of time are converted into harmonic frequency components expressed in cycles per second, or Hz. For

optical systems, objects are described by harmonic spatial frequency components typically expressed in cycles per millimeter.

The filtering aspect is dictated by the transfer function of the physical system. The filtering process is complex in that there are real and imaginary components. Think of the real part of the transfer function as an amplitude filter, and the imaginary part of the transfer function as a phase filter. The job of the transfer function is to determine the magnitude and relative phase of the response to each harmonic input. This is achieved by multiplying the transfer function by the harmonic input, which yields the spectral content of the output. An inverse Fourier transform is performed to convert back into the temporal or spatial domain.

For example, the image of a bar target of infinite extent, shown in Fig. 2.28, is computed using the frequency domain. The object is described using harmonic spatial frequencies computed using a Fourier series. The Fourier series representation of the object is given as

$$f(x) = \frac{A}{2} + \frac{2A}{\pi} \left[\cos 2\pi\xi x - \frac{1}{3} \cos 2\pi(3\xi)x + \frac{1}{5} \cos 2\pi(5\xi)x - \frac{1}{7} \cos 2\pi(7\xi)x + \dots \right]. \quad (2.8)$$

Several of the individual frequency components are plotted and graphically summed to illustrate how spatial frequencies may be used to represent the object in Fig. 2.29. An abbreviated object spectrum is plotted in Fig. 2.30.

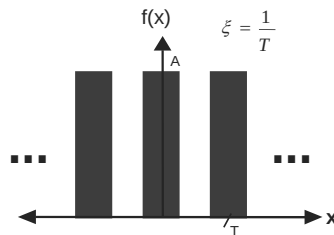


Figure 2.28 Bar target of infinite extent.

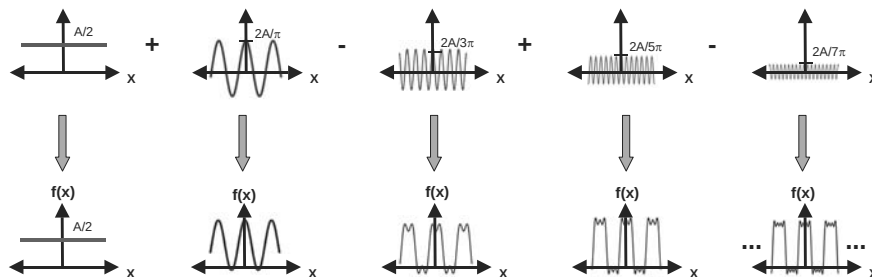


Figure 2.29 Harmonic spatial frequency components used to describe a bar target of infinite extent.

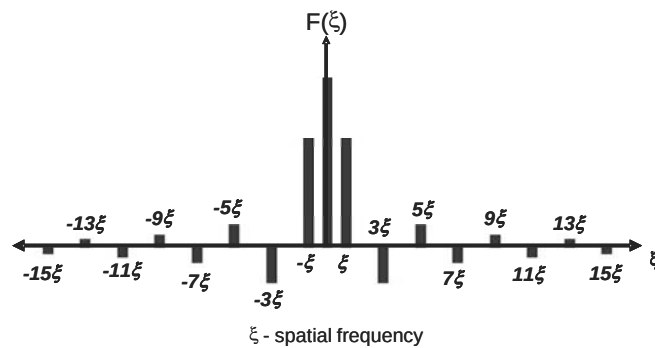


Figure 2.30 Spatial frequency content of a bar target of infinite extent.

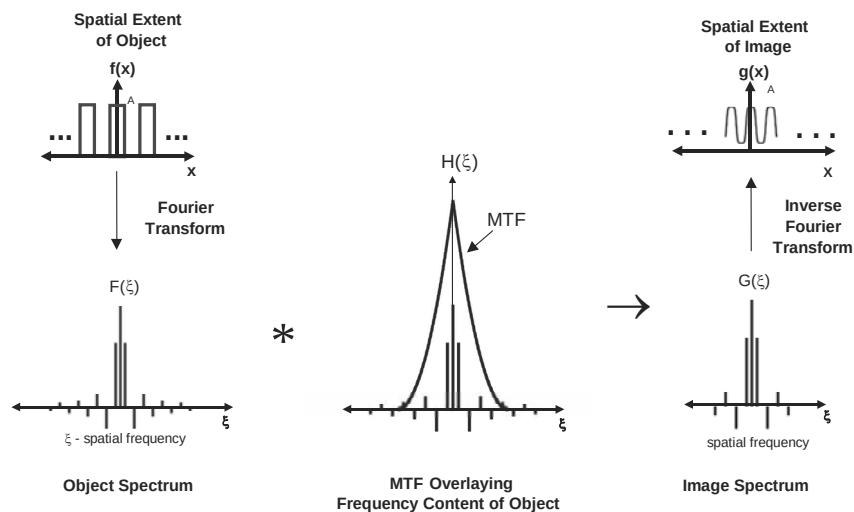


Figure 2.31 Image formation in the frequency domain.

The response or image of the optical system is controlled by the OTF, which determines how the system responds to each of the harmonic spatial frequencies that make up the object. The real part of the OTF or the amplitude filter determines the magnitude of the response for each component. The amplitude filter is just the MTF. The phase filter or *phase transfer function* (PTF) dictates the relative phase of each of the components. Computationally, image formation is computed by multiplying the optical transfer function by the object spectrum as shown in Fig. 2.31.

Note how the image spectrum is a truncated version of the object spectrum, which is a consequence of the filtering effect of the optical system. The higher-frequency components, which are responsible for the fine detail in the object, are cut off. This leads to an image that is a rounded or smoothed version of the object. The image in the spatial domain (the domain where the image can be “seen”) is computed by performing an inverse Fourier transform.

2.8 Imaging System Fundamentals

Several important imaging system definitions are highlighted in Fig. 2.32. The aperture stop is a physical aperture within the optical system that limits the amount of light that hits the detector plane. The image of the aperture stop in object space is known as the entrance pupil, which determines the on-axis size of the cone of light entering the optical system. The image of the aperture stop in image space is known as the exit pupil and determines the on-axis size of the cone of light exiting the optical system. Pupils are important properties of an optical system in that they represent the minimum diameter in a given space to pass all field angles of the optical system. It is common to locate optical elements at or near pupil locations to minimize the size of the optical elements including primary mirrors, steering mirrors, and deformable optics.

The field stop is the aperture that limits the field of view or the angular extent that the optical system can view. A field stop may be a window or the edges of a detector or a mechanical aperture added at an intermediate image to reduce stray light. The image of the field stop in object space is the entrance window, and in image space is the exit window.

The f -number ($f/\#$) of an optical system is defined as the ratio of the focal length of the optical system divided by the entrance pupil diameter. The f -number is a measure of the light-collecting properties of an optical system that dictates the illuminance of the image (power per area) along with many other characteristics of the optical system including depth of focus, size of the diffraction image, cut-off frequency, and allowable mechanical tolerances. The terms “fast” and “slow” $f/\#$ come from photography. For an optical system with a fixed focal length, a larger entrance pupil diameter (small or fast $f/\#$) lets in more light and requires a shorter exposure time than a smaller entrance pupil diameter (large or slow $f/\#$) that requires a longer period of exposure, as shown in Fig. 2.33.

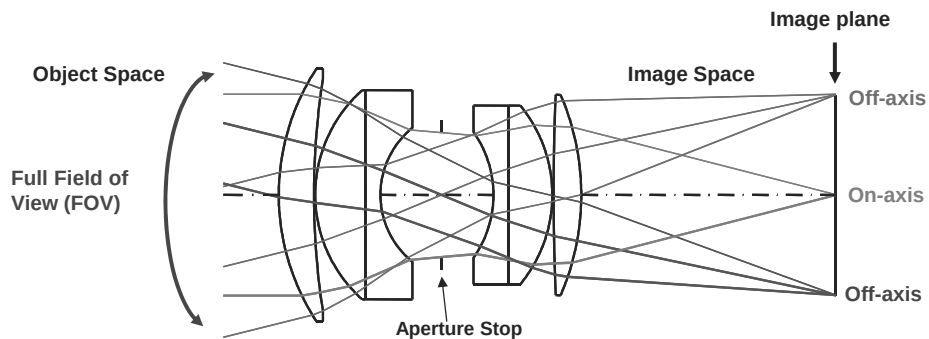


Figure 2.32 Optical system imaging definitions.

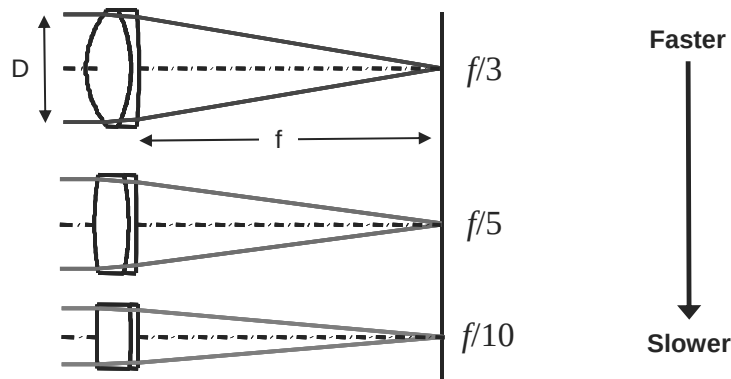


Figure 2.33 Optical system f -numbers, fast and slow.

2.9 Conic Surfaces

Conic surfaces are commonly employed in reflective systems such as Cassegrain-type telescopes that provide for aberration control and ease of testing. The classical Cassegrain configuration uses a parabolic primary and hyperbolic secondary, whereas the Ritchey–Chrétien Cassegrain design uses a hyperbolic surface for both the primary and secondary mirror surfaces. Conic surfaces are created by intersecting a plane with a cone, as shown in Fig. 2.34.

Conics have two foci with the property that a ray going through one focus F passes through the other focus F' with no aberrations, as shown in Fig. 2.35.

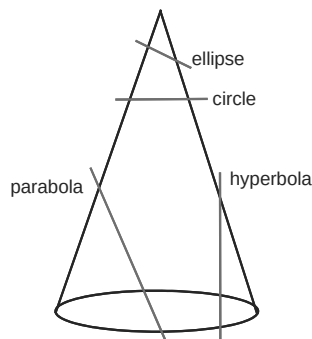


Figure 2.34 Conic surfaces are created by intersecting a plane with a cone.

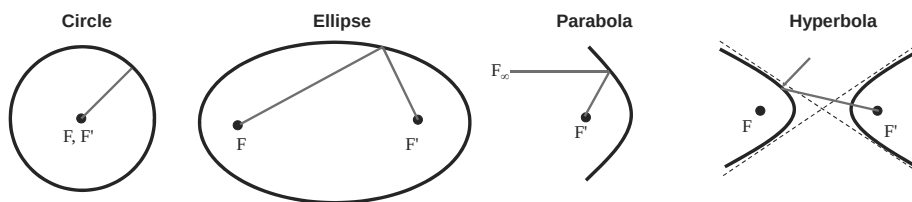


Figure 2.35 For a conic surface, rays from one focus point pass through the second focus point free of aberrations.

Table 2.1 Conic constants and surface types.

Surface	k
Sphere	0
Paraboloid	-1
Ellipsoid	$-1 < k < 0$
Hyperboloid	$k < -1$

The conic surface may be represented by the conic equation,

$$\text{sag} = \frac{cr^2}{1 + \sqrt{1 - (1+k)c^2r^2}}. \quad (2.9)$$

The sag is a measure of the distance between the optical surface and the tangent plane at the vertex, c is the base curvature at the vertex, and k is the conic constant. The value of k determines the type of surface, as listed in Table 2.1.

2.10 Optical Design Forms

There are a variety of design parameters that optical engineers evaluate when determining a suitable optical design form. Optical design forms are classified by the type of surfaces that are employed. *Dioptric* systems use all refractive elements, *catoptric* systems use all reflective surfaces or mirrors, and *catadioptric* systems use both refractive and reflective surfaces.

The advantage of dioptric or refractive systems is that systems can be designed with no obscurations, which results in greater image quality and no loss in throughput. In addition, refractive systems may be designed with faster $f/\#$ s and larger fields-of-view than reflective systems. The disadvantages of refractive systems are that they tend to be longer and heavier than reflective systems, need to address the effect of chromatic aberrations, and are difficult to athermalize.

The advantages of catoptrics or all-reflective systems is that they may be smaller (shorter) than refractive designs, there are no chromatic aberrations, and they can be made athermal. On-axis design forms, such as Cassegrain and Schmidt telescopes shown in Fig. 2.36(a)–(b), have central obscurations that reduce the power and degrade image quality. The on-axis reflective telescopes tend to have higher $f/\#$ s and smaller fields-of-view than corresponding refractive designs.

Off-axis reflective designs such as the three-mirror anastigmat (TMA) shown in Fig. 2.36(c) provide several advantages over their on-axis counterparts. This includes no central obscuration, a larger field-of-view, and superior stray-light rejection. Disadvantages of off-axis design forms such as the TMA include requiring an additional mirror and use of non-rotationally symmetric aspheric surfaces that are difficult to manufacture and test. These designs also tend to be larger and heavier.

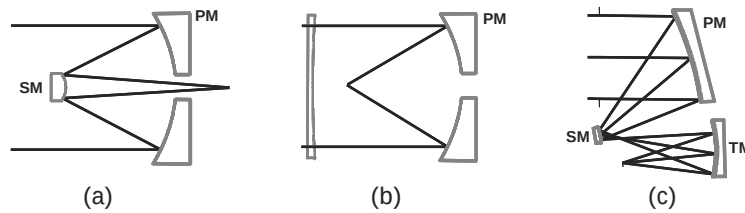


Figure 2.36 Common reflective telescope design forms: (a) Cassegrain, (b) Schmidt, and (c) three-mirror anastigmat.

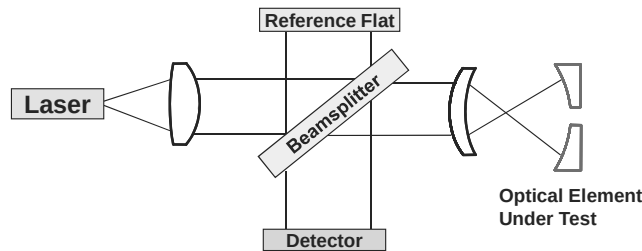


Figure 2.37 Twyman–Green interferometer test setup.

2.11 Interferometry and Optical Testing

Interferometry is used to measure the deviation of an optical surface from its prescribed shape that is based on the interference of two wavefronts: one from the optical surface under test and the other from a reference surface. An example of the Twyman–Green interferometric test setup is shown in Fig. 2.37. The interference pattern produced by the interfering wavefronts is known as an interferogram. The interferogram serves as a topographical map with each contour level or fringe representing a half wavelength of surface error. The surface error is measured normal to the optical surface. The discretized data from interferogram files is typically represented using Zernike polynomials or in a uniform grid array.

2.12 Mechanical Obscurations

In many instances, optical elements and optical support structures block a portion of the incident light passing through the optical system. For example, the secondary mirror and metering structure for a Cassegrain telescope obstructs light from reaching the primary mirror, as shown in Fig. 2.38. These obscurations increase the blur diameter of an image point by scattering light normal to the boundary of the obscuration. It is important in the mechanical design effort to be able to compare the resulting image degradation due to various mechanical configurations in addition to typical mechanical design response quantities such as gravity sag and natural frequency. This section describes an approximate technique to predict the effects of obscurations on optical performance as measured by the encircled energy function.

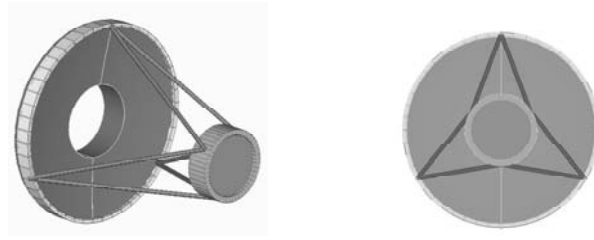


Figure 2.38 Obscurations in a Cassegrain telescope assembly.

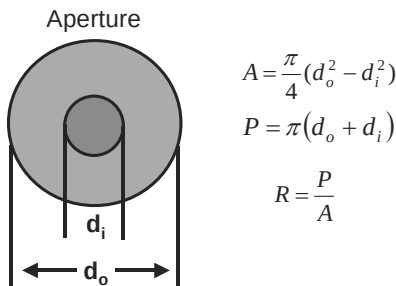


Figure 2.39 Ratio of the aperture periphery to the transmitting area controls the percentage of diffracted light.

2.12.1 Obscuration periphery, area, and encircled energy

There are two primary factors controlling the percentage of light that is diffracted by an obscuration. The periphery of the obscuration dictates the amount of energy that is diffracted. As the periphery increases, the amount of diffracted energy increases. The area of the obscuration controls the amount of energy transmitted through the optical system. As the area of the obscuration increases, the transmitted energy decreases, and a larger percentage of the light is diffracted for the same periphery. The exact mathematical formulation to compute the effects of diffraction is typically complex. However, a simple approximation based on the ratio of the total obscuration periphery P to the total area of the transmitting aperture A , given as R , can be used to compare mechanical design concepts.¹ An example calculation of this ratio is illustrated in Fig. 2.39. The normalized encircled energy EE as a function of R is given as

$$EE(r_o) = 1 - \frac{\lambda f}{2\pi^2 r_o} R, \quad (2.10)$$

where λ is the wavelength, f_{eff} is the effective focal length of the optical system, and r_o is the radial coordinate on the focal plane. This approximation allows the encircled energy to be computed as a function of radial extent and is valid for arbitrary aperture shapes for most practical imaging applications assuming a uniformly illuminated aperture. Mechanical design trades may then be performed to evaluate mechanical support and mounting structures as a function of image

quality. In addition, the ratio R may be used as a structural constraint in design optimization solutions. Other spider design equations are discussed by Harvey.² A more-detailed evaluation of the effects of obscurations on image quality may be performed using optical design software.

2.12.2 Diffraction effects for various spider configurations

The encircled energy approximation is used to compare five spider configurations for a Cassegrain telescope support structure. Each of the spider configurations has the same total cross-sectional area. The encircled energy is plotted for each configuration in Fig. 2.40.

The three-vane design provides the best optical performance, with the three-tangential and four-vane designs exhibiting a slight decrease in performance. The six- and eight-vane configurations show a significant decrease in performance due to a significant increase in diffracted energy. This result is expected given the increase in periphery versus area for each additional vane.

2.12.3 Diffraction spikes

Each spider configuration produces its own characteristic diffraction pattern as shown in Fig. 2.41. The diffraction pattern of the three vanes produces six spikes in the diffraction pattern, whereas the four-vane configuration produces only

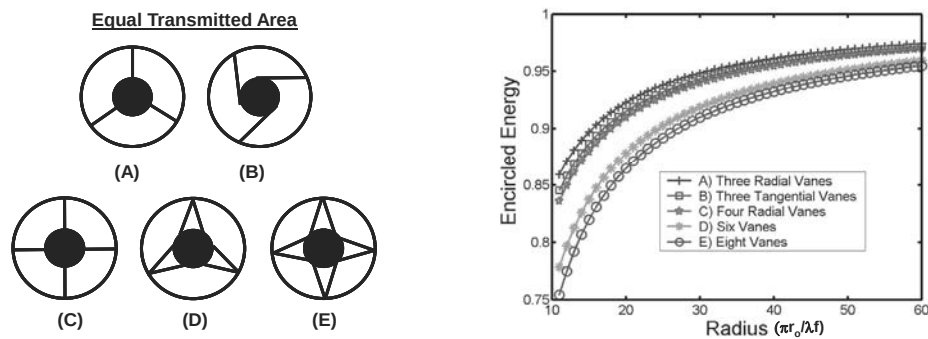


Figure 2.40 Comparison of encircled energy vs. spider configurations of constant area.

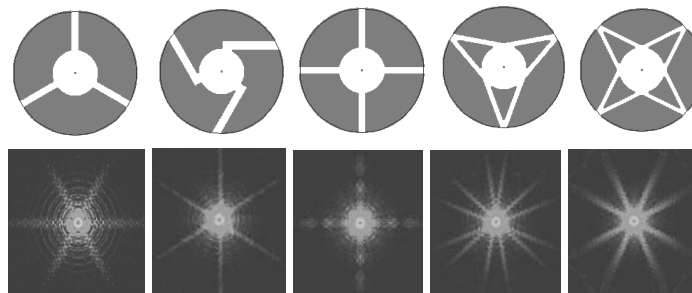


Figure 2.41 Support structure configurations and the resulting PSF.

four spikes. This is explained by the fact that light is diffracted in both directions normal to the vane. Hence, for the three-vane configuration with vanes at 0, 120, and 240 deg, light is scattered in six directions. The vane at 0 deg scatters light at 90 and 270 deg, the vane at 120 deg scatters light at 30 and 210 deg, and the vane at 240 deg scatters light at 150 and 330 deg. The reason there are only four diffraction spikes with four vanes and not eight is that half of the directions overlap. Use of curved spider legs eliminates the diffraction spikes resulting in a rotationally symmetric diffraction image.³ However, this does not necessarily result in improved optical performance.

2.13 Optical-System Error Budgets

Optical-system error budgets or performance budgets are common optical-design tools used to establish design requirements on the system, subassemblies, and components. An example optical-system wavefront error budget is shown in Fig. 2.42. Top-down allocations are performed initially based in part on past designs and experience. These determine a starting point for the fabrication, assembly, and environmental design considerations. A bottoms-up roll-up of the errors may be performed to rebalance and redistribute allocations after initial calculations. Error budget contributions are typically assumed to be uncorrelated and combined using the RSS method. For errors that are known to be correlated, these values may be added in the error budget.

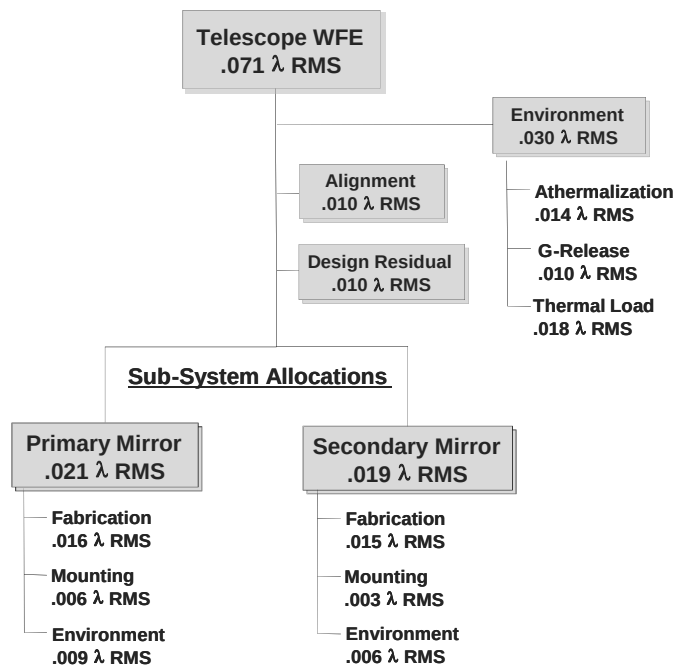


Figure 2.42 An example optical-system wavefront error budget.

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◀Chapter 3▶

Zernike and Other Useful Polynomials

The use of polynomials offers several benefits in optomechanical analysis including improving data interpretation and providing efficient means of data transfer. Zernike polynomials are a popular form and are well suited for use with optical systems. Fitting Zernike polynomials to FEA-derived mechanical response quantities provides a compact representation of hundreds or thousands of data points whose individual terms may be readily interpreted for insight into the mechanical and optical behavior. Use of an orthogonal set of polynomials such as the Zernike set allows the ability to remove terms that may be correctable such as during optical alignment and for systems that have active focus control. Polynomials also serve as an effective vehicle to transfer data between mechanical and optical software tools facilitating integration of the mechanical and optical analysis models. The Zernike polynomials are the most commonly used polynomials; however, other useful polynomial forms include annular Zernikes, X-Y, Legendre–Fourier, and aspheric polynomials.

3.1 Zernike Polynomials

Zernike polynomials¹ are used for a variety of purposes in optical engineering, including the description of aberrations, interferometric test data, and adaptive optics. Their popularity is derived from several benefits that are particularly useful to optical systems including base terms comprising radial and azimuthal variables suitable for descriptions of circular apertures and for their condition of orthogonality over a unit circle. Two sets of Zernike polynomials commonly used in optical engineering are the Standard and the Fringe polynomial definitions.

3.1.1 Mathematical description

The mathematical description for a given surface, $\Delta Z(r, \theta)$, is provided by Eq. (3.1), where A_{nm} and B_{nm} are the Zernike coefficients:

$$\Delta Z(r, \theta) = A_{00} + \sum_{n=2}^{\infty} A_{n0} R_n^0(r) + \sum_{n=1}^{\infty} \sum_{m=1}^n R_n^m \left[A_{nm} \cos(m\theta) + B_{nm} \sin(m\theta) \right]. \quad (3.1)$$

The radial dependence of the Zernike polynomials is given by the following expression:

$$R_n^m(r) = \sum_{s=0}^{\frac{n-m}{2}} (-1)^s \frac{(n-s)!}{s! \left(\frac{n+m}{2} - s\right)! \left(\frac{n-m}{2} - s\right)!} r^{(n-2s)}. \quad (3.2)$$

The variables n and m in Eqs. (3.1) and (3.2) are integer values known as the radial and circumferential wave number, respectively. In deriving the individual Zernike terms from the Zernike equations listed above, $n - m$ must be an even number, and $n \geq m$.

The Zernike polynomials form an orthogonal set over a normalized circular aperture or unit circle. Each of the higher-order polynomials contains an appropriate amount of the lower-order polynomial to preserve this condition. The condition of orthogonality allows each of the Zernike terms to be independent providing separation between the orders of the polynomial terms.

3.1.2 Individual Zernike terms

The first term of the Zernike series, piston, is a constant term that represents a best-fit average to the data. The next two terms represent tilt of the data along perpendicular planes. Focus represents a quadratic or parabolic change in the radial extent of the surface shape. Astigmatism is best described as the shape of a horse's saddle or a potato chip, possessing unequal curvatures along perpendicular axes. Coma is a surface with a pair of humps, where one of the humps is inverted. 3D contour plots of several Zernike polynomials are shown in Fig. 3.1.

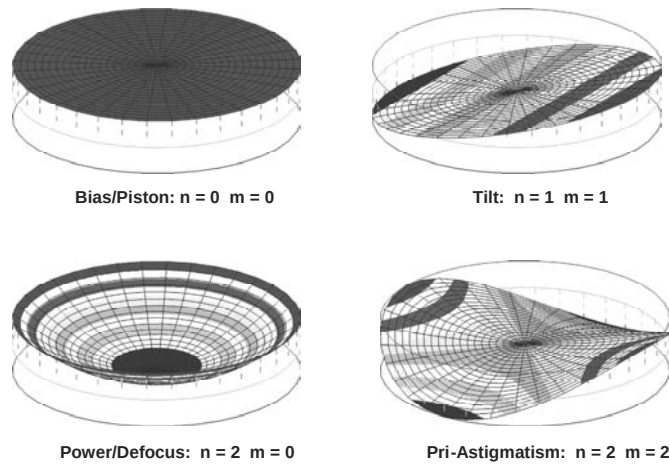
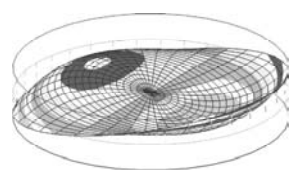
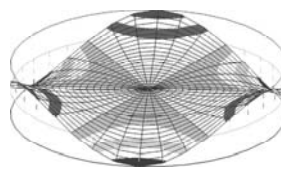
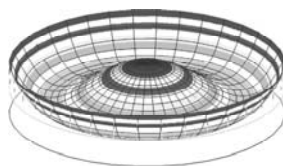
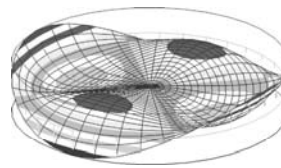
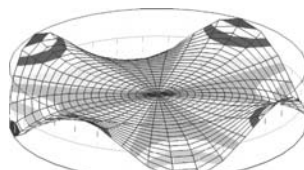
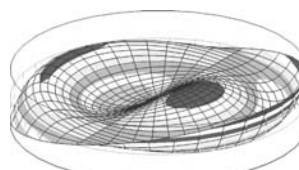
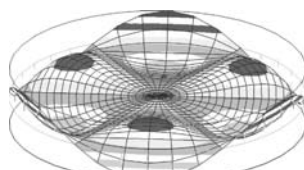
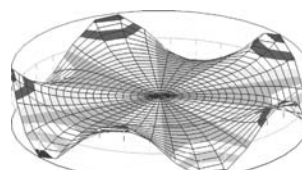
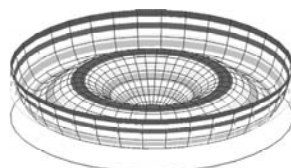
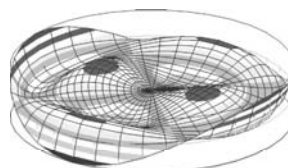
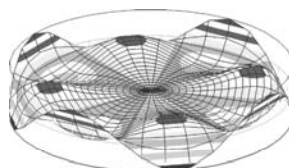
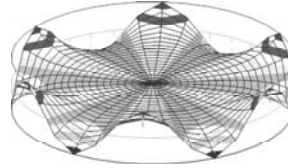


Figure 3.1 Zernike polynomials (continued, next page).

Pri-Coma: $n = 3$ $m = 1$ Pri-Trefoil: $n = 3$ $m = 3$ Pri-Spherical: $n = 4$ $m = 0$ Sec-Astigmatism: $n = 4$ $m = 2$ Pri-Tetrafoil: $n = 4$ $m = 4$ Sec-Coma: $n = 5$ $m = 1$ Sec-Trefoil: $n = 5$ $m = 3$ Pri-Pentafoil: $n = 5$ $m = 5$ Sec-Spherical: $n = 6$ $m = 0$ Ter-Astigmatism: $n = 6$ $m = 2$ Sec-Tetrafoil: $n = 6$ $m = 4$ Pri-Hexafoil: $n = 6$ $m = 6$ **Figure 3.1** Zernike polynomials (continued).

3.1.3 Standard Zernike polynomials

The Standard set of Zernike polynomials is a popular form used in optical system design. There are two common approaches to normalize and order the Standard Zernike polynomials: the convention outlined in Born and Wolf² uses amplitude normalization where the peak amplitude for a unit term is one; the convention outlined in Noll³ uses RMS normalization where the RMS over the unit circle has a value of one. A comparison of the two normalization approaches for focus and spherical terms are shown in Fig. 3.2.

The RMS value for each of the unit amplitude normalized terms may be computed using the following relationships:

$$\text{For the axisymmetric terms: } \left[\sqrt{(n+1)} \right]^{-1} \quad (3.3)$$

$$\text{For the non-axisymmetric terms: } \left[\sqrt{2(n+1)} \right]^{-1} \quad (3.4)$$

These expressions result in the normalization factors for the unit RMS Zernike terms:

$$\text{For the axisymmetric terms: } \sqrt{n+1} \quad (3.5)$$

$$\text{For the non-axisymmetric terms: } \sqrt{2(n+1)} \quad (3.6)$$

Pyramid charts are useful to compare the Zernike ordering schemes between the two Standard Zernike approaches as shown in Figs. 3.3 and 3.4. The pyramid charts show the numbering scheme of the different sets as a function of the radial and circumferential wave numbers n and m . A listing of the first 37 terms of the Standard Zernike using amplitude normalization is presented in Table 3.1.

Normalization		
<u>Zernike Term</u>	<u>Unit Amplitude</u>	<u>Unit RMS</u>
Focus ($n = 2$)	$2r^2 - 1$	$\sqrt{3}(2r^2 - 1)$
Spherical ($n = 4$)	$6r^4 - 6r^2 + 1$	$\sqrt{5}(6r^4 - 6r^2 + 1)$

Figure 3.2 Amplitude and RMS normalization for focus and spherical Zernike terms.

	sin													cos											
n/m	12	11	10	9	8	7	6	5	4	3	2	1	0	1	2	3	4	5	6	7	8	9	10	11	12
0													1												
1												3		2											
2											6		5		4										
3										10		9		8		7									
4									15		14		13		12		11								
5								21		20		19		18		17		16							
6							28		27		26		25		24		23		22						
7						36		35		34		33		32		31		30		29					
8					45		44		43		42		41		40		39		38		37				
9				55		54		53		52		51		50		49		48		47		46			
10			66		65		64		63		62		61		60		59		58		57		56		
11		78		77		76		75		74		73		72		71		70		69		68		67	
12	91		90		89		88		87		86		85		84		83		82		81		80		79

Figure 3.3 Standard Zernike pyramid chart (using the Born and Wolf convention).

	sin												cos												
n/m	12	11	10	9	8	7	6	5	4	3	2	1	0	1	2	3	4	5	6	7	8	9	10	11	12
0													1												
1												3	2												
2											5	4	6												
3										9	7	8	10												
4							15	13	11	12	14														
5						21	19	17	16	18	20														
6					27	25	23	22	24	26	28														
7				35	33	31	29	30	32	34	36														
8			45	43	41	39	37	38	40	42	44														
9			55	53	51	49	47	46	48	50	52	54													
10		65	63	61	59	57	56	58	60	62	64	66													
11	77	75	73	71	69	67	68	70	72	74	76	78													
12	91	89	87	85	83	81	79	80	82	84	86	88	90												

Figure 3.4 Standard Zernike pyramid chart (using the Noll convention).

Table 3.1 Standard Zernike polynomials (first 37 terms listed below).

	<u>n</u>	<u>m</u>	<u>POLYNOMIAL</u>	<u>NAME</u>
<1>	0	0	1	Piston
<2>	1	1	$r\cos(\theta)$	A-Tilt
<3>	1	1	$r\sin(\theta)$	B-Tilt
<4>	2	2	$r^2\cos(2\theta)$	Pri Astigmatism-A
<5>	2	0	$2r^2 - 1$	Focus
<6>	2	2	$r^2\sin(2\theta)$	Pri Astigmatism-B
<7>	3	3	$r^3\cos(3\theta)$	Pri Trefoil-A
<8>	3	1	$(3r^3 - 2r)\cos(\theta)$	Pri Coma-A
<9>	3	1	$(3r^3 - 2r)\sin(\theta)$	Pri Coma-B
<10>	3	3	$r^3\sin(3\theta)$	Pri Trefoil-B
<11>	4	4	$r^4\cos(4\theta)$	Pri Tetrafoil-A
<12>	4	2	$(4r^4 - 3r^2)\cos(2\theta)$	Sec Astigmatism-A
<13>	4	0	$6r^4 - 6r^2 + 1$	Pri Spherical
<14>	4	2	$(4r^4 - 3r^2)\sin(2\theta)$	Sec Astigmatism-B
<15>	4	4	$r^4\sin(4\theta)$	Pri Tetrafoil-B
<16>	5	5	$r^5\cos(5\theta)$	Pri Pentafoil-A
<17>	5	3	$(5r^5 - 4r^3)\cos(3\theta)$	Sec Trefoil-A
<18>	5	1	$(10r^5 - 12r^3 + 3r)\cos(\theta)$	Sec Coma-A
<19>	5	1	$(10r^5 - 12r^3 + 3r)\sin(\theta)$	Sec Coma-B
<20>	5	3	$(5r^5 - 4r^3)\sin(3\theta)$	Sec Trefoil-B
<21>	5	5	$r^5\sin(5\theta)$	Pri Pentafoil-B
<22>	6	6	$r^6\cos(6\theta)$	Pri Hexafoil-A
<23>	6	4	$(6r^6 - 5r^4)\cos(4\theta)$	Sec Tetrafoil-A
<24>	6	2	$(15r^6 - 20r^4 + 6r^2)\cos(2\theta)$	Ter Astigmatism-A
<25>	6	0	$20r^6 - 30r^4 + 12r^2 - 1$	Sec Spherical
<26>	6	2	$(15r^6 - 20r^4 + 6r^2)\sin(2\theta)$	Ter Astigmatism-B
<27>	6	4	$(6r^6 - 5r^4)\sin(4\theta)$	Sec Tetrafoil-B
<28>	6	6	$r^6\sin(6\theta)$	Pri Hexafoil-B
<29>	7	7	$r^7\cos(7\theta)$	Pri Septafoil-A
<30>	7	5	$(7r^7 - 6r^5)\cos(5\theta)$	Sec Pentafoil-A
<31>	7	3	$(21r^7 - 30r^5 + 10r^3)\cos(3\theta)$	Ter Trefoil-A
<32>	7	1	$(35r^7 - 60r^5 + 30r^3 - 4r)\cos(\theta)$	Ter Coma-A
<33>	7	1	$(35r^7 - 60r^5 + 30r^3 - 4r)\sin(\theta)$	Ter Coma-B
<34>	7	3	$(21r^7 - 30r^5 + 10r^3)\sin(3\theta)$	Ter Trefoil-B
<35>	7	5	$(7r^7 - 6r^5)\sin(5\theta)$	Sec Pentafoil-B
<36>	7	7	$r^7\sin(7\theta)$	Pri Septafoil-B
<37>	8	8	$r^8\cos(8\theta)$	Pri Octafoil-A

3.1.4 Fringe Zernike polynomials

The Fringe⁴ Zernike polynomials are a second popular set of terms commonly used in optical system design. The Fringe set is a reordered subset of the Standard Zernike terms with a total of 37 terms that are unit normalized as listed in Table 3.2. The Fringe set includes higher-order radially symmetric terms while excluding the higher-order azimuthal terms.

Table 3.2 Fringe Zernike polynomials.

	<u>n</u>	<u>m</u>	<u>POLYNOMIAL</u>	<u>NAME</u>
<1>	0	0	1	Piston
<2>	1	1	$r\cos(\theta)$	Tilt-A
<3>	1	1	$r\sin(\theta)$	Tilt-B
<4>	2	0	$2r^2 - 1$	Focus
<5>	2	2	$r^2\cos(2\theta)$	Pri Astig.-A
<6>	2	2	$r^2\sin(2\theta)$	Pri Astig.-B
<7>	3	1	$(3r^3 - 2r)\cos(\theta)$	Pri Coma-A
<8>	3	1	$(3r^3 - 2r)\sin(\theta)$	Pri Coma-B
<9>	4	0	$6r^4 - 6r^2 + 1$	Pri Spherical
<10>	3	3	$r^3\cos(3\theta)$	Pri Trefoil-A
<11>	3	3	$r^3\sin(3\theta)$	Pri Trefoil-B
<12>	4	2	$(4r^4 - 3r^2)\cos(2\theta)$	Sec Astig.-A
<13>	4	2	$(4r^4 - 3r^2)\sin(2\theta)$	Sec Astig.-B
<14>	5	1	$(10r^5 - 12r^3 + 3r)\cos(\theta)$	Sec Coma-A
<15>	5	1	$(10r^5 - 12r^3 + 3r)\sin(\theta)$	Sec Coma-B
<16>	6	0	$20r^6 - 30r^4 + 12r^2 - 1$	Sec Spherical
<17>	4	4	$r^4\cos(4\theta)$	Pri Tetrafoil-A
<18>	4	4	$r^4\sin(4\theta)$	Pri Tetrafoil-B
<19>	5	3	$(5r^5 - 4r^3)\cos(3\theta)$	Sec Trefoil-A
<20>	5	3	$(5r^5 - 4r^3)\sin(3\theta)$	Sec Trefoil-B
<21>	6	2	$(15r^6 - 20r^4 + 6r^2)\cos(2\theta)$	Ter Astig.-A
<22>	6	2	$(15r^6 - 20r^4 + 6r^2)\sin(2\theta)$	Ter Astig.-B
<23>	7	1	$(35r^7 - 60r^5 + 30r^3 - 4r)\cos(\theta)$	Ter Coma-A
<24>	7	1	$(35r^7 - 60r^5 + 30r^3 - 4r)\sin(\theta)$	Ter Coma-B
<25>	8	0	$70r^8 - 140r^6 + 90r^4 - 20r^2 + 1$	Ter Spherical
<26>	5	5	$r^5\cos(5\theta)$	Pri Pentafoil-A
<27>	5	5	$r^5\sin(5\theta)$	Pri Pentafoil-B
<28>	6	4	$(6r^6 - 5r^4)\cos(4\theta)$	Sec Tetrafoil-A
<29>	6	4	$(6r^6 - 5r^4)\sin(4\theta)$	Sec Tetrafoil-B
<30>	7	3	$(21r^7 - 30r^5 + 10r^3)\cos(3\theta)$	Ter Trefoil-A
<31>	7	3	$(21r^7 - 30r^5 + 10r^3)\sin(3\theta)$	Ter Trefoil-B
<32>	8	2	$(56r^8 - 105r^6 + 60r^4 - 10r^2)\cos(2\theta)$	Qua Astig.-A
<33>	8	2	$(56r^8 - 105r^6 + 60r^4 - 10r^2)\sin(2\theta)$	Qua Astig.-B
<34>	9	1	$(126r^9 - 280r^7 + 210r^5 - 60r^3 + 5r)\cos(\theta)$	Qua Coma-A
<35>	9	1	$(126r^9 - 280r^7 + 210r^5 - 60r^3 + 5r)\sin(\theta)$	Qua Coma-B
<36>	10	0	$252r^{10} - 630r^8 + 560r^6 - 210r^4 + 30r^2 - 1$	Qua Spherical
<37>	12	0	$924r^{12} - 2772r^{10} + 3150r^8 - 1680r^6 + 420r^4 - 42r^2 + 1$	Qin Spherical

	sin							cos					
n/m	5	4	3	2	1	0	1	2	3	4	5		
0						1							
1					3		2						
2				6		4		5					
3			11		8		7		10				
4		18		13		9		12		17			
5	27		20		15		14		19		26		
6		29		22		16		21		28			
7			31		24		23		30				
8				33		25		32					
9					35		34						
10						36							
11													
12						37							

Figure 3.5 Fringe Zernike pyramid chart.

A pyramid chart of the Fringe Zernike polynomials is presented in Fig. 3.5.

3.1.5 Magnitude and phase

An alternate format to present Zernike polynomials is use of the magnitude and phase convention. Each pair of the Zernike terms that are a function of the angle θ , such as tilt and astigmatism (represented by the Zernike coefficients A_{nm} and B_{nm}), may be expressed as a single term with an associated magnitude and phase as given below:

$$M = \sqrt{A_{nm}^2 + B_{nm}^2}, \quad (3.7)$$

$$\text{Phase} = \frac{1}{m} \tan^{-1} \frac{B_{nm}}{A_{nm}}. \quad (3.8)$$

where the phase is the circumferential orientation of the Zernike term. This convention provides advantages in interpretation and a more compact format. For example, the set of Fringe Zernike terms may be reduced from a listing of 37 to 22 terms.

3.1.6 Orthogonality of Zernike polynomials

Zernike polynomials form a set of orthogonal surface descriptors that provides several favorable characteristics⁵ in the optomechanical design process. This property allows individual Zernike terms to be subtracted or added to the polynomial series without changing the value of the other coefficients. Practically, this allows Zernike terms that may be corrected within the optical system such as focus to be removed from the fit without affecting the value of the other terms, allowing design efforts to concentrate on minimizing the uncorrectable terms.

Zernike polynomials are orthogonal for continuous data over a unit circle if the area of the product of the two Zernike functions Φ_1 and Φ_2 is zero:

$$\int_0^1 \int_0^{2\pi} \Phi_1 \Phi_2 \rho d\Theta d\rho = 0, \quad (3.9)$$

For axisymmetric functions, Eq. (3.9) reduces to

$$2\pi \int_0^1 \Phi_1 \Phi_2 \rho d\rho = 0. \quad (3.10)$$

Using Eq. (3.10), the orthogonality of the piston and focus terms and the focus and spherical terms is shown below:

$$2\pi \int_0^1 (1)(2\rho^2 - 1)\rho d\rho = 2\pi \left(\frac{2}{4} - \frac{1}{2} \right) = 0, \quad (3.11)$$

$$2\pi \int_0^1 (2\rho^2 - 1)(6\rho^4 - 6\rho^2 + 1)\rho d\rho = 2\pi \left(\frac{12}{8} - \frac{18}{6} + \frac{8}{4} - \frac{1}{2} \right) = 0. \quad (3.12)$$

Orthogonality is met only for continuous data. Because finite element data is discrete, the condition of orthogonality is only approximated when fitting Zernike polynomials. The condition is best approximated for a highly dense, uniformly spaced mesh. Orthogonality degrades significantly as the data becomes irregular and when fit to noncircular apertures. These practical cases are discussed below.

3.1.6.1 Noncircular apertures

Fitting Zernike polynomials to data over noncircular apertures requires that the pupil be sized to the radius that encloses the full area of the aperture. This is shown for an elliptical aperture and circular aperture with a central hole in Fig. 3.6. Within the full pupil radius there will be points in which no data exists, resulting in loss of orthogonality.

For example, consider the primary mirror of a Cassegrain telescope that includes a central hole of $\rho = 0.2$. The orthogonality of the Zernike terms is now lost, as demonstrated using the piston and focus terms:

$$2\pi \int_{0.2}^1 (1)(2\rho^2 - 1)\rho d\rho = 2\pi \left[\left(\frac{2}{4} - \frac{1}{2} \right) - \left(\frac{.0032}{4} - \frac{.04}{2} \right) \right] = 0.12 \neq 0. \quad (3.13)$$

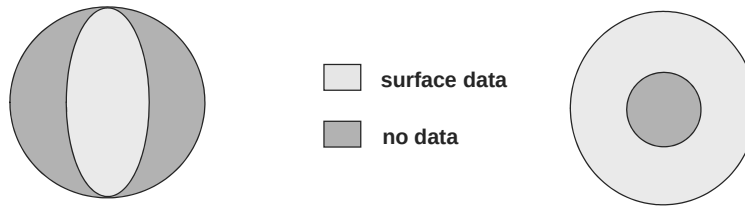


Figure 3.6 Noncircular apertures: elliptical and circular with central hole.

Orthogonality is also lost on a noncircular geometry, such as a square optic:

$$\int_{-1}^{+1} \int_{-1}^{+1} (1)(2\rho^2 - 1) dx dy \neq 0. \quad (3.14)$$

Variations of the Zernike polynomials exist that are orthogonal over noncircular apertures.⁶ The annular Zernikes are an example of such a set that are discussed in Section 3.2. However, their general treatment is beyond the scope of this text.

3.1.6.2 Discrete data

The orthogonality of Zernike polynomials is also lost when fitting terms to discrete data. For discrete data evaluated at node k , the condition of orthogonality becomes

$$\sum_k \Phi_{1k} \Phi_{2k} A_k = 0, \quad (3.15)$$

where A_k is the area associated with node k .

A comparison of orthogonality using numerical integration for the piston, focus, and spherical terms fit to varying mesh densities is shown in Table 3.3, where the residual error verses the number of equally spaced radial integration points K over the unit circle are listed. The diagonal terms $\Phi_i \Phi_j$ represent the square of the RMS, and the off-diagonal terms $\Phi_i \Phi_j$ represent the coupling or nonorthogonality. As the number of radial node points increase in the mesh density, the polynomial terms become increasingly orthogonal.

Table 3.3 Numerical integration on a unit circle.

K	$\Phi_0 \Phi_0$	$\Phi_1 \Phi_1$	$\Phi_2 \Phi_2$	$\Phi_0 \Phi_1$	$\Phi_0 \Phi_2$	$\Phi_1 \Phi_2$
10	1.0000	.34660	.23838	.00500	.01990	.02460
20	1.0000	.33666	.20990	.00125	.00499	.00623
50	1.0000	.33387	.20160	.00020	.00080	.00100
100	1.0000	.33347	.20040	.00005	.00020	.00025
200	1.0000	.33337	.20010	.00001	.00005	.00006
500	1.0000	.33334	.20002	.00000	.00001	.00001
1000	1.0000	.33333	.20000	.00000	.00000	.00000

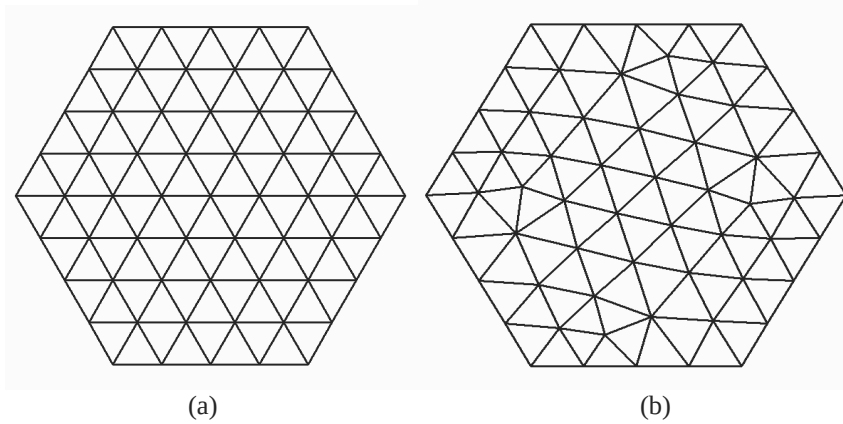


Figure 3.7 (a) Regular “isomesh” model, and (b) irregular “automesh” model.

Orthogonality is also a function of the uniformity of the data. For instance, coupling of the Zernike terms increases when the finite element mesh is nonuniform. A comparison of a uniform isomesh and irregular “automesh” is shown in Fig. 3.7.

In the isomesh, axisymmetric terms are coupled only to other axisymmetric terms. In the irregular mesh, axisymmetric terms pick up additional coupling with the nonaxisymmetric terms, such as astigmatism, coma, and trefoil.

The degree of nonorthogonality that is acceptable is dependent on the application. A set of nonorthogonal Zernike terms can be an excellent representation of the data. The nonorthogonality between the Zernike terms means that the terms are coupled and hence the meaning of each term is lost to some degree; if a term is removed, the value of the other terms will change. A method to check the degree of nonorthogonality of a Zernike fit is to fit the data to a varying number of Zernike terms. The difference in the coefficients for each fit is indicative of the nonorthogonality. Another option is to evaluate the degree of cross-coupling in the off-diagonal terms of the $[H]$ matrix developed in the following section.

3.1.7 Computing the Zernike polynomial coefficients

The individual Zernike terms represent a set of discrete surface data Δ by a series of base surfaces ϕ_i , each multiplied by a coefficient a_i and then summed:

$$\Delta = \sum_i a_i \phi_i = a_0 + a_1 \phi_1 + a_2 \phi_2 + a_3 \phi_3 + \dots + a_i \phi_i. \quad (3.16)$$

This is depicted graphically in Figure 3.8. The coefficients for the Zernike terms to describe a set of discrete surface data such as finite element results may be determined by using a least-squares fit.⁷ Consider a grid of node points i

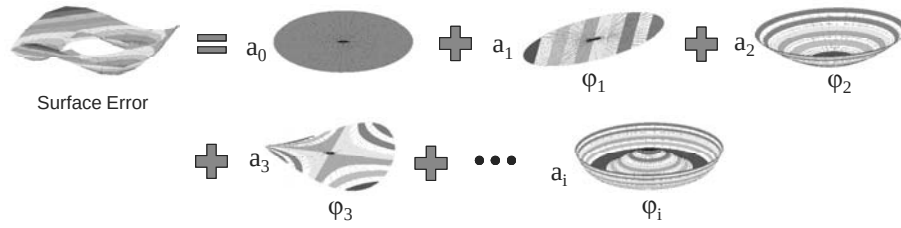


Figure 3.8 Representing surface errors as a combination of base surfaces.

representing an optical surface in a finite-element model. A least-squares error function is defined as the difference between the polynomial description of the deformation Z_i and the actual finite-element computed deformation δ_i . A weighting function W_i may be applied that is proportional to the area that each node represents on the optical surface. This accounts for the variation in nodal density and allows for an equitable contribution of each node point in the overall fit. The area may be computed as a fraction of the total normal surface area or projected surface area. Typically, use of the projected area yields a more representative fit, as usage of the normal surface area increases the contribution of the nodes on the edges of the surface. The least-squares error function E is given as

$$E = \sum W_i (\delta_i - Z_i)^2. \quad (3.17)$$

The Zernike polynomial approximation Z_i is given by the summation of the Zernike coefficients c_j that are being solved and the Zernike polynomial ϕ_{ji} :

$$Z_i = \sum c_j \phi_{ji}. \quad (3.18)$$

This yields the following least-squares error function:

$$E = \sum W_i \left(\delta_i - \sum c_j \phi_{ji} \right)^2. \quad (3.19)$$

To compute the best-fit Zernike coefficients, the error function is minimized with respect to the coefficients. This is done mathematically by taking the derivative of the error function with respect to the coefficients and setting it equal to zero:

$$\frac{\partial E}{\partial c_j} = 2 \sum W_i \left(\delta_i - \sum c_j \phi_{ji} \right) \phi_{ji} = 0. \quad (3.20)$$

The resulting expression is in linear matrix form, allowing the coefficients $\{c\}$ to be solved using Gaussian elimination:

$$[H]\{c\} = \{p\}, \quad (3.21)$$

where

$$p_j = \sum W_i \delta_i \phi_{ji}, \quad (3.22)$$

and

$$H_{jk} = \sum W_i \phi_{ji} \phi_{ki}. \quad (3.23)$$

Once the Zernike coefficients have been computed, the RMS fit error should be computed to determine how well the polynomial set represents the actual data. The RMS fit error is computed as the RMS of the difference between the polynomial representation and the actual data. The required accuracy depends on the specific application but, generally, the RMS fit error should be a small fraction of the RMS surface error.

3.2 Annular Zernike Polynomials

Annular Zernike polynomials⁸ are a modified set of Zernike polynomials that are orthogonal over an annular aperture useful for Cassegrain class systems that use a primary mirror with a central hole. The annular Zernike's order and normalization follow the convention of Noll. The terms include an annulus ratio ϵ , which is defined as the ratio of the inner annular radius to the outer radius of the aperture. When the annulus ratio is zero, the terms reduce to the Standard Zernike form. Several of the terms are shown in Table 3.4.

3.3 X-Y Polynomials

The X-Y polynomials are useful for fitting data with rectangular content, such as a rectangular mirror or a mirror with a rectangular grid of stiffening ribs. Most optical software codes provide a surface definition that includes use of X-Y polynomials on top of a base surface. This surface definition may be used to represent finite element surface deformations as discussed in Chapter 4. The mathematical representation of the X-Y polynomial is expressed as

$$z(x, y) = A_{00} + A_{10}x + A_{01}y + A_{20}x^2 + A_{11}xy + A_{02}y^2 + \dots + A_{nm}x^n y^m. \quad (3.24)$$

A disadvantage of the X-Y polynomials is that the polynomial terms are not orthogonal. Thus, when fitting surface distortions, the higher-order terms tend to alternate in sign and increase rapidly in magnitude. The addition or deletion of a term causes large changes in the magnitude of the other terms.

Table 3.4 The first 11 Annular Zernike terms.

Term	N	M	Annular Zernike Polynomial Term
1	0	0	1
2	1	1	$\frac{2r \cos \theta}{[1 + \varepsilon^2]^{1/2}}$
3	1	1	$\frac{2r \sin \theta}{[1 + \varepsilon^2]^{1/2}}$
4	2	0	$\left(\frac{\sqrt{3}}{1 - \varepsilon^2}\right) [2r^2 - (1 + \varepsilon^2)]$
5	2	2	$\left[\frac{6}{1 + \varepsilon^2 + \varepsilon^4}\right]^{1/2} r^2 \sin 2\theta$
6	2	2	$\left[\frac{6}{1 + \varepsilon^2 + \varepsilon^4}\right]^{1/2} r^2 \cos 2\theta$
7	3	1	$\left[\frac{8(1 + \varepsilon^2)}{1 + 2\varepsilon^2 - 6\varepsilon^4 + 2\varepsilon^6 + \varepsilon^8}\right]^{1/2} \left[3r^2 - 2\left(\frac{1 + \varepsilon^2 + \varepsilon^4}{1 + \varepsilon^2}\right)\right] r \sin \theta$
8	3	1	$\left[\frac{8(1 + \varepsilon^2)}{1 + 2\varepsilon^2 - 6\varepsilon^4 + 2\varepsilon^6 + \varepsilon^8}\right]^{1/2} \left[3r^2 - 2\left(\frac{1 + \varepsilon^2 + \varepsilon^4}{1 + \varepsilon^2}\right)\right] r \cos \theta$
9	3	3	$\left[\frac{8}{1 + \varepsilon^2 + \varepsilon^4 + \varepsilon^6}\right]^{1/2} [x^2 - 3y^2] \sin \theta$
10	3	3	$\left[\frac{8}{1 + \varepsilon^2 + \varepsilon^4 + \varepsilon^6}\right]^{1/2} [3x^2 - y^2] \cos \theta$
11	4	0	$\frac{\sqrt{5}}{(1 - \varepsilon^2)^2} [6r^4 - 6(1 + \varepsilon^2)r^2 + (1 + 4\varepsilon^2 + \varepsilon^4)]$

3.4 Legendre Polynomials

Legendre polynomials are an alternative set of polynomials for rectangular surfaces and apertures that offers the advantage of being orthogonal. Due to the orthogonality condition, the coefficients of the higher-order terms tend towards zero, and the addition or deletion of a term has little effect on the other terms. The polynomial set, however, is not typically used in optical design codes. The low-order Legendre terms are displayed in Fig. 3.9 and the mathematical representation of the Legendre polynomials is expressed below:

$$z(x, y) = \sum_{n=0}^N \sum_{m=0}^M c_{nm} P_n(x) P_m(y), \quad (3.25)$$

where

$$P_n(x) = \sum_{k=0}^n (-1)^k \frac{(2n-2k)!}{2^n k! (n-k)! (n-2)!} x^{n-2k}, \quad (3.26)$$

$$P_m(y) = \sum_{k=0}^m (-1)^k \frac{(2m-2k)!}{2^m k! (m-k)! (m-2)!} y^{m-2k}.$$

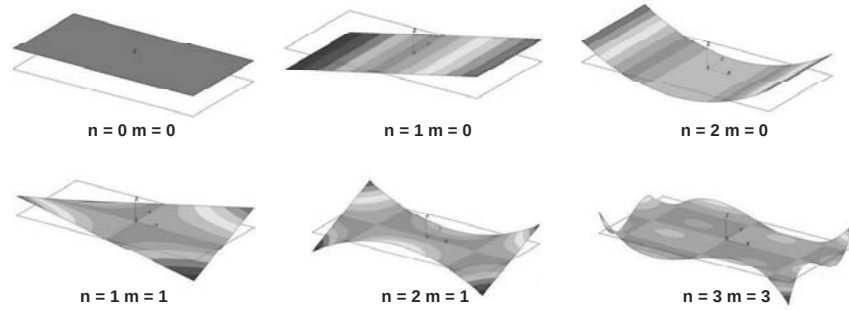


Figure 3.9 Legendre polynomial low-order base surfaces.

3.5 Legendre–Fourier Polynomials

The Legendre–Fourier polynomials form an orthogonal set of surface descriptors useful in the description of surface errors for cylindrical optics.^{9,10} A notable example of an optical system using cylindrical optics is NASA’s Chandra X-Ray Observatory. The Legendre–Fourier polynomials are a product of two sets of functions, where the Legendre polynomial represents the axial direction, and the Fourier series represents the azimuthal direction. The mathematical description of the Legendre–Fourier polynomials $f(z, \theta)$ is shown below, where a_{nm} are the coefficients, and G_{nm} are the polynomials:

$$f(z, \theta) = \sum_{n=0}^{\infty} \left[a_{n0} G_{n0} + \sum_{m=1}^{\infty} \left(a_{nm}^C G_{nm}^C + a_{nm}^S G_{nm}^S \right) \right], \quad (3.27)$$

where

$$G_{n0}(z, \theta) = \sqrt{2n+1} P_n(z), \quad (3.28)$$

$$G_{nm}^C(z, \theta) = \sqrt{2(2n+1)} P_n(z) \cos(m\theta), \quad (3.29)$$

and

$$G_{nm}^S(z, \theta) = \sqrt{2(2n+1)} P_n(z) \sin(m\theta). \quad (3.30)$$

Several of the Legendre–Fourier polynomial base surfaces are shown in Fig. 3.10. Azimuthally symmetric terms along with decenter, tilt, and out-of-roundness are illustrated.

The RMS of the Legendre–Fourier polynomials may be computed using the relationship below:

$$\sigma = \sqrt{\sum_{n=0}^{\infty} \left[a_{n0}^2 + \sum_{m=1}^{\infty} \left(a_{nm}^{(C)^2} + a_{nm}^{(S)^2} \right) \right]}. \quad (3.31)$$

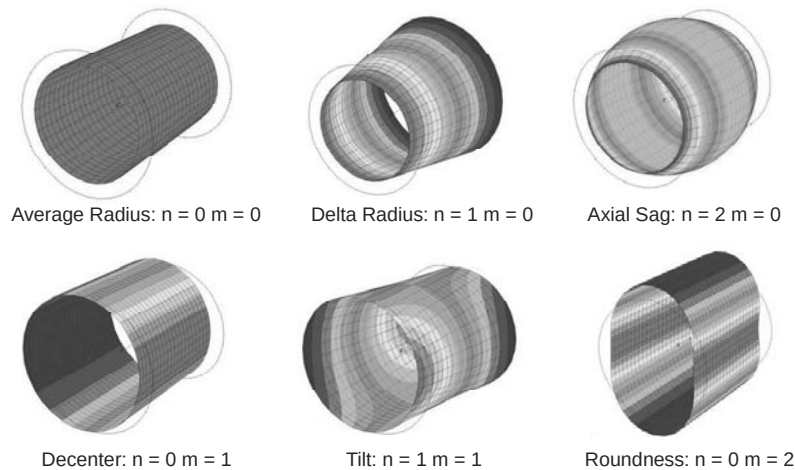


Figure 3.10 Legendre–Fourier low-order base surfaces.

3.6 Aspheric Polynomials

In optical design, aspheric polynomials are commonly used in conjunction with a base conic definition to describe the shape of aspheric surfaces. There are many forms of aspheric polynomials including the even, odd, toroidal, anamorphic, superconic, and Forbes definitions.

The even asphere polynomials have been a standard definition for rotationally symmetric aspheric surfaces that use the even powers of the radial coordinate and are expressed as

$$z(r) = Ar^2 + Br^6 + Cr^8 + Dr^{10} + Er^{12} + Fr^{14} + Gr^{16} + Hr^{18} + Jr^{20}. \quad (3.32)$$

The even aspheric polynomials may be used to fit axisymmetric distortions to a very high order. However, they have limited usefulness in representing most mechanical displacements since they cannot represent nonaxisymmetric behavior. Another disadvantage is that the even aspheric terms do not define an orthogonal set.

The Forbes¹¹ polynomials define an axisymmetric orthogonal set and simplify the design, test, and fabrication of rotationally symmetric optical elements as compared to the even asphere polynomials. The Forbes polynomials are described mathematically as

$$z(r) = (r/r_{\max})^4 \sum_{j=0}^J a_j Q_j^{\text{con}} [(r/r_{\max})^2], \quad (3.33)$$

where Q_j^{con} are the base surfaces of the polynomial. Radial plots of the lower-order base terms are shown in Fig. 3.11.

The Forbes polynomials have limited use in fitting surface displacements due to their inability to represent nonaxisymmetric errors. In addition, the terms start with an r^4 term, with no constant or r^2 term. Therefore, the polynomials alone cannot represent typical axisymmetric errors.

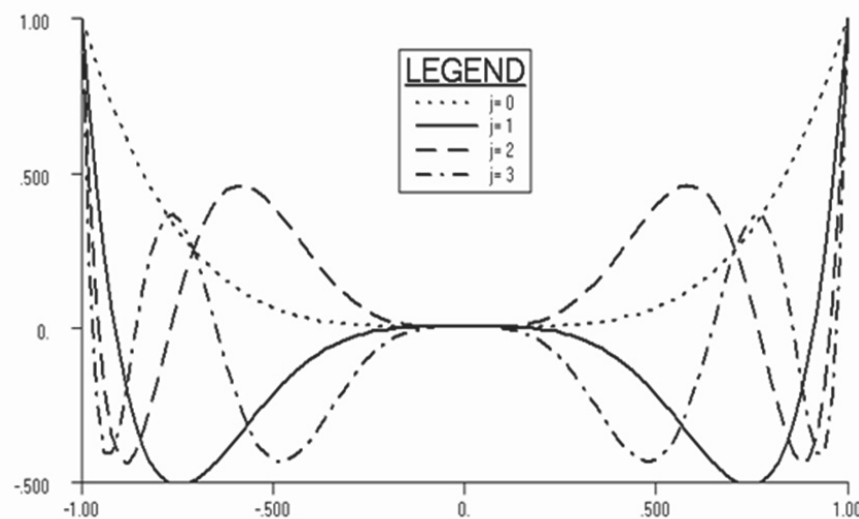


Figure 3.11 Radial plots of the lower-order Forbes polynomial terms.

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◀Chapter 4▶

Optical Surface Errors

Optical elements of high-performance imaging systems must meet demanding surface-error requirements to maintain precision pointing and overall image quality. For example, surface figure requirements typically must be maintained to fractions of a wavelength, and positional errors must meet micron and microradian tolerances. Finite element analysis is typically used to evaluate surface errors due to mechanical and environmental loads including inertial, dynamic, thermo-elastic, assembly loads, coating effects, adhesive shrinkage, CTE inhomogeneity, and others.

Integrating the FEA-derived optical surface errors into optical design software provides a means to predict optical behavior that can account for optical surface errors due to complex environmental conditions and concurrent disturbances. This class of analysis can be performed to predict optical performance as a function of time and provides insights beyond which is achievable with performance budget estimates. The impact of optical surface errors on optical performance can also be predicted using optical sensitivity coefficients. Use of optical sensitivity coefficients and matrices are convenient to perform “closed-loop” design trades and sensitivity studies that are beneficial early in the design process.

4.1 Optical-Surface Rigid-Body Errors

Mechanical and thermal loads that act on an optical system can significantly degrade optical performance by changing the position of optical elements and creating optical element misalignments. Positional or rigid-body errors include translations and rotations of a surface in six DOF. Translation of the optic along the optical axis is called despace, changes in lateral position are called decenter, and tip and tilt refer to rotations about the lateral axes as shown in Fig. 4.1. For non-rotationally symmetric optics, rotation about the optical axis must also be considered. These rigid-body errors result in optical system pointing errors and wavefront aberrations.

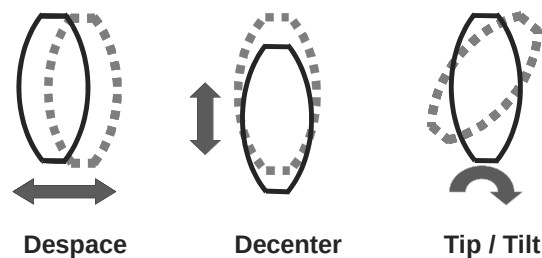


Figure 4.1 Rigid-body optical element motions.

4.1.1 Computing rigid-body motions

Computing the rigid-body motions of an optical element or surface using FEA depends upon the application and the desired model fidelity. For small optical elements where elastic deformations are considered insignificant, the rigid-body motions can be determined using a single node coupled with a lumped mass representation. This approach is illustrated in Fig. 4.2.

Using shell or solid elements to model an optical element where multiple nodes represent the optical surface requires post-processing of the FEA data to extract the rigid-body motions. One approach internal to FEA codes is the use of an interpolation element that is tied to the optical surface nodes to compute the average rigid-body motions. The method may be employed for static and dynamic mechanical loads but is not recommended for thermal loading. Use of the interpolation element for thermal loads does not account for the radial motion of the node when computing axial or despace rigid-body displacements (this is described in more detail in Section 4.2.1).

An alternative approach for computing rigid-body motions of an optical surface that is represented by multiple nodes is to perform a least-squares best-fit. This requires exporting the FEA surface displacements into an auxiliary software algorithm for post-processing. The equations for performing the least-squares fit are presented below.

For an optical surface that is represented by a grid of nodes, the rigid-body motion of the surface (three translations, T_x , T_y , T_z , and three rotations, R_x , R_y , R_z) may be computed as the area-weighted average motion. The rigid-body nodal displacements $d\tilde{x}_i$, $d\tilde{y}_i$, and $d\tilde{z}_i$, at a given node position x_i , y_i , and z_i , due to optical element rigid-body motions in six DOF are expressed as

$$\begin{aligned} d\tilde{x}_i &= T_x + z_i R_y - y_i R_z \\ d\tilde{y}_i &= T_y - z_i R_x + x_i R_z \\ d\tilde{z}_i &= T_z + y_i R_x - x_i R_y. \end{aligned} \quad (4.1)$$

The squared error E between the actual optical-surface nodal displacements dx_i , dy_i , and ds_i and the rigid-body nodal displacements $d\tilde{x}_i$, $d\tilde{y}_i$, and $d\tilde{z}_i$ is defined as

$$E = \sum_i w_i \left[(dx_i - d\tilde{x}_i)^2 + (dy_i - d\tilde{y}_i)^2 + (ds_i - d\tilde{z}_i)^2 \right]. \quad (4.2)$$

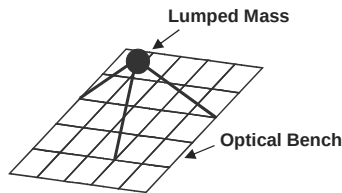


Figure 4.2 Single-node representation of an optical element with a lumped mass.

Note that the sag displacement ds is used in these calculations. This calculation is discussed in more detail in Section 4.2.1. The best-fit motions are found by taking partial derivatives with respect to each term and setting the result to zero. For example, the resulting equation for translation in the x direction is

$$\sum_i w_i T_x + \sum_i w_i z_i R_y - \sum_i w_i y_i R_z = \sum_i w_i dx_i. \quad (4.3)$$

Repeating this for each of the six rigid-body equations results in six simultaneous equations to solve for the average rigid-body motions.

4.1.2 Representing rigid-body motions in the optical model

The rigid-body errors computed from the FEA model may be represented in the optical model by using standard tilt and decenter commands that are commonly used to develop folded optical systems. These commands may be applied to perturb individual or groups of surfaces. Rigid-body errors applied to a double Gauss lens assembly are shown in Fig. 4.3. Adding FEA-derived surface errors to an optical model requires consistency between the mechanical and optical models in regards to units, geometry, and coordinate systems. In the FEA model, the displacement of nodes may be defined using either local or global coordinate systems. In an optical model, the coordinate system of an optical surface is nominally defined by a local coordinate system at the vertex. For on-axis optics, where the vertex is at the geometric center of the optic, maintaining consistency between the mechanical and optical coordinate systems is straightforward. For off-axis optics where the vertex is off-center or not physically on the optical substrate where typically the mechanical coordinate system is located, it is more challenging. In this instance, coordinate systems may be defined within the optical model using dummy surfaces and coordinate breaks that are located at the physical center of the substrate consistent with the mechanical model. Alternatively, within the FEA model, the vertex motions of an off-axis surface may be determined by adding a rigid link that relates the average rigid-body motions of the optical surface to the vertex location.

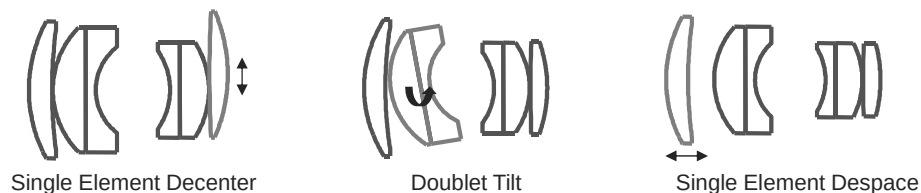


Figure 4.3 Rigid-body motions of optical elements in a double Gauss lens assembly.

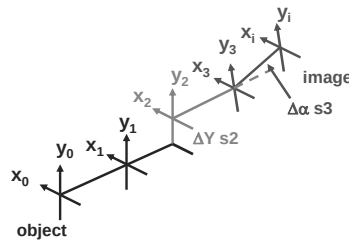


Figure 4.4 In the optical model, decenters and tilts are nominally applied to the local coordinate system defining the surface.

Nominally applying rigid-body errors to optical surfaces in the optical model is done by tilting or decentering the local coordinate system that defines the surface, as illustrated in Fig. 4.4. This results in cumulative errors since each local coordinate system is defined relative to the local coordinate system of the preceding surface. A common method to uncouple the perturbations is to specify a decenter and return, which, as the name implies, returns the local coordinate system of the surface following the tilted and decentered surface to the original coordinate frame. Repeating this command for each of the surfaces allows rigid-body errors to be defined independently. Other methods to uncouple the rigid-body errors include the use of global coordinates as well as coordinate breaks and/or dummy surfaces. In general, applying rotations to an optical surface in the optical model is order dependent. However, for small rotations such as those typically computed by a linear finite element analysis, the order of the rotations can generally be neglected.

It is recommended that the rigid-body surface errors be separated from the higher-order elastic optical surface deformations and represented in the optical model using tilts and decenters. The residual surface deformations can be represented through polynomial fits or interpolated arrays. This approach affords the greatest accuracy and in addition provides greater insight into the behavior of the optical system.

4.2 Optical-Surface Shape Changes

Mechanical and thermal loads acting on an optical instrument may elastically deform the shape of the optical surface. Peak-to-valley (P-V) and root-mean-square (RMS) values are typically used to quantify a discrete set of surface displacements. The relationship between P-V and RMS is dependent upon the deformed shape. Rule-of-thumb estimates are shown for select surface errors in Fig. 4.5.

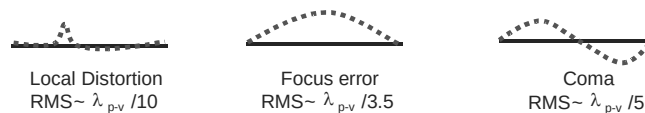


Figure 4.5 Relationship between P-V and RMS surface error is dependent on the deformed surface shape.



Figure 4.6 Sag displacements on the left and surface normal displacements on the right.

Optical-surface shape changes may be characterized by changes in the sag of the optical surface or changes in the surface normal. Sag and surface normal displacements are illustrated in Fig. 4.6. The sag displacement is defined as the distance from the vertex tangent plane to the optical surface. Perturbations in the sag of an optical surface may be added as changes to the nominally defined surface. The use of surface normal displacements are based on interferometric testing that measures surface errors normal to the optical surface. Both the sag and surface normal errors may be computed from the finite element displacement vector and used to represent deformed optical surfaces within optical design software.

4.2.1 Sag displacements

The nominal shape of an optical surface is typically defined by the sag of the surface as a function of radial position r . Perturbations to the optical surface shape may be represented by changes in the nominal sag definition. In general, the sag deformation is not equal to the finite element computed displacement vector measured along the optical axis since the node position may also be radially displaced. This is illustrated for an optical surface supported at the vertex undergoing a uniform increase in temperature in Fig. 4.7. The temperature increase causes the radius of curvature to increase; thus, the change in the sag value for any position on the optical surface is negative. However, the z displacement dz , as computed by the finite element model, is positive. In the case where the loading causes small radial motions of the node as compared to the axial motion, such as under inertial or dynamic loads, dz is a very close approximation to the sag displacement.

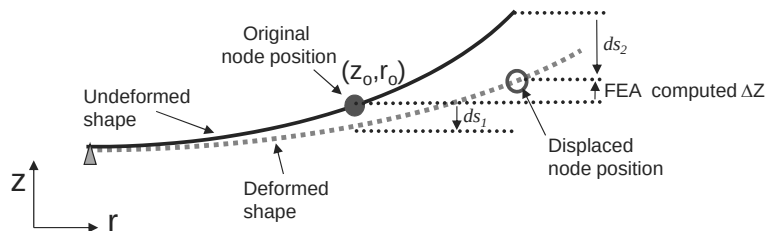


Figure 4.7 Two approaches to compute sag displacements that account for radial motion.

Two methods to compute the sag change ds in an optical surface that accounts for radial motion are discussed below. The first approach computes the change in the sag ds_1 , based on small-displacement theory¹ using the nominal node position and the perturbations dx , dy , and dz :

$$ds_1 = dz - \frac{\partial z(r_0)}{\partial r} \sqrt{dx^2 + dy^2}. \quad (4.4)$$

This method to compute the sag displacement is linear and thus the values may be scaled. This provides advantages in the form of computational efficiencies for trade studies, sensitivity analyses, and the combining of multiple load cases such as unit g-loads, thermal soaks, and thermal gradients. The computation also enables the use of modal techniques for surface error calculations due to dynamic loading and also active control simulation that rely on linear calculations. The calculation is an excellent approximation and practically starts to degrade for highly curved surfaces faster than $f/1$.

The second approach uses the displaced node position to determine the change in the sag position^{2,3} ds_2 . The sag is computed as the difference between the total sag at the displaced node location and the sag defined by the nominal optical surface at the displaced radial location, expressed as

$$ds_2 = dz + \text{sag}(x_i, y_i) - \text{sag}(x_i + dx, y_i + dy). \quad (4.5)$$

This approach is an exact solution to the sag of the optical surface and is recommended for use on highly curved surfaces faster than $f/1$. In general, this calculation is nonlinear and the sag values ds_2 may be not be linearly scaled.

4.2.2 Surface normal deformations

Optical-surface shape changes may also be represented by surface errors normal to the optical surface. Surface normal displacements may be determined for each point on a given surface by the dot product of the finite element displacement vector (dx , dy , dz) with the unit surface normal vector. (The surface normal vector is computed by taking the gradient at a given point and normalizing.) For a spherical surface, the surface normal displacement d_{sn} at a given (x, y) position and surface curvature ρ is computed using the following relationship:

$$d_{sn} = dz \sqrt{1 - \rho^2(x^2 + y^2)} - \rho(xdx + ydy). \quad (4.6)$$

It is assumed that the z axis is parallel to the optical axis.

4.3 Relating Surface Errors to Wavefront Error

There are simple expressions that relate optical surface errors to the wavefront error of the optical system for both refractive and reflective surfaces. These relationships are based on the surface normal error of the optical surface.

4.3.1 Refractive surfaces

The relationship between a surface normal error d_{sn} and the wavefront error WFE for a refractive surface is given by

$$WFE = (n \cos \theta - n' \cos \theta') d_{sn}, \quad (4.7)$$

where n represents the index of refraction of the medium, n' is the index of the optical element, θ is the angle of incidence, and θ' is the angle of refraction. This is depicted in Fig. 4.8.

The nomenclature for a ray intersecting an optical surface is shown in Fig. 4.8(a), and the ray paths for the nominal and perturbed ray with the normal surface error d_{sn} are depicted in Fig. 4.8(b). The difference in the two optical paths is the OPD and the resulting wavefront error.

The impact of a bump on the surface of a window for a planar wavefront in air ($n' = 1$) is illustrated in Fig. 4.9. In this case, the wavefront error simplifies to

$$WFE = (n - 1)d_{sn}. \quad (4.8)$$

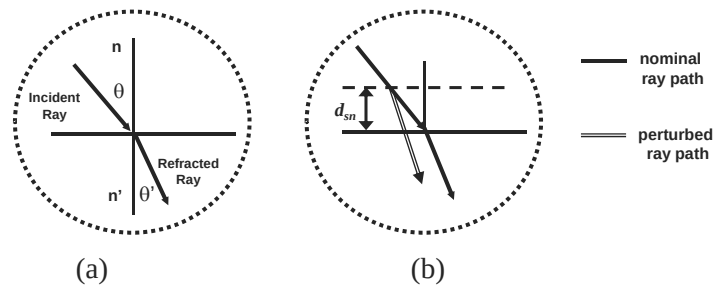


Figure 4.8 (a) Nomenclature for ray hitting refractive surface, and (b) ray paths for both nominal and perturbed ray paths.

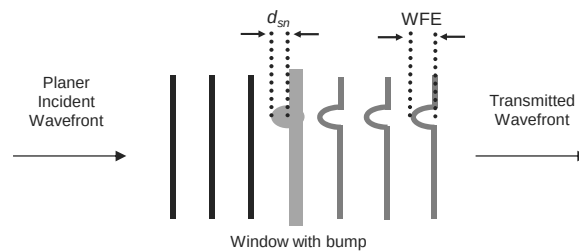


Figure 4.9 Wavefront error due to bump on window.

For visible optical systems, common window materials include BK7 and fused silica (index of refraction ~ 1.5) that result in wavefront errors on the order of half the surface error. For IR materials with higher indices of refraction that range from 1.8 to 4, such as sapphire, zinc selenide, silicon, and germanium, a bump can create appreciable wavefront errors.

Mechanical loads that act on transmissive optical elements tend to deform both the front and rear surfaces. For a ray travelling through a deformed element, wavefront error is created by the difference in the surface error between the front and rear surfaces. For a normally incident wavefront on an optical element in bending, the front and rear displacements can compensate for each other. For light entering at non-normal incidence on an optical element, the wavefront intersects the front and rear surfaces at different locations, and thus both pointing and wavefront errors can result even if the front and rear surface deformations are equal.

4.3.2 Reflective surfaces

The wavefront error for a ray reflecting off a deformed optical surface is given by

$$WFE = 2d_{sn} \cos \theta, \quad (4.9)$$

where θ is the angle of incidence.

The nominal ray path and the perturbed ray path for a surface error on a reflective surface is shown in Fig. 4.10. The OPD created by the surface error is shown by the dotted line and is computed using a reference plane that is normal to the reflected ray direction.

The impact of a bump on the surface of a mirror for a wavefront at normal incidence is illustrated in Fig. 4.11. In this case, the wavefront error simplifies to

$$WFE = 2d_{sn}. \quad (4.10)$$

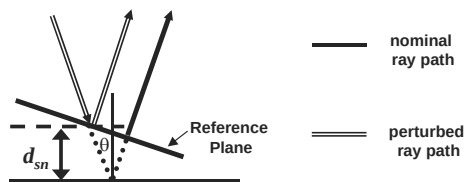


Figure 4.10 Optical path error for a ray hitting an optical surface with a surface normal error.

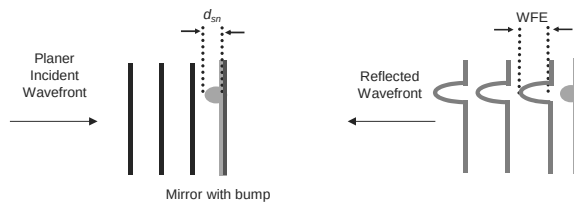


Figure 4.11 Wavefront error due to a bump on a reflective surface.

4.4 Optical Surface Deformations and Zernike Polynomials

A popular method to represent optical surface deformations is the use of Zernike polynomials. Data interpretation is simplified by representing hundreds or thousands of FEA-computed surface displacements by a few significant Zernike terms. Also, Zernike polynomials provide the ability to pass data to optical design software, and the ability to assess residual errors after rigid-body and/or elastic error correction via alignment of downstream optical elements or active control.

4.4.1 Optical-surface error analysis example

An optical-surface error analysis is performed on a primary mirror of a Cassegrain telescope. The telescope is subject to gravity acting perpendicular to the optical axis and a uniform temperature change of 40 °C. The finite element model and resulting displacement contour map is shown in Fig. 4.12. The average rigid-body errors of the surface include a translation in the z direction and a tilt about the x axis that are listed in Table 4.1. The surface error contour plot of the sag displacements with the rigid-body errors removed is shown in Fig. 4.13(a). The higher-order sag displacements are fit to Zernike polynomials and are listed in Table 4.2. The dominant Zernike terms representing the deformed surface are focus, spherical, and trefoil.

An advantage of using Zernike polynomials with finite element data is that they form an approximate set of orthonormal terms, which means terms may be removed from the data with little effect on the value of the other terms. For optical systems with active control or compensating elements that can correct the rigid-body and/or focus errors of a given surface, the remaining Zernike terms

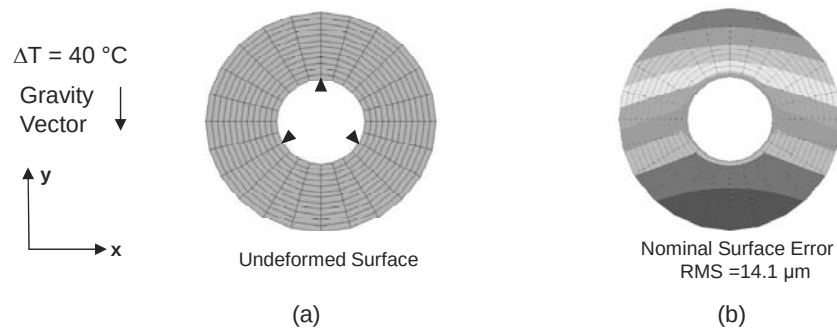


Figure 4.12 (a) Gravity and thermal loads acting on a primary mirror, and (b) resulting surface deformations.

Table 4.1 Average rigid-body errors.

Translations (μm)			Rotations (urad)		
Tx	Ty	Tz	Rx	Ry	Rz
0	0	2.4	100	0	0

Table 4.2 Optical surface deformations represented by Zernike polynomials after rigid-body terms removed.

Aberration Type	Magnitude (waves)	Phase (deg)	Residual RMS	Residual P-V
			1.04	4.3
Piston	0	0	1.04	4.3
Tilt	0	0	1.04	4.3
Focus	2.5	0	0.33	1.5
Pri Astigmatism	0	0	0.33	1.5
Pri Coma	0	0	0.33	1.5
Pri Spherical	-0.5	0	0.26	1
Pri Trefoil	0.6	30	0.1	0.4
Sec Astigmatism	0	0	0.1	0.4
Sec Coma	0	0	0.1	0.4
Sec Spherical	0.1	0	0.09	0.4
Pri Tetrafoil	0	0	0.09	0.4
Sec Trefoil	0.2	-30	0.04	0.25
Ter Astigmatism	0	0	0.04	0.25
Ter Coma	0	0	0.04	0.25
Ter Spherical	0	0	0.04	0.25
Pri Pentafoil	0	0	0.04	0.25
Sec Tetrafoil	0	0	0.04	0.25
Ter Trefoil	0.1	30	0.02	0.15
Qua Astigmatism	0	0	0.02	0.15
Qua Coma	0	0	0.02	0.15
Qua Spherical	0	0	0.02	0.15
Qin Spherical	0	0	0.02	0.15

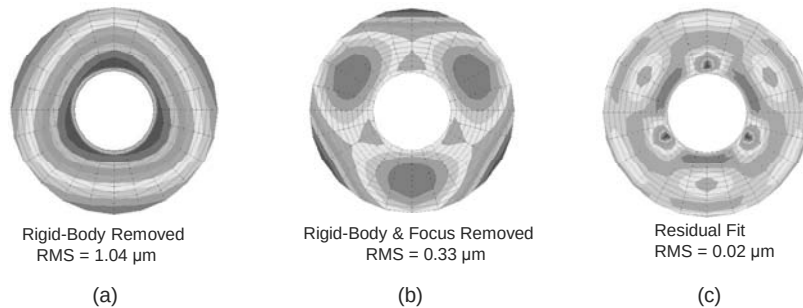


Figure 4.13 Residual optical surface deformations after (a) rigid-body errors are removed, (b) rigid-body and focus Zernike terms are removed, and (c) residual error plot showing the data not fit by the Zernike polynomials.

represent the uncorrectable errors. Design modifications may then concentrate on minimizing the residual surface errors.

The residual RMS and peak-to-valley columns in the Zernike table list the remaining surface errors after each of the Zernike terms above and in the designated row are removed. For example, after piston, tilt, and focus terms are removed from the data, the remaining RMS surface error is 0.33 μm and the peak-to-valley error is 1.5 μm . In the final row of the table, the residual error is the difference between the Zernike fit and the actual data and is a measure of how well the polynomials fit the data. The residual surface error contour plot is shown in Fig. 4.13(c). The polynomials are unable to fit the high frequency spatial errors and in particular around the mounting locations.

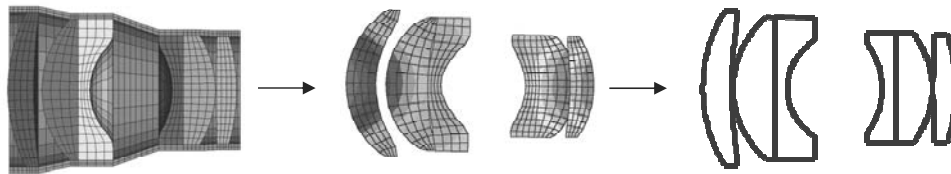


Figure 4.14 Data flow of finite-element-computed surface deformations into an optical model.

Since the Zernike polynomials are fit to a single vector quantity (surface normal or sag data), they do not represent the full rigid-body motion of an optical surface in six DOF. For instance, fitting Zernike terms to the surface displacements of a flat optical surface yields no information about whether the surface was laterally displaced or rotated about the optical axis. When computing optical element errors, it is common practice to remove the rigid-body errors and represent the higher-order surface deformations in the optical model using techniques discussed in Section 4.5.

4.5 Representing Elastic Shape Changes in the Optical Model

There are several commonly utilized methods to represent finite-element-derived optical surface displacements within commercial optical design software (such as Code V⁴ and Zemax⁵). This process is depicted in Fig. 4.14. These methods include polynomial surface definitions, surface interferogram files, and uniform arrays of data that use either sag or surface normal displacements. General discussion and application of representing finite-element surface displacements using the above optical modeling techniques is discussed by Doyle et al.⁶

Engineering judgment determines the “best” modeling approach for a specific application and is dependent on the optical system, optical model, and the desired accuracy. Uncertainties such as material properties, boundary conditions, and load conditions along with understanding limitations and approximations in the accuracy of the models should also enter into the decision as to the most applicable approach.

4.5.1 Polynomial surface definition

Polynomial surface definitions use a base surface definition plus the addition of polynomial terms to describe the shape of an optical surface. This definition allows finite-element displacement data to be fit to polynomials and added as perturbations to the base surface for ray tracing in the optical model. Polynomial options include Zernike polynomials, X-Y polynomials, aspheric polynomials, and others. The user can select the polynomial set that best represents the FEA displacements. The shape of an optical surface is defined by the sag displacement from the tangent plane. Thus, the polynomials must be fit to optical surface sag displacements. Finite-element-derived sag deformations, for example, can be

represented by Zernike coefficients as perturbations to the base surface shown below:

$$\text{sag} = \frac{cr^2}{1 + \sqrt{1 - (1+k)c^2r^2}} + \sum a_i Z_i. \quad (4.11)$$

The first term is the nominal conic surface definition, and the second term represents the perturbations to the base surface represented by the Zernike coefficients a_i and the Zernike polynomials Z_i .

The accuracy of this approach is dependent on the accuracy of the polynomial fit to the surface displacements. Fitting to a larger number of terms provides the potential of an improved fit and hence accuracy. The maximum number of terms allowed in the fit is dependent upon the optical design software.

4.5.2 Interferogram files

Surface interferogram files are 2D data sets that represent surface normal deviations that are assigned to optical surfaces in the optical model. This file format is also used to represent interferometrically measured topographical fringe maps created during optical testing. Use of an interferogram file is an approximate technique to represent a deformed surface shape, as compared to ray tracing off a deformed optical surface represented using a polynomial surface definition. The approximation lies in the computation of the optical errors for a given ray. A ray is traced to the undeformed surface, and the intersection coordinates are used to determine the surface error as defined by the interferogram file from which ray deviations and OPD are computed. The error associated with this approximation is a function of the ray angle and the spatial variation and magnitude of the displacement field. The error in this approximation in representing FEA optical-surface deformations consistent with mechanical perturbations is typically negligible for most applications.

Interferogram file data can be represented in two formats: Zernike polynomials (Standard or Fringe) or as a uniform rectangular array (or grid array) and require finite element displacements to be converted into surface-normal displacements.

The Zernike polynomial format provides a more accurate representation relative to a grid array if an accurate fit is achieved. Code V places no limit on the number of Zernike polynomial terms that may be used to represent the surface normal displacements. Surface deformations and slope data is computed directly from the polynomial representation. The grid format is useful when an accurate Zernike fit cannot be achieved.

The interferogram file data may be scaled in the optical model, which is useful in performing design trades by scaling surface errors due to unit g-loads, thermal soaks, or thermal gradients. As with assigning rigid-body perturbations to an optical surface, understanding and relating the finite element coordinate system to the optical surface coordinate system is necessary for a successful surface-error representation. For instance, in Code V, a positive surface

deformation represents a “bump” on the optical surface, as shown in Fig. 4.15. This is consistent with measuring the surface from the “air” side of the element.

In addition, it is necessary to align and place the interferogram file at the correct location and with the proper orientation on the optical surface. Commands are available to scale, mirror (reverse or flip), rotate, and decenter the interferogram file to the correct position. Test cases should always be run to verify that the position and orientation of the interferogram files are correct.

4.5.3 Uniform grid arrays of data

Uniform grid arrays of data are useful in representing optical surface displacements when an accurate polynomial fit cannot be achieved. Grid arrays are able to represent high-frequency spatial variations seen in edge roll-off, localized mounting effects, or quilting of a lightweight optic. For example, two residual surface-displacement maps after adaptive correction (gravity loading on the left and thermal loading on the right) are shown in Fig. 4.16. The percent of the RMS surface error represented by a 66-term and 231-term Standard Zernike polynomial is shown in Table 4.3. A large fraction of the surface displacements is not included in the Zernike fit for each of these two cases. A uniform array provides a much more accurate representation. For example, a 51×51 array represents over 98% and 99% of the RMS surface error for the two cases, respectively.

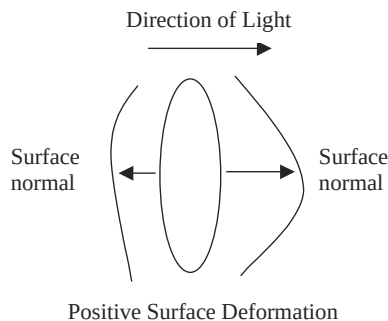


Figure 4.15 Sign convention for Code V surface interferogram files.

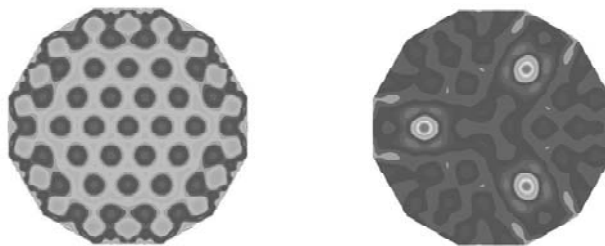


Figure 4.16 Surface displacements due to gravity (left) and thermal soak (right) after adaptive correction.

Table 4.3 Percent of RMS surface error represented by 66- and 231-term Standard Zernike polynomial and a 51 x 51 uniform grid array.

	66-Term Fit	231-Term Fit	Grid 51x51
Gravity	5%	32%	98%
Thermal Soak	4%	40%	99%

The loss in accuracy in representing surface displacements with uniform arrays of data is two-fold; first, interpolation is required to create a uniform rectangular array from a non-uniform FEA mesh; and second, errors result from ray tracing in the optical model for incident rays that do not coincide with a data point. In this case, a second interpolation step is used within the optical model to compute the surface errors. Two common uniform-array formats are Code V's surface interferogram files and the Zemax Grid Sag surface.

4.5.3.1 Grid Sag surface

The Grid Sag surface is a Zemax surface definition that uses a uniform array of sag displacements and/or slope data to define perturbations to a base surface. The base surface has a shape defined by a base plane, sphere, conic asphere, or polynomial plus additional sag terms defined by a rectangular array of sag values, defined as

$$\text{sag} = \frac{cr^2}{1 + \sqrt{1 - (1+k)c^2r^2}} + z(x_i, y_i). \quad (4.12)$$

Zemax offers two interpolation routines, linear and bicubic, to determine the surface errors during optical ray tracing. If only sag displacements are provided, the linear interpolation routine is used to compute the slope terms using finite differences. If, in addition to the sag displacements, the first derivatives in the x and y directions $\partial(ds)/\partial x$, $\partial(ds)/\partial y$, and the cross-derivative terms $\partial(ds)^2/\partial x \partial y$ are supplied by the user, then Zemax's bicubic interpolation may be used. The rotation values help ensure a smooth fit over the boundary points.

4.5.3.2 Interpolation

In general, creating a grid interferogram file or Grid Sag surface requires the surface displacements computed at the finite-element grid points to be interpolated to a uniform grid, as shown in Fig. 4.17. The accuracy of the interpolation method is critical for high-performance optical systems, such as near mounting locations where regions of rapidly varying displacements commonly exist.

One method to interpolate surface data to a uniform grid uses Delaunay triangulation techniques including nearest neighbor, linear, and cubic. Another method to interpolate data to a uniform grid is to use the finite element shape

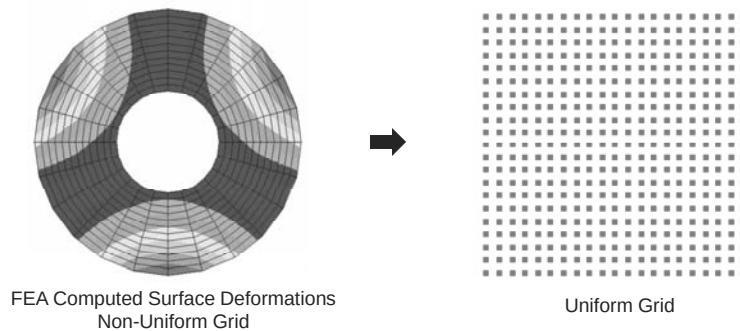


Figure 4.17 Interpolating FEA displacements to a uniform grid.

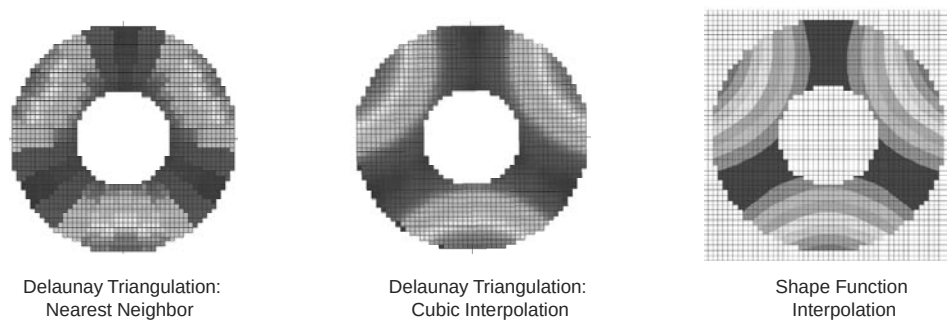


Figure 4.18 Interpolation using Delaunay triangulation and FE shape functions.

functions.^{7,8} In this approach, values are interpolated to the grid points using the shape functions from the surface element in which the grid point falls.

For 3D models, interpolation may be performed by creating a set of “dummy” plate elements to be modeled on the optical surface. The structural thickness of the plate elements can be made arbitrarily small. Alternatively, the 2D shape functions on the solid element face can be used to perform the interpolation.

An example of interpolating a finite-element mesh to a uniform grid is shown for Delaunay triangulation techniques (nearest neighbor and cubic) and cubic finite element shape functions in Fig. 4.18. The interpolation from a FEA mesh to a rectangular array is more accurate using cubic interpolation (as compared to linear interpolation). In this case, a surface of “dummy” plate elements is required to provide the nodal rotations. Note for accurate edge effects, a surface coat of “dummy” plate elements should wrap around the optic to avoid erroneous edge effects (see Section 10.2.3.3 for more detail).

4.6 Predicting Wavefront Error Using Sensitivity Coefficients and Matrices

An approximate technique to predict the wavefront error in an optical system due to surface errors is through the use of wavefront error sensitivity coefficients.

Wavefront error sensitivities are computed by applying a unit surface error perturbation within the optical model and computing the resulting RMS wavefront error. These sensitivities are then used to multiply the actual FEA-derived optical surface errors to compute the optical system wavefront error. This approach has the advantage of allowing the mechanical engineer to perform “closed-loop” design trades without requiring the additional step of importing surface errors into the optical model. This approach is suitable during the preliminary design stages and works as long as the optical design does not change. Two approaches are commonly used including use of rigid-body and radius-of-curvature sensitivity coefficients, and the use of Zernike polynomial sensitivity matrices.

4.6.1 Rigid-body and radius-of-curvature sensitivity coefficients

This method computes the optical system wavefront error based on the rigid-body and radius-of-curvature changes for each optical element. Wavefront error optical sensitivity coefficients are computed by applying a unit rigid-body perturbation in six DOF and a radius-of-curvature perturbation for each optical element in the system. Wavefront error is computed by multiplying the optical sensitivity coefficients by the corresponding finite-element-derived surface errors to determine the RMS wavefront error contribution from each effect. (Computing the radius-of-curvature change from a set of FEA-computed optical surface displacements is discussed in Section 4.6.1.2.) The system wavefront error is computed by root-sum-squaring (RSS) the individual RMS values. This may be performed for multiple environmental effects including mechanical and thermal loading. This analysis technique is approximate but has the advantage that as long as the optical design does not change, efficient design trades may be performed in the mechanical design space. This method ignores the higher-order surface shape changes of the optical element and assumes that the optical errors are uncorrelated. An example where utilization of wavefront error sensitivity coefficients is not appropriate is for an optical system made of a single material experiencing uniform temperature changes where the errors are correlated. In this case, the changes in the position and shape of the optical elements compensate for each other resulting in zero wavefront error.

4.6.1.1 Wavefront sensitivity coefficients example

The wavefront error for a Cassegrain telescope subject to gravity using rigid-body and radius-of-curvature sensitivity coefficients is shown in Fig. 4.19. A spreadsheet is used to multiply the sensitivity coefficients by the FEA-computed optical surface displacements. The errors are combined using the root sum square method that assumes the errors are independent. For comparison, these same optical errors were added to the optical model for direct calculation of the system wavefront error that resulted in a 19% difference. Whereas this approach is approximate, the technique can be effective in getting an “80%” solution appropriate for early design trades and sensitivity studies.

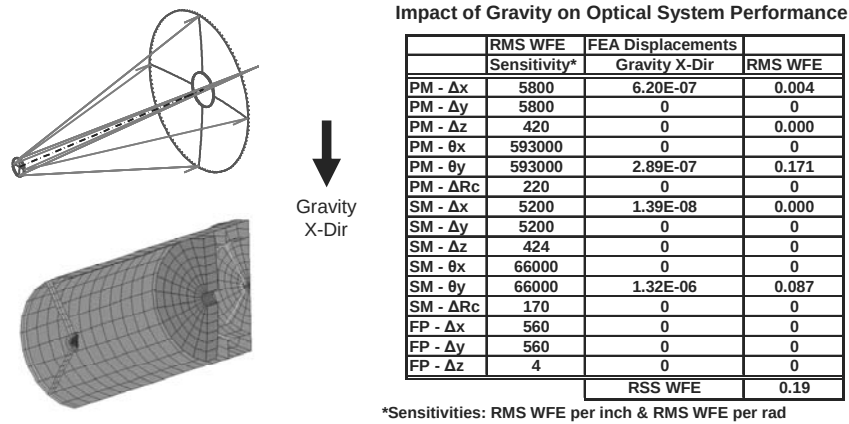


Figure 4.19 Computing optical system wavefront error using rigid-body and radius-of-curvature wavefront error sensitivities.

4.6.1.2 Computing radius of curvature changes

Two methods to compute the change in the radius of curvature of an optical surface from a set of FEA displacements are discussed. The first approach is an approximate technique using the Zernike focus term that has been fit to the sag deformations of the optical surface. This is an approximation because the Zernike focus term is parabolic and a function of r^2 , whereas a spherical surface includes higher-order radial terms as shown in the series expansion s :

$$s = \frac{r^2}{2R} + \frac{r^4}{8R^3} + \frac{r^6}{16R^5} + \frac{r^8}{128R^7} + \dots, \quad (4.13)$$

where r is the radial extent of the surface, and R is the radius of curvature of the optical surface, as illustrated in Fig. 4.20. This approximation may be demonstrated by fitting Zernike polynomials to an optical surface with a pure radius of curvature change. The focus term and the higher-order rotationally symmetric terms are used to describe the deformed shape. For optical surfaces that are not highly curved, the sag contribution of a spherical surface is dominated by the parabolic term.

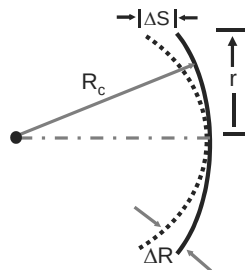


Figure 4.20 Radius of curvature change.

The Zernike focus term is a “best-fit” quadratic to the deformed shape and may be used to estimate a change in the radius of curvature ΔR of an optical surface as given below:

$$\Delta R = 4Z_{20} \left(\frac{R}{r} \right)^2, \quad (4.14)$$

where Z_{20} is the coefficient of the amplitude-normalized Zernike focus term.

An alternate approach to compute the change in the radius of curvature of an optical surface is using a least-squares approach with Newton’s method. The following approach is iterative and solves for two variables (c^* and b^*) to find the best-fit change in the vertex radius of curvature R for a set of FEA surface deformations.

The error term E to be minimized is computed as the sum of the squared errors at each node:

$$E = \sum_j w_j \left[(s_j + d_j) - (s_j^* + b^*) \right]^2, \quad (4.15)$$

where w_j is the area weighting, s_j is the nominal sag position, s_j^* is the sag position based on the best-fit radius of curvature, d_j is the sag displacement of node j , and b^* is the axial motion of the center of curvature.

The original sag position of node j , s_j , is found from

$$s_j = \frac{cr_j^2}{1 + \sqrt{1 - (1+k)c^2r_j^2}}, \quad (4.16)$$

and the sag position of node j using a new curvature c^* is given by

$$s_j^* = \frac{c^* r_j^2}{1 + \sqrt{1 - (1+k)c^{*2}r_j^2}}, \quad (4.17)$$

where c^* is the new curvature ($c^* = 1/R^*$). Newton’s method can then be used to find the best-fit change in the radius of curvature using c^* and b^* .

4.6.2 Use of Zernike sensitivity matrices

The use of Zernike sensitivity matrices allows both rigid-body and higher-order, elastic optical surface errors to be included in the wavefront calculation. This technique requires applying both unit rigid-body and individual Zernike surface perturbations to each optical surface in the optical model and computing a set of Zernike coefficients that describe the optical system wavefront error for each

perturbation. The system wavefront error is then computed using linear superposition by multiplying the actual optical surface errors from the applied loads by the set of Zernike wavefront error sensitivity matrices as illustrated in Fig. 4.21. This method assumes that the optical surface errors may be accurately represented by Zernike polynomials and that the system behaves linearly. This approach may account for multiple environmental effects and has the advantage of accurately accounting for dependent and correlated errors such as thermal soaks.

4.7 Finite-Element-Derived Spot Diagrams

Optical surface quality may be evaluated using a spot diagram computed directly from the finite element model.^{9,10} Here, collimated light is assumed incident on a finite element surface. Rays are modeled as rigid bars from each node on the optical surface to unmerged nodes located at the image point. The image point is located at twice the focal length because the angle of the reflected rays off of the deformed optical surface is twice the angle error as computed by the finite element model. This computes the correct ray displacement. A plot of the location of the displaced nodes on the image plane gives the spot diagram; a corresponding RMS spot size may then be computed.

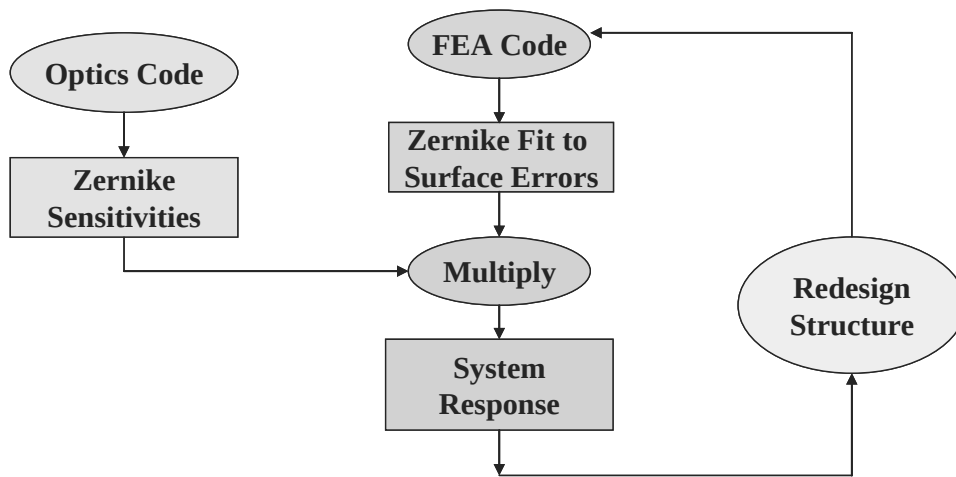


Figure 4.21 Computing optical-system wavefront error using Zernike sensitivity coefficients.

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◀Chapter 5▶

Optomechanical Displacement Analysis Methods

This chapter presents guidelines relevant to finite-element-model construction and analysis methods for predicting the motion and deformation of optics. A key idea to be conveyed is that an analyst's choice of how to model optical components is dependent on several factors. The most obvious factor, of course, is that the mechanical behavior of the hardware will require that certain modeling features and methods be used in order to accurately predict a system's true behavior. However, consideration of this factor alone would lead to the construction of finite element models that capture the mechanical detail of every fillet and stress riser in the system. This approach is certainly not practical when schedule and cost constraints are prohibitive of such an effort. Fortunately, predicting most optomechanical performance metrics do not require models capable of such extensive mechanical representation. Often, only first-order mechanical behavior is needed to provide sufficient accuracy in the prediction of optical performance. Another important factor in the choice of a modeling method is how the analysis results will be used. For example, if the goal of an analysis is to compare several design concepts in the early phases of a feasibility study, then simple models that may not accurately predict the absolute behavior may nevertheless be effective in providing relative performance predictions among the various design concepts. By presenting an array of modeling methods, each with their own limitations and strengths, it is hoped that the reader becomes better able to make the best modeling decisions to meet the technical, schedule, and cost requirements of any optomechanical displacement analysis task.

5.1 Displacement FEA Models of Optical Components

5.1.1 Definitions

In discussing the displacement models of optics, it is helpful to define a few terms relevant to optic motion and deformation. These definitions aid future discussions of the limitations of the various modeling methods.

Component rigid-body motion is the set of average translations and rotations of the optical component. This quantity can also be thought of as the motion of the center of mass of the optical component as illustrated in Fig. 5.1.

Optical surface rigid-body motion is the set of average translations and rotations of the optical surface of an optical component. Fig. 5.1 illustrates that this motion may be different from the component rigid-body motion.

Global surface deformation is the component of the total surface deformation that is exhibited over most or the entire optical surface. Such deformations are

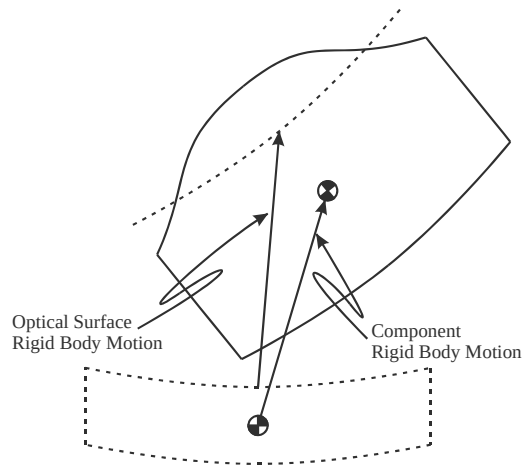


Figure 5.1 Component rigid-body motion and optical surface rigid-body motion are distinct quantities.

well predicted, in general, by both coarse and detailed finite element models and are reasonably approximated by low-order surface polynomials.

Local surface deformation is the component of total surface deformation of an optical surface that is confined to local regions. Such deformations generally require more detailed finite element models to be accurately predicted. Local surface deformations also require very high order surface polynomials to be described, or they may not be representable by polynomials at all. Such deformations usually result from mount-induced effects.

Quilting deformation is a specific type of local surface deformation seen in lightweighted mirrors that have a relatively thin optical facesheet backed by a cellular core structure. Sources of quilting deformation include thermoelastic deformation of the optical facesheet caused by nonuniform thermal gradients through the thickness of the optical facesheet and elastic deformation of the optical facesheet due to an applied gravity load or polishing pressure.

5.1.2 Single-point models

The simplest of all displacement models is the single-point model where the optic is represented by a single node as shown in Fig. 5.2. In such a model only the component rigid-body motions are predicted. Therefore, such a model is used when the elastic deformation of the optic is not important to the goal of the analysis. Common applications for single-point optic models are for small mirrors and lenses whose elastic deformations do not significantly contribute to optical performance degradation. It is also assumed that the optical surface rigid-body motion can be sufficiently approximated by the component rigid-body motion, or it is not of interest to results of the analysis. If thermoelastic effects or other mechanical behaviors cause the component rigid-body motion to be measurably different from the optical surface rigid-body motion as shown in Fig. 5.1, a model of this type may not be acceptable.

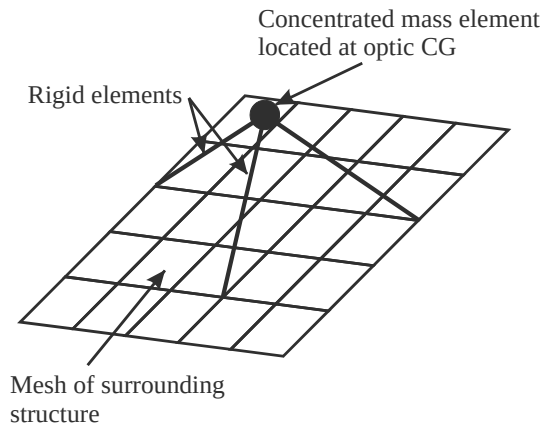


Figure 5.2 Single-point model of an optic connected to a surrounding mesh with rigid elements.

The connection of the single-point model to its supporting structure is an important consideration. The selected element or elements used to perform the connection may be rigid, interpolation or elastic. Rigid elements are multipoint-constraint equation elements that employ rigid kinematic formulations to link the degrees of freedom of a single independent node to the degrees of freedom of one or more dependent nodes. Interpolation elements are multipoint-constraint equation elements that employ averaging formulations to link the degrees of freedom of a single dependent node to the degrees of freedom of a group of independent nodes. While rigid elements allow no relative deformation between the degrees of freedom they connect, interpolation elements add no stiffness to the set of nodes used to compute the average of the dependent node. Therefore, rigid elements and interpolation elements offer two opposite extremes with regard to the stiffness added to the model. This makes them convenient tools to bound mechanical predictions for situations in which an optical mount is not yet designed but must be included in an analysis of the system in which it is used.

Rigid element and interpolation element formulations, however, may have no thermal expansion capabilities depending on the features of the finite-element tool being employed. Therefore, erroneous deformations may be predicted by a single-point model of an optic connected by rigid or interpolation elements if thermoelastic loads are applied. A rigid element with no thermoelastic growth can introduce radically erroneous results due to the resulting fictitious thermoelastic mismatch. Even if thermoelastic properties are specified for a rigid element, erroneous thermoelastic mismatch results can result from the misrepresentation in stiffness associated with rigid elements. Use of an interpolation element is often the best approximation if no representative elastic model is to be developed. An alternative to the use of rigid and interpolation elements is the use of beam elements with representative properties and thermoelastic expansion properties. In addition, the use of zero-length rigid

elements or pin flags at the connection points attached to the supporting structure can be used to link the single-point model in only specific degrees of freedom. Such a connection may be used to represent a kinematic interface.

When single-point models are used in dynamics analyses, it is important to include a complete description of the optic's mass. This mass description should include mass moments and mass products of inertia in addition to the translational mass. Such mass properties are defined on a concentrated mass element available in most finite element codes. These mass properties can be computed from analytical equations for simple geometries or by solid modeling tools for more complicated shapes. Analytical equations for simple solid geometries can be found in most mechanical design, vibrations, or dynamics textbooks. Since mass properties must be located at the center-of-gravity, an offset from the node on the optical face is required.

Although single-point models are limited in the output they provide, they can be an excellent choice for including the mass of an optic and predicting its component rigid-body motion. In addition, single-point models are very easy to alter, making them excellent tools for early design trades and concept studies.

5.1.3 Models of solid optics

Solid optics are characterized by geometric topology that lacks lightweighting or discrete stiffening. Examples are lenses, solid mirrors, prisms, and windows.

5.1.3.1 Two-dimensional models of solid optics

Some solid optics exhibit mechanical behavior that can be well approximated under the assumptions of plate or shell behavior. In such cases, the elastic stiffness of a 2D, solid optic model is defined by membrane, bending, and transverse-shear stiffnesses. The dimensional parameters on which these stiffnesses depend are the thickness of the optic and the transverse shear factor. For solid optics, the transverse shear factor should be specified as 0.8333.

2D models can provide excellent predictions of global elastic behavior for static and vibration analyses. An important limitation of 2D-element optic models, however, is that they do not predict deformation effects in the direction through the thickness of the optic. Therefore, their rigid-body motions and global elastic deformations are represented by the midplane of the optic and not necessarily that of the optical surface. Differences between the behavior of the midplane of an optic and its optical surface can be caused by mount-induced loads and thermoelastic growth through the thickness of the optic. Furthermore, mount-induced loads will show greater local deformations in 2D-element models than may actually exist at the optical surface of the actual hardware. Therefore, the analyst should choose this method of modeling a solid optic only when it is reasonable to assume that such effects are not significant to the overall goal of the analysis.

Plate-element meshes can also be used to model components where a reasonable representation of stiffness is desired but accurate displacement

predictions are not required. This is often the case when the components being modeled are far enough from the regions of primary interest that accurate representation of their elasticity is not required. A lens to be modeled as part of a lens barrel model is shown in Fig. 5.3(a). However, suppose displacements are not required of the lens shown in the figure.

A model that correctly represents its stiffness may be required to obtain useable displacement results elsewhere in the system. The lens has a relatively constant thickness approximately equal to t_0 , as shown in Fig. 5.3(a), and can be reasonably represented by the plate mesh shown in Fig. 5.3(b). The stiffness of the lens shown in Fig. 5.3(c), however, may not be well represented by a plate mesh due to the inability of such a model to predict potential deformations such as those shown. Such a model may have to be constructed of 3D solid elements, as described in the next section, in order to provide a reasonable approximation of its stiffness.

5.1.3.2 Three-dimensional element models of solid optics

Components whose elastic behavior cannot be accurately represented by plate assumptions require solid-element formulations that use the full 3D representation of Hooke's law. Examples of such components are thick lenses, thick solid mirrors, and prisms. Fig. 5.4 shows some examples of such models.

The construction of solid-element models deserves a few guidelines to be followed in most cases. Solid-element models of lenses and mirrors should have at least four trilinear elements through their thicknesses. Such a minimum resolution is required in most cases to provide a reasonably accurate prediction of the variation in stress states through the thickness of the component. In many cases, more than four elements will be required. The number of elements required is dictated by the variation of displacements through the thickness of the component and the elements' ability to represent them.

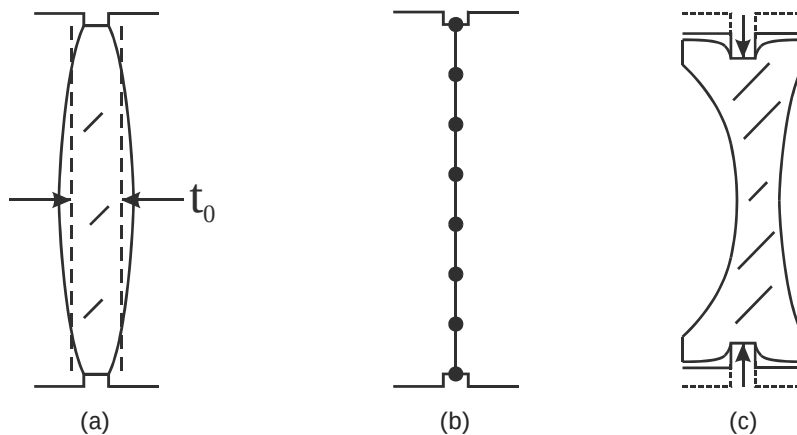


Figure 5.3 Modeling of lenses with 2D models: (a) lens with relatively constant thickness t_0 , (b) corresponding 2D-element mesh, and (c) elastic behavior in a lens that would not be represented by a 2D-element mesh.

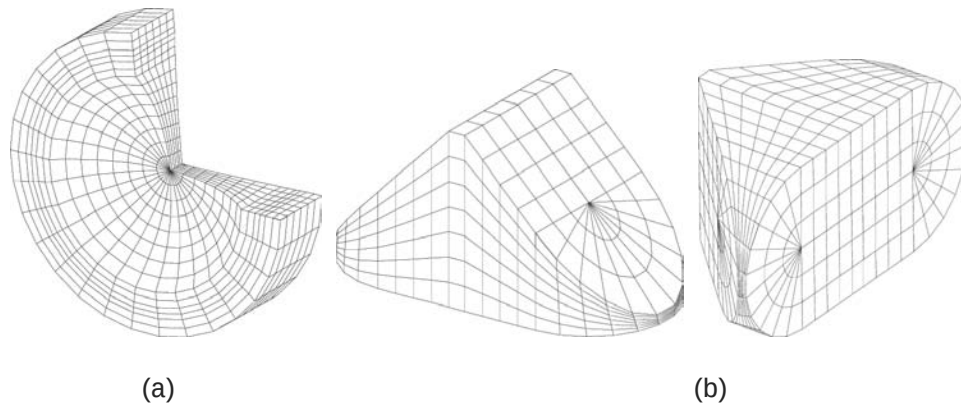


Figure 5.4 Examples of 3D solid models: (a) lens and (b) Porro prism.

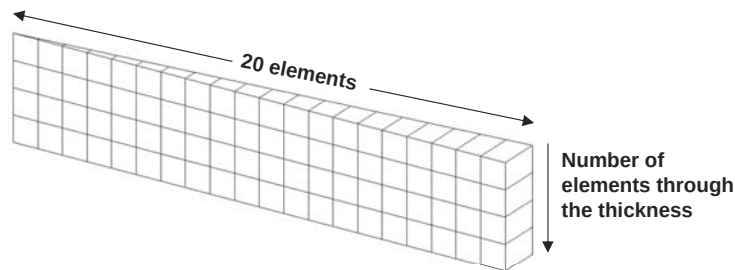


Figure 5.5 Axisymmetric wedge model.

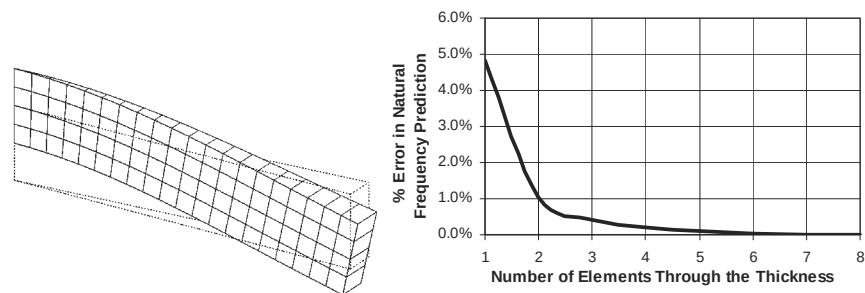


Figure 5.6 Frequency error verses resolution.

An axisymmetric solid mirror with a diameter-to-thickness ratio of 10 is modeled as a 5-degree wedge as shown in Fig. 5.5. In MSC.Nastran, a model of 8-noded hexahedron elements with a constant radial mesh resolution of 20 and a variable through-the-thickness mesh resolution of 1 through 8 elements was used. The plot of the model shown in Fig. 5.5 illustrates four elements through the thickness. In Fig. 5.6, the percent error in the first axisymmetric free-free natural frequency is plotted verses mesh resolution. In Fig. 5.7, the gravity-induced

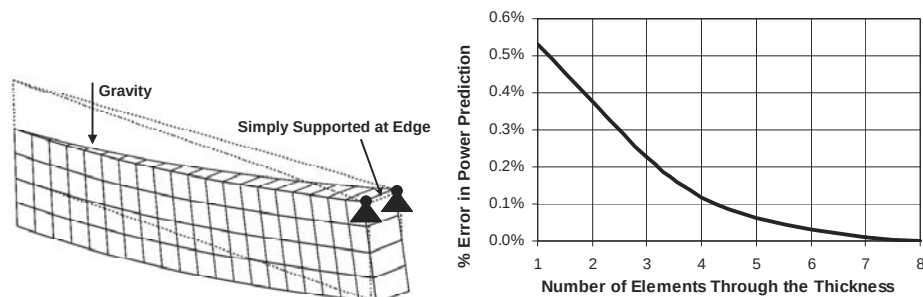


Figure 5.7: Power error verses resolution.

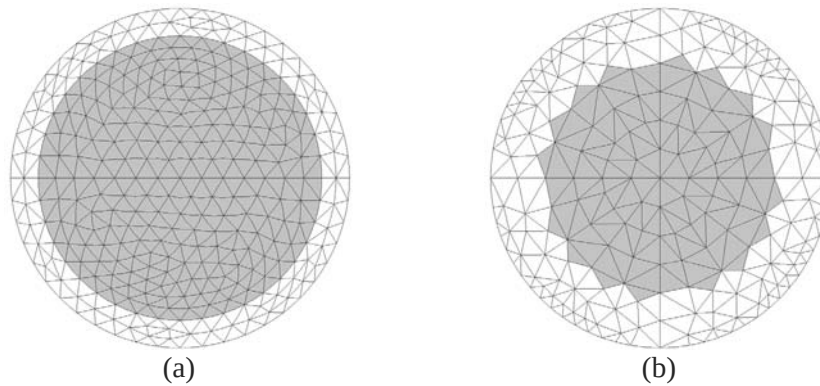


Figure 5.8: (a) Resulting aperture with mesh lines on aperture, and (b) resulting aperture without mesh lines on aperture for a mesh of an optic created by an automeshing technique.

amplitude of the error in the Zernike power term computed with a simply supported edge condition is plotted verses mesh resolution. From the results shown in Figs. 5.6 and 5.7 the use of four or five elements through the thickness gives around 0.1% error in the natural frequency and static displacement results.

The use of automeshing algorithms to generate meshes of highly symmetric optical components, as shown in Fig. 5.4, has shortcomings in practice. Automeshing routines will commonly generate nonsymmetrical meshes for even the most symmetric structures. Such asymmetries in element meshes can generate nonsymmetrical results for problems with symmetric behavior. Automeshing routines, on the other hand, are not without usefulness—they can be useful in situations involving very complicated geometry not meshable by six-sided and five-sided solid elements.

When automeshing any optical model, extra care should be taken to give forethought to any aperturing that may be applied in the processing of the results. If the mesh layout does not contain mesh lines along such aperture or obstruction shapes as shown in Fig. 5.8(a), then chopping as shown in Fig. 5.8(b) will occur if aperturing or obstructing of the finite element results is performed. Chopping will cause misrepresentation of optomechanical behavior and result plots that

appear of questionable validity. Enforcing such mesh lines, however, may be difficult in some software that does not easily allow manipulation of CAD geometry.

The use of the four-noded, constant-strain tetrahedron element should be strictly avoided. The formulation of this element assumes a constant state of strain throughout its volume, resulting in a mesh that is too stiff for useable displacement results. If tetrahedron elements must be used, then ten-noded tetrahedron elements should be employed.

5.1.4 Lightweight mirror models

Modeling of lightweight mirrors is a common application that deserves special attention. Three types of lightweight mirror displacement models are discussed in this section. Each type of model has its own strengths and weaknesses, and the analyst is encouraged to keep the goals of the analysis in mind while choosing which type of model to use.

A lightweight mirror may have one of the various core-cell shapes as shown in the mirrors in Fig. 5.9. These mirrors are examples of open-back lightweight mirror construction. In addition, lightweight mirrors may include a back facesheet that provides increased plate-bending stiffness. Lightweight mirrors are fabricated with varying diameter-to-depth ratios to suit particular applications. All of these constructions can be modeled by the techniques discussed in this section. Of course, nonoptical structures similar in construction to lightweight mirrors may also be modeled by these methods.

5.1.4.1 Two-dimensional equivalent-stiffness models of lightweight mirrors

In a 2D equivalent-stiffness model such as that shown in Fig. 5.10, effective plate properties are assigned to a plate mesh, representing the lightweight optic's construction. A ring of beam elements should also be included around the inner

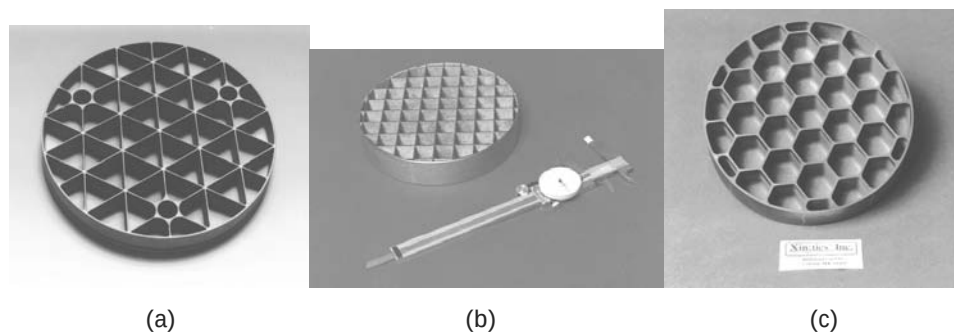


Figure 5.9 Examples of lightweight mirror construction using silicon carbide: (a) triangular core, (b) square core, and (c) hexagonal core (courtesy of AOA Xinetics, Inc., Devens, Massachusetts).

and outer edges of the mirror to represent the edge walls of the core. The properties of these beam elements should be computed with conventional beam-section equations for the inner and outer wall. The grid plane of the 2D-element mesh may be placed at the neutral plane of the optic, or at any other convenient location, with the use of an offset definition. The definition of variables used in the equations for computing the effective properties are defined with Figs. 5.11 and 5.12 as follows:

t_f = front-faceplate thickness,
 t_b = back-faceplate thickness,
 t_c = core-wall thickness,
 h_c = core height,
 ρ = mass density, and
 B = midplane-to-midplane inscribed-circle cell size.

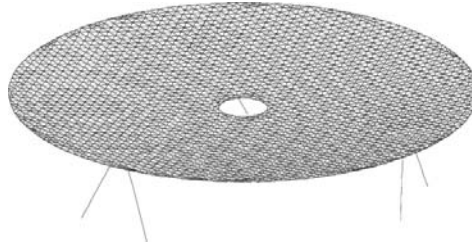


Figure 5.10 2D equivalent-stiffness model of a lightweight mirror.

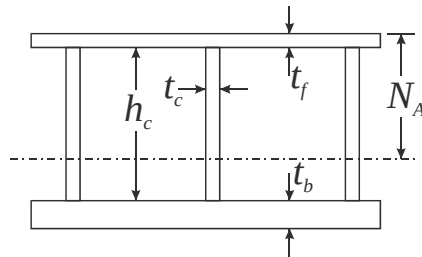


Figure 5.11 Variable definition for 2D effective model equations.

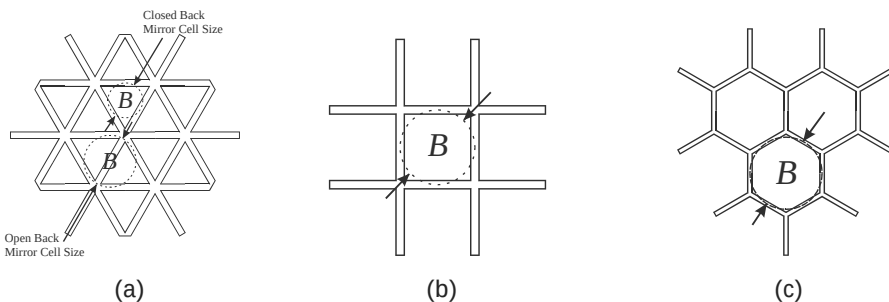


Figure 5.12 Cell-size B definitions for various cell geometries: (a) triangular cells, (b) square cells, and (c) hexagonal cells. Notice the different cell-size definitions for an open-back lightweight mirror vs. a closed-back lightweight mirror.

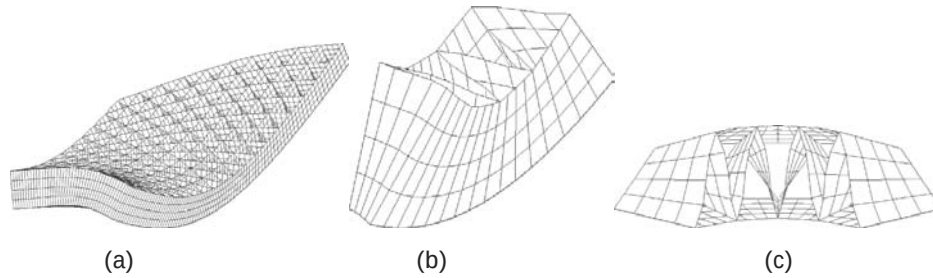


Figure 5.13 The core walls of an open-back triangular core lightweight mirror display twisting deformation when the optic is loaded in bending; (a) an open-back lightweight mirror in bending, (b) isometric view of some of the core cells, and (c) top view of some of the core cells showing twisting of core walls.

Fig. 5.12 illustrates the definition of B for various cell shapes. If the mirror includes only an optical facesheet with an open-back triangular cell core, then the analyst is advised to use the distance between parallel core webs for B instead of the inscribed circle diameter used for closed-back mirrors. The rationale behind this method can be best illustrated by studying the bending deformation of the open-back mirror shown in Fig. 5.13. Notice that the bending stiffness is dominated only by core walls, which are perpendicular to the moment axis. Core walls that are not oriented perpendicular to the axis of an applied plate-bending moment only twist around and do not significantly contribute to the stiffness of the bending section. Thus, the inscribed circle between parallel walls is chosen for open-back mirrors. Note that the only cell geometry that should be used for open-back construction is triangular; this is because any other cell geometry allows bending deformation of the core walls, significantly contributing to the compliance of the mirror.

The solidity ratio α is computed first from

$$\alpha = \frac{t_c}{B}. \quad (5.1)$$

The effective membrane thickness T_m is computed by summing the front- and back-facesheet thicknesses with the core depth scaled by the solidity ratio:

$$T_m = t_f + t_b + \alpha h_c, \quad t_f \neq t_b \quad [5.2(a)]$$

or

$$T_m = 2t + \alpha h_c, \quad t_f = t_b = t. \quad [5.2(b)]$$

With the solidity ratio and other dimensions defined in Fig. 5.11, the distance of the neutral plane from the optical surface can be found from

$$N_A = \frac{1}{T_m} \left[t_f \frac{t_f}{2} + t_b \left(\frac{t_b}{2} + h_c + t_f \right) + \alpha h_c \left(\frac{h_c}{2} + t_f \right) \right], \quad t_f \neq t_b, \quad [5.3(a)]$$

$$N_A = \frac{h_c}{2} + t, \quad t_f = t_b = t. \quad [5.3(b)]$$

The plate-bending moment of inertia is then computed from

$$I_b = \frac{1}{12} t_f^3 + t_f \left(N_A - \frac{t_f}{2} \right)^2 + \frac{1}{12} t_b^3 + t_b \left(N_A - t_f - h_c - \frac{t_b}{2} \right)^2 + \frac{1}{12} \alpha h_c^3 + \alpha h_c \left(N_A - t_f - \frac{h_c}{2} \right)^2, \quad t_f \neq t_b, \quad [5.4(a)]$$

or

$$I_b = \frac{1}{12} \left[(2t + h_c)^3 - (1 - \alpha) h_c^3 \right], \quad t_f = t_b = t. \quad [5.4(b)]$$

Because some finite element codes require the bending moment of inertia be given as a scale factor on the quantity $T_m^3/12$, a bending ratio R_b can be defined as

$$R_b = \frac{12 I_b}{T_m^3}. \quad (5.5)$$

The effective plate shear depth S can be found from

$$S = \frac{12 \alpha I_b}{(t_f + t_b + h_c)^2 - (1 - \alpha) h_c^2}, \quad t_f \neq t_b, \quad [5.6(a)]$$

or

$$S = \frac{12 \alpha I_b}{(2t + h_c)^2 - (1 - \alpha) h_c^2}, \quad t_f = t_b = t. \quad [5.6(b)]$$

As was done for the bending moment of inertia, a shear ratio can be expressed as

$$R_s = k_s \frac{S}{T_m}, \quad (5.7)$$

where k_s is a shear factor. While a common shear factor for rectangular sections is between 0.822 to 0.870, it has been found by the authors that the value of 0.667 given by Timoshenko yields results that are most accurate for lightweight mirror models.^{1,5}

The effective membrane thickness T_m and the mass density ρ will not generate the correct mass representation in the 2D equivalent-stiffness model. Therefore, the model mass can be corrected for closed-back mirror models by adding nonstructural mass (NSM), defined as

$$\text{NSM} = \rho \alpha h_c. \quad (5.8)$$

For open-back triangular core mirrors, the nonstructural mass must be twice the value computed by Eq. (5.8). The stress-recovery points as distances from the neutral plane are defined as

$$\begin{aligned} c_1 &= N_A \\ c_2 &= N_A - t_f - t_b - h_c, \quad t_f \neq t_b, \end{aligned} \quad [5.9(a)]$$

or

$$\begin{aligned} c_1 &= N_A \\ c_2 &= N_A - 2t - h_c, \quad t_f = t_b = t. \end{aligned} \quad [5.9(b)]$$

These equations assume that the element normals are directed from the back of the mirror toward the optical surface.

The 2D model representation does not have the ability to predict quilting deformation of the optical surface. An estimation of the peak-to-valley of quilting can be independently computed by

$$\delta_{\text{Quilting}} = \frac{12\lambda p B^4 (1 - \nu^2)}{E t_f^3}, \quad (5.10)$$

where δ_{Quilting} is the peak-to-valley deformation, p is the applied pressure, and λ is a shape-dependent constant found in Table 5.1.² A surface root-mean-square (RMS) value is found by scaling the peak-to-valley prediction by conversion

Table 5.1 Constants for use with Eq. (5.10).

<u>CELL SHAPE</u>	<u>λ</u>	<u>P-V TO RMS</u>
Triangle	0.00151	0.3087
Square	0.00126	0.2964
Hexagon	0.00111	0.2982

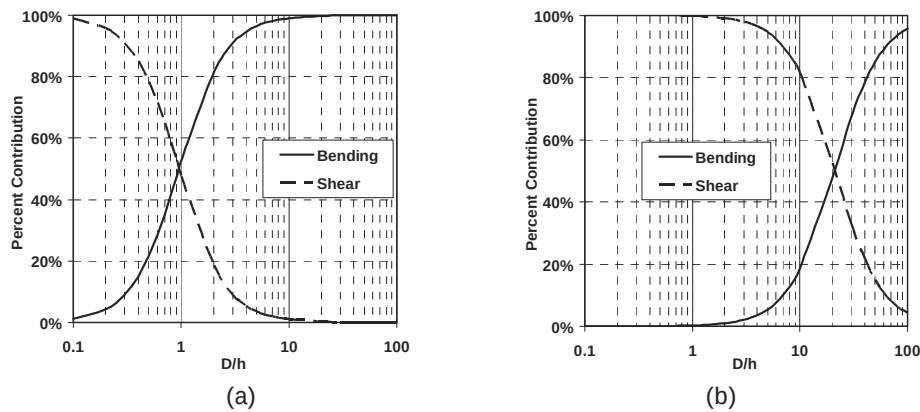


Figure 5.14 Bending- and shear-deformation contributions: (a) solid constant-thickness mirror and (b) lightweight mirror.

factors developed by the authors shown in Table 5.1. This prediction can be combined to the model-predicted surface RMS error by the root-sum-square (RSS) method.

Transverse shear deformations can be a much more important effect on lightweight mirrors than on conventional solid mirrors. Fig. 5.14 shows comparisons of the transverse displacement contributions from bending and transverse-shear compliances as a function of diameter-to-depth ratios (D/h) for a simply supported constant thickness mirror and a simply supported lightweight mirror with uniform pressures applied. Notice that since the fractional contribution of transverse-shear deformation does not become insignificant compared to the bending deformation in a lightweight mirror until diameter-to-depth ratios approach 100, it is extremely important to include an appropriate effective shear factor in order to develop an accurate representation of the mirror compliance.

The limitations of this 2D lightweight mirror model are very similar to those discussed in Section 5.1.3.1 for 2D models of solid optics. In general, the global deformations of this type of model are reasonable for static and dynamic analyses. Most local effects such as mount dimpling are not well represented, and others such as quilting are not represented at all. In addition, because the stiffness through the depths of lightweight mirrors can be small and their depths can be high compared to solid mirrors, the assumption that the through-the-thickness deformations are negligible may not be applicable for more strict analysis goals. For example, the axial optical surface rigid-body motion of a deep lightweight mirror subject to axial inertial loads may be very dependent on how much local deformation develops around the back surface mount points as shown in Fig. 5.15. A 2D effective model lacks the ability to include these effects.

A unique advantage of the 2D equivalent-stiffness model is that it is easily implemented in a design optimization study. All of the effective property

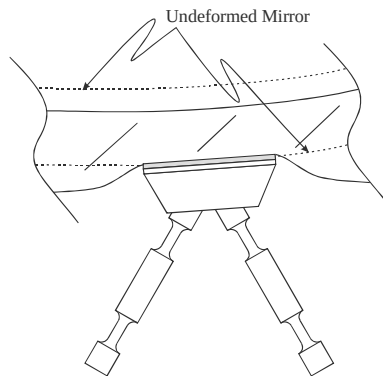


Figure 5.15 Highly exaggerated local deformation due to loads at the mounts.

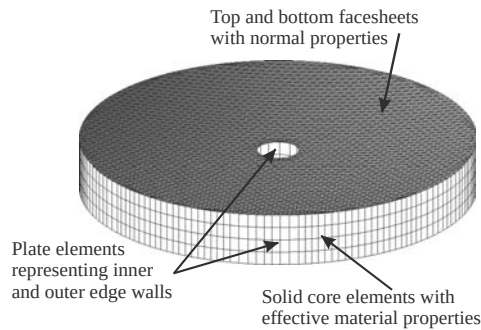


Figure 5.16 3D equivalent-stiffness model of a lightweight mirror.

equations and the quilting estimate shown above may be included in a property-sizing design optimization run to assist in the development of a lightweight mirror design to meet optical performance, weight, and other requirements. Although the predictive accuracy of this model is not as favorable as the model types discussed below, it is the most superior model type for the purpose of quickly developing an optimum mirror design to be used in subsequent more detailed verification analyses.

5.1.4.2 Three-dimensional equivalent-stiffness models

The 3D equivalent-stiffness model of a lightweight mirror, shown in Fig. 5.16, has predictive accuracy capabilities superior to the 2D equivalent-stiffness model, but it is slightly more complex. The front and back faceplates are represented by a mesh of plate elements that reside at the appropriate midsurfaces. They each reference unmodified material properties and the thickness of the faceplates. The lightweighted core, however, is represented by solid elements that share the nodes of the faceplate meshes and reference effective, transversely orthotropic material properties calculated from equations. In addition, as shown in Fig. 5.16, the 3D equivalent-stiffness model should

include a representation of the core edge wall with shell elements at the inner and outer mesh faces of the solid elements that represent the core.

The equations for computing the effective core properties, developed by the authors, are given below in two forms with Eqs. (5.11) and (5.12). The first set of equations gives the engineering constants while the second set gives the elastic Hooke's law matrix, which relates the stresses to the strains. Both definitions are given to accommodate the requirements of different finite element codes.

$$\begin{aligned}
 E_x^* &= E_y^* = \alpha E, & E_z^* &= 2\alpha E, \\
 \nu_{zx}^* &= \nu_{zy}^* = \nu, & \nu_{xy}^* &= \nu_{yx}^* = 0, \\
 \nu_{xz}^* &= \nu_{yz}^* = \frac{\nu}{2}, & G_{xz}^* &= G_{yz}^* = \alpha G, \\
 G_{xy}^* &= 0, & \rho^* &= \frac{2\alpha\rho h_c}{\frac{t_f}{2} + \frac{t_b}{2} + h_c}.
 \end{aligned} \tag{5.11}$$

where E is the Young's modulus, G is the shear modulus, ν is the Poisson's ratio, ν_{ij} equals $-\varepsilon_j/\varepsilon_i$ due to a uniaxial stress applied in the i direction, ρ is the mass density, and $*$ indicates an effective material property. Notice that the effective core density ρ^* includes a correction factor to account for the overlap in core mesh with half of each facesheet thickness.

When Eq. (5.11) is substituted into the orthotropic form of Hooke's law as found in Jones,³ the following matrix relation results:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} = \begin{bmatrix} \left(1 - \frac{\nu^2}{2}\right) \frac{\alpha E}{(1 - \nu^2)} & \frac{\nu^2 \alpha E}{2(1 - \nu^2)} & \frac{\nu \alpha E}{(1 - \nu^2)} & 0 & 0 & 0 \\ \frac{\nu^2 \alpha E}{2(1 - \nu^2)} & \left(1 - \frac{\nu^2}{2}\right) \frac{\alpha E}{(1 - \nu^2)} & \frac{\nu \alpha E}{(1 - \nu^2)} & 0 & 0 & 0 \\ \frac{\nu \alpha E}{(1 - \nu^2)} & \frac{\nu \alpha E}{(1 - \nu^2)} & \frac{2\alpha E}{(1 - \nu^2)} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\alpha E}{2(1 + \nu)} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\alpha E}{2(1 + \nu)} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix}. \tag{5.12}$$

Notice the ordering of the elements of the stress and strain vectors in Eq. (5.12): the order of these elements varies throughout the literature and in the definition of Hooke's Law matrix specifications in finite element software.

Since the effective material properties of the core are dependent on direction, it is important for the analyst to make sure that the material coordinate system of the solid-element mesh is correctly defined so that the material description will be properly oriented. Since the x and y directions are identical in the above formulations, either a cylindrical or rectangular material coordinate system may be employed as long as the z direction is defined parallel to the direction defined by the intersection of the core walls.

The 3D equivalent-stiffness model predicts some deformation behaviors not represented in the 2D equivalent-stiffness model, but it still displays some shortcomings in predictive accuracy. Global-elastic behavior through the thickness of the mirror is well represented. Deformation effects such as thermoelastic growth through the thickness and elastic isolation of the optical surface from the mount points are represented quite well. However, highly localized effects at the mount points are not fully represented. Therefore, while the optical surface rigid-body motion is better predicted with a 3D equivalent-stiffness model compared to the 2D equivalent-stiffness model, some inaccuracies are, nevertheless, to be expected. In addition, quilting deformation is not represented at all. Eq. (5.10) can be employed to estimate quilting effects as was suggested for the 2D equivalent-stiffness model.

The 3D equivalent-stiffness model has many of the same benefits of simplicity as the 2D equivalent-stiffness model, but it has increased predictive capability. Its use in design optimization, however, requires features that allow the analyst to define material properties as design variables. In addition, the consideration of mirror depth as a design variable requires a shape optimization feature. Employment of such capabilities makes the 3D equivalent-stiffness model an excellent choice for preliminary design trade studies where through-the-thickness effects may be very important.

5.1.4.3 Three-dimensional plate/shell model

The 3D plate/shell model has the most superior deformation prediction capabilities, but it is the most complicated and time-consuming model type to construct. It is also the most difficult model type to alter, often making it a poor choice for early design-trade studies. The model is composed entirely of plate elements located at the midsurfaces of each facesheet and core-wall segment. An example model is shown in Fig. 5.17. Effective properties can be given to the facesheets using the 2D equivalent-stiffness method if the facesheets contain their own cathedral-rib stiffening. Cathedral ribs are illustrated in Fig. 5.18.

Due to the geometry of most lightweight mirror cores, it is unlikely that the analyst will be able to mesh the mirror faceplates of quadrilateral elements without some degree of warping. Since warping is an extremely detrimental distortion for four-noded quadrilateral elements, it is advised that three-noded triangular or quadratic elements be employed. Some finite element codes have

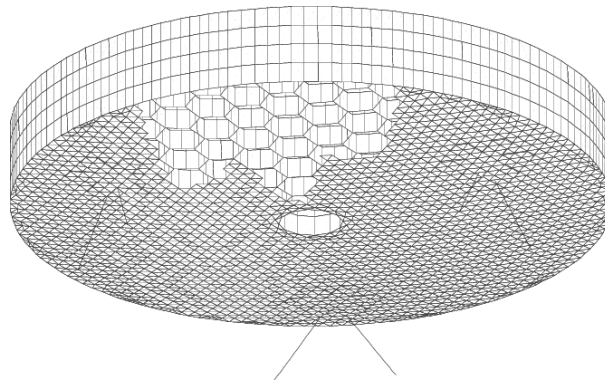


Figure 5.17 3D plate/shell model of a lightweight mirror.

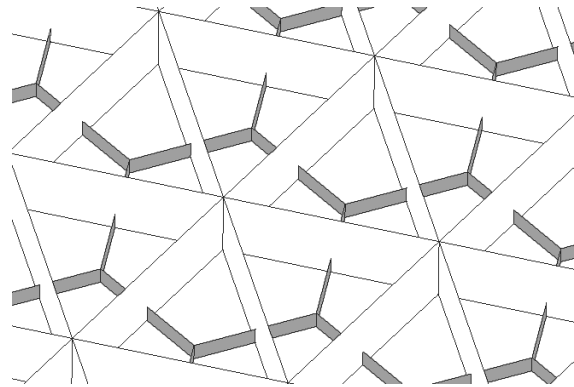


Figure 5.18 Cathedral ribs (shaded) in the design of an open-back lightweight mirror.

developed three-noded elements with formulations superior to the constant strain formulation, but such elements are not always fully featured. The three-noded constant-strain formulation elements give adequate results if a sufficient number of elements are used.

As was done in the 3D equivalent-stiffness model, the mass density of the core elements can be adjusted to account for the overlap of the core elements with half of each facesheet thickness. This adjustment is performed by scaling the true mass density by $h_c / (t_f / 2 + t_b / 2 + h_c)$.

5.1.4.4 Example: gravity deformation prediction comparison of a lightweight mirror

Predictions of natural frequencies, weight, and static deformation due to a gravity load from each of the three model types discussed above are to be compared for a lightweight mirror design fabricated of ULE. The mirror has an outer diameter of 71.12 cm and an inner diameter of 7.62 cm. The core depth is 5.00 cm, and the cells are hexagonal in shape with a midplane-to-midplane inscribed-circle diameter of 5.00 cm. The facesheets are 4.6-mm thick, and the core-wall

thickness is 1.5 mm. The mirror is mounted on three sets of bipod flexures that are bonded to the back surface of the optic.

5.1.4.4.1 Two-dimensional effective property calculations

By Eq. (5.1), we compute the solidity ratio:

$$\alpha = \frac{t_c}{B} = \frac{1.5 \text{ mm}}{50.0 \text{ mm}} = 0.03. \quad (5.13)$$

The effective-membrane thickness T_m is computed by Eq. [5.2(b)]:

$$\begin{aligned} T_m &= 2t + \alpha h_c, \\ T_m &= 2(4.6 \text{ mm}) + (0.03)(50. \text{ mm}) = 10.7 \text{ mm}. \end{aligned} \quad (5.14)$$

Find the location of the neutral plane, N_A , using Eq. [5.3(b)]:

$$\begin{aligned} N_A &= \frac{h_c}{2} + t, \\ N_A &= \frac{50.0 \text{ mm}}{2} + 4.6 \text{ mm} = 29.6 \text{ mm}. \end{aligned} \quad (5.15)$$

The plate-bending moment of inertia is computed with Eq. [5.4(b)]:

$$\begin{aligned} I_b &= \frac{1}{12} \left[(2t + h_c)^3 - (1 - \alpha) h_c^3 \right] \\ I_b &= \frac{1}{12} \left\{ [2(4.6 \text{ mm}) + (50.0 \text{ mm})]^3 - [1 - (0.03)](50.0 \text{ mm})^3 \right\} \\ &= \frac{1}{12} [207,474.688 \text{ mm}^3 - 121,250.0 \text{ mm}^3] \\ &= 7185.39 \text{ mm}^3. \end{aligned} \quad (5.16)$$

The bending ratio I_b is computed by Eq. (5.5):

$$R_b = \frac{12I_b}{T_m^3},$$

$$R_b = \frac{12(7185.39 \text{ mm}^3)}{(10.7 \text{ mm})^3} = 70.385. \quad (5.17)$$

The effective shear depth S is computed from Eq. [5.6(b)]:

$$S = \frac{12\alpha I_b}{(2t + h_c)^2 - (1 - \alpha)h_c^2},$$

$$S = \frac{12(0.03)(7185.39 \text{ mm}^3)}{[2(4.6 \text{ mm}) + 50.0 \text{ mm}]^2 - (1 - 0.03)(50.0 \text{ mm})^2} = 2.400 \text{ mm}. \quad (5.18)$$

By Eq. (5.7), the shear-factor ratio is then

$$R_s = \frac{2}{3} \frac{S}{T_m} = \frac{2}{3} \frac{(2.400 \text{ mm})}{(10.7 \text{ mm})} = 0.150. \quad (5.19)$$

Eq. (5.8) is used to compute the nonstructural mass that corrects the model mass with a material density of 2.187 g/cm^3 as

$$\text{NSM} = \rho \alpha h_c = (0.002187 \text{ g/mm}^3)(0.03)(50.0 \text{ mm}) = 0.00328 \text{ g/mm}^2. \quad (5.20)$$

The 2D equivalent-stiffness model is shown in Fig. 5.10.

5.1.4.4.2 Three-dimensional effective property calculations

Because the facesheets are modeled with their true thicknesses, effective properties for the 3D equivalent-stiffness model are computed only for the core. As shown in Eq. (5.12), these properties are in the form of a Hooke's law matrix whose elements are G_{ij} for the i^{th} row and the j^{th} column. The computations for the nonzero values of the Hooke's law matrix are shown as

$$G_{11} = G_{22} = \left(1 - \frac{\nu^2}{2}\right) \frac{\alpha E}{1 - \nu^2} = \left[1 - \frac{(0.17)^2}{2}\right] \frac{(0.03)(6.757 \times 10^{10} \mu\text{N/mm}^2)}{1 - (0.17)^2}$$

$$= 2.057 \times 10^9 \mu\text{N/mm}^2,$$

$$\begin{aligned}
G_{12} = G_{21} &= \left(\frac{v^2}{2} \right) \frac{\alpha E}{1 - v^2} = \left[\frac{(0.17)^2}{2} \right] \frac{(0.03)(6.757 \times 10^{10} \mu\text{N} / \text{mm}^2)}{1 - (0.17)^2} \\
&= 3.016 \times 10^7 \mu\text{N} / \text{mm}^2, \\
G_{13} = G_{23} = G_{31} = G_{32} &= v \frac{\alpha E}{1 - v^2} = (0.17) \frac{(0.03)(6.757 \times 10^{10} \mu\text{N} / \text{mm}^2)}{1 - (0.17)^2} \\
&= 3.549 \times 10^8 \mu\text{N} / \text{mm}^2, \\
G_{33} &= 2 \frac{\alpha E}{1 - v^2} = 2 \frac{(0.03)(6.757 \times 10^{10} \mu\text{N} / \text{mm}^2)}{1 - (0.17)^2} = 4.175 \times 10^9 \mu\text{N} / \text{mm}^2, \\
G_{55} = G_{66} &= \frac{\alpha E}{2(1 + v)} = \frac{(0.03)(6.757 \times 10^{10} \mu\text{N} / \text{mm}^2)}{2(1 + 0.17)} = 8.663 \times 10^8 \mu\text{N} / \text{mm}^2.
\end{aligned} \tag{5.21}$$

The effective core density is given by Eq. (5.11):

$$\begin{aligned}
\rho^* &= \frac{2\alpha\rho h_c}{\frac{t_f}{2} + \frac{t_b}{2} + h_c} \\
&= \frac{2(0.03)(0.002187 \text{ g} / \text{mm}^3)(50.0 \text{ mm})}{\frac{4.6 \text{ mm}}{2} + \frac{4.6 \text{ mm}}{2} + 50.0 \text{ mm}} = 1.202 \times 10^{-4} \text{ g} / \text{mm}^3.
\end{aligned} \tag{5.22}$$

The 3D equivalent-stiffness model is shown in Fig. 5.16.

5.1.4.4.3 Three-dimensional plate/shell model effective property calculations

The only effective property to compute for the 3D plate/shell model is the core density. Since the mesh of the core extends through half of the dimension of the faceplates, the nominal density is scaled as follows:

$$\begin{aligned}
\rho^* &= \rho \frac{h_c}{\left(\frac{t_f}{2} + \frac{t_b}{2} + h_c \right)} \\
&= (0.002187 \text{ g} / \text{mm}^3) \frac{50.0 \text{ mm}}{\frac{4.6 \text{ mm}}{2} + \frac{4.6 \text{ mm}}{2} + 50.0 \text{ mm}} = 0.002003 \text{ g} / \text{mm}^3.
\end{aligned} \tag{5.23}$$

The 3D plate/shell model is shown in Fig. 5.17.

5.1.4.4.4 Comparison of results

A comparison of the deformation results due to gravity is shown in Table 5.2. The deformation results have been formatted in nonzero Zernike polynomial coefficients with units of nanometers. Plots of the deformations for each model are shown in Fig. 5.19.

Notice that all three models yield similar displacement predictions, but the differences in the predictions and the plotted deformations illustrate the limitations of each. The inability of the 2D equivalent stiffness model to represent through the thickness deformation is illustrated by a greater trefoil prediction and by a lower bias prediction as compared to the 3D equivalent stiffness and 3D plate/shell models. The global deformation predictions are very similar, however, as evidenced by comparable residual RMS predictions after bias is removed.

The weight and natural frequency predictions are shown in Table 5.3.

Table 5.2 Gravity deformation results.

	2D EQUIVALENT STIFFNESS MODEL			3D EQUIVALENT STIFFNESS MODEL			3D PLATE/SHELL MODEL		
	MAG. (NM)	RESI- DUAL RMS (NM)	RESIDUAL P-V (NM)	MAG. (NM)	RESIDUAL RMS (NM)	RESIDUAL P-V (NM)	MAG. (NM)	RESIDUAL RMS (NM)	RESI- DUAL P-V (NM)
Input Surface		933.7	432.5		942.3	414.4		945.4	422.0
Bias	-927.8	112.0	432.5	-937.0	108.6	414.4	-939.8	110.8	422.0
Power (Defocus)	37.3	110.1	458.4	34.4	107.0	440.8	36.8	109.0	450.2
Pri Trefoil	286.9	41.7	219.3	278.4	40.7	218.4	283.1	41.9	227.1
Pri Spherical	-44.4	37.0	161.5	-42.9	36.2	160.2	-44.0	37.2	166.1
Sec Trefoil	111.5	17.6	99.1	110.8	16.3	94.3	113.8	16.9	97.4
Sec Spherical	8.2	17.2	87.3	10.3	15.8	79.5	10.7	16.4	82.1
Pri Hexafoil	38.9	13.9	90.3	34.9	12.9	88.3	36.0	13.4	91.8
Ter Trefoil	22.6	12.6	60.7	25.2	11.1	57.2	25.8	11.6	60.6
Ter Spherical	8.8	12.3	67.1	7.0	10.9	61.5	7.5	11.3	65.3
Sec Hexafoil	38.7	7.8	44.0	35.0	6.7	38.9	36.3	7.0	41.3

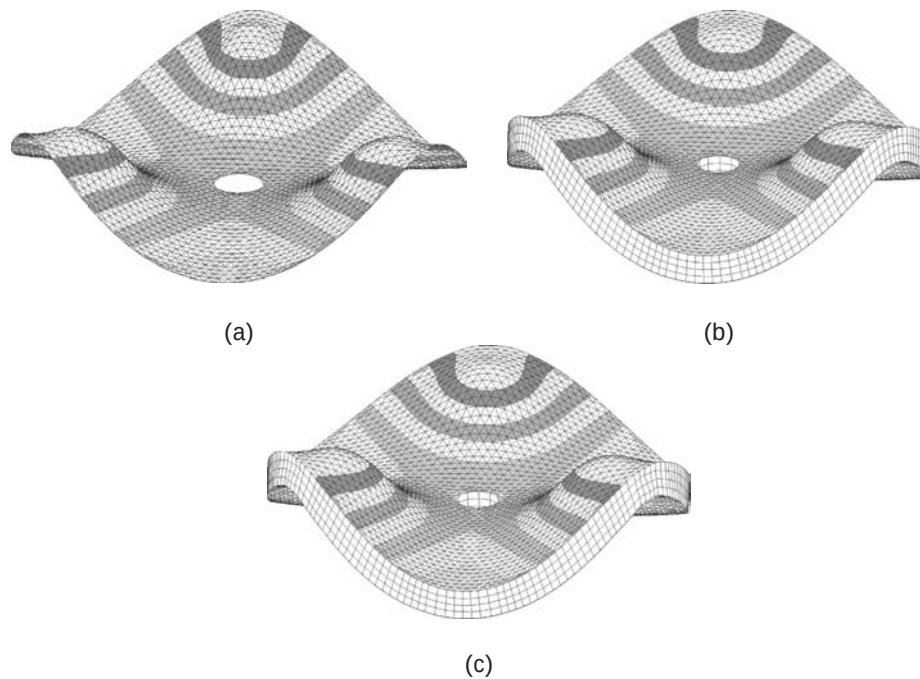


Figure 5.19 Highly exaggerated deformed plots of mounted, lightweight mirror models loaded by gravity: (a) 2D equivalent-stiffness model, (b) 3D equivalent-stiffness model, and (c) 3D plate/shell model.

Table 5.3 Weight and natural frequency predictions.

	2D <u>EFFECTIVE</u>	3D <u>EFFECTIVE</u>	3D <u>PLATE</u>
Weight	10.94 kg	10.96 kg	10.86 kg
Unmounted			
Natural frequency	813 Hz	812 Hz	809 Hz
Mounted			
Natural frequency	129 Hz	131 Hz	131 Hz

5.1.4.5 Example: Lightweight mirror with significant quilting

This example involves a lightweight mirror with a thin faceplate that exhibits significant gravity-induced quilting. The purpose of this example is to show the RSS combination of two uncorrelated surface errors: global surface deformation and quilting surface deformation. The mirror design geometry is as follows:

Material = Fused silica
 Outside diameter = 1 m
 Overall height = 0.1025 m
 Faceplate thickness = 0.0025 m

Core thickness = 0.0015 m
 Cell spacing = 0.10 m
 Radius of curvature = 3.0 m
 Mount radius = 0.35 m

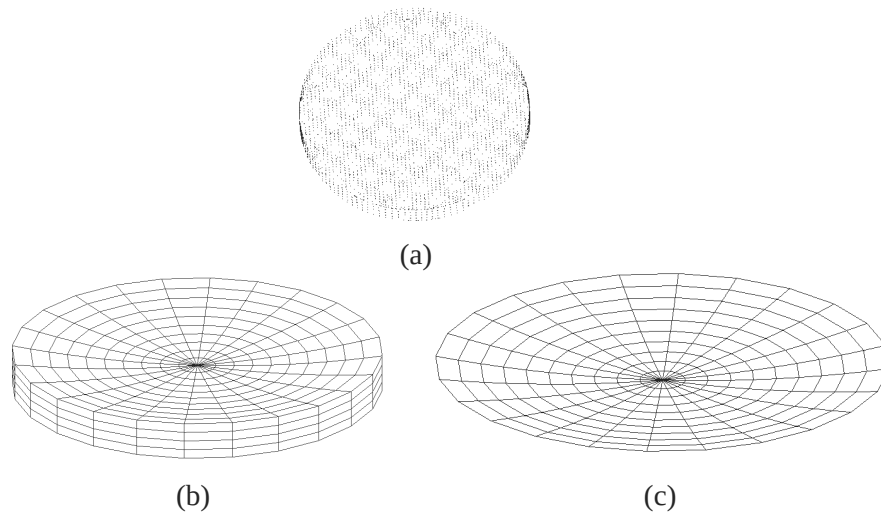


Figure 5.20 (a) 3D plate/shell model shown with faceplate mesh removed, (b) 3D equivalent stiffness model, and (c) 2D equivalent stiffness model.

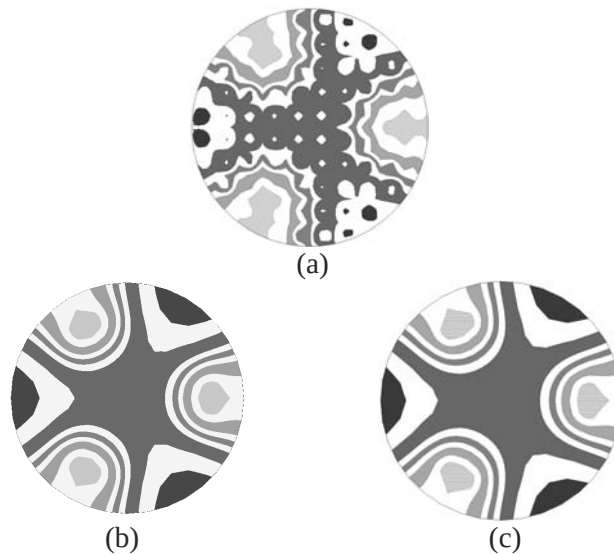


Figure 5.21 Gravity-induced residual surface deformation after best-fit plane removed: (a) 3D plate/shell model, (b) 3D equivalent stiffness, and (c) 2D equivalent stiffness.

Plots of three finite-element models of the mirror are shown in Fig. 5.20: (a) shows a 3D plate/shell model, whereas (b) and (c) show the 3D effective and 2D effective models, respectively, of the same mirror.

The mirror is supported by a three-point kinematic mount at the seven-tenths radial location with gravity acting along the optical axis. Fig. 5.21 shows contour plots of residual surface error after best-fit plane has been removed for each of the three model types.

Table 5.4 Zernike fit to lightweight mirror models.

N	M	Aberration	3D PLATE/SHELL MODEL		3D EQUIVALENT STIFFNESS MODEL		2D EQUIVALENT STIFFNESS MODEL	
			Magnitude (um)	Residual RMS (um)	Magnitude (um)	Residual RMS (um)	Magnitude (um)	Residual RMS (um)
-	-	After BFP	-	0.1470	-	0.1447	-	0.1486
2	0	Power (Defocus)	0.039	0.1452	0.033	0.1434	0.038	0.1470
3	3	Pri Trefoil	0.370	0.0635	0.381	0.0496	0.388	0.0543
4	0	Pri Spherical	-0.058	0.0582	-0.059	0.0422	-0.065	0.0463
5	3	Sec Trefoil	0.161	0.0345	0.129	0.0172	0.139	0.0196
6	0	Sec Spherical	0.025	0.0329	0.013	0.0155	0.011	0.0184
6	6	Pri Hexafoil	0.044	0.0309	0.030	0.0139	0.036	0.0166
7	3	Ter Trefoil	0.039	0.0292	0.031	0.0100	0.030	0.0138
8	0	Ter Spherical	0.018	0.0285	0.009	0.0092	0.013	0.0125
8	6	Sec Hexafoil	0.048	0.0259	0.022	0.0065	0.028	0.0094

**Figure 5.22** Residual surface deformation after all Zernike terms through hexafoil are subtracted: (a) 3D plate/shell model and (b) 3D effective model.

Zernike polynomial fits to the surface deformations shown in Fig. 5.21 are given in Table 5.4. The results show fair agreement between all three models. However, a principal difference in the results is the quilting, which is predicted by the 3D plate/shell model but not by the equivalent stiffness models. The residual surface error after all Zernikes have been removed, shown in Fig. 5.22(a), is principally cell quilting with some additional local mount effect. The quilting portion is highly uncorrelated with the global surface deformation predicted by the equivalent stiffness models, and, therefore, can be combined with the surface RMS error predictions of the equivalent models by the RSS method. Using the equation for the surface RMS due to quilting,

$$\delta_{\text{QuiltRMS}} = 0.3 \frac{12C_s p B^4 (1 - \nu^2)}{E t_f^3} = 0.021 \mu\text{m}. \quad (5.24)$$

The quilting RMS can be added to the 3D equivalent stiffness RMS by the RSS technique and then compared to the prediction from the 3D plate/shell model, which includes the global surface deformation and quilting deformation. For RMS after best-fit plane:

$$\text{Combined Surface RMS} = \sqrt{0.1447^2 + 0.021^2} = 0.1462, \quad (5.25)$$

which is 0.5% below the prediction of 0.1470 from the full shell model. For the residual surface error after subtraction of all Zernikes through secondary hexafoil,

$$\text{Combined Surface RMS} = \sqrt{0.0065^2 + 0.021^2} = 0.0222, \quad (5.26)$$

which is 14% below the prediction of 0.0259 full shell model. Fig. 5.18 shows for both the 3D plate/shell model and the 3D equivalent stiffness model that the residual deformations after all Zernikes have been subtracted. Notice that the residual deformation of the 3D plate/shell model shown in Fig. 5.18(a) contains some asymmetric mount effect due to the interaction of the rectangular core pattern with the three-fold mount configuration. This behavior is not predicted by the 3D equivalent stiffness model because it lacks the representation of the individual core cells. This difference in predictive ability is the reason for the 14% difference in prediction for the residual surface error after subtraction of the Zernike terms through hexafoil.

This example shows that equivalent-stiffness models can be effective tools for early design concepts to easily perform design trade studies with many mirror design parameters. Once a design has been chosen, the full 3D plate/shell model should be created for more accurate performance predictions.

If the surface deformation data of the full shell model were characterized by a Zernike polynomial fit for import to an optical code, this representation alone would lack the quilting and mount-induced deformations, which are impossible to be accurately represented by the finite sets of Zernike polynomials used by commercially available optical analysis tools. However, if the Zernike polynomial representation were expressed as surface interferogram files, then the residual surface after subtraction of the Zernike polynomial fit could be interpolated via finite element shape functions to a second surface interferogram file in the format of a rectangular array. In some optical codes, multiple surface-interferogram files may be applied to the same surface, allowing both

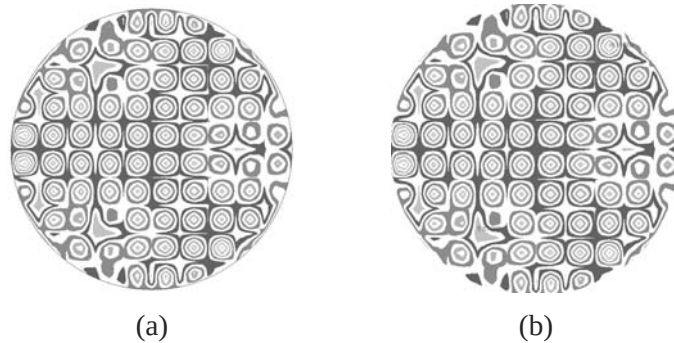


Figure 5.19 Comparison between (a) original FE model and (b) interpolated array of residual surface deformation after subtraction of best-fit Zernike representation.

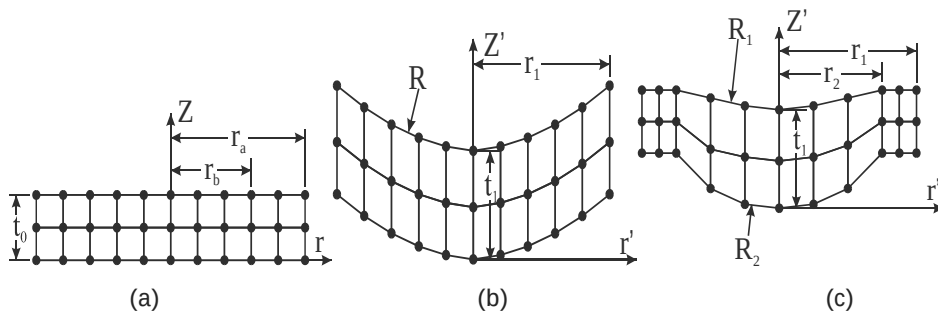


Figure 5.20 Definition of variables for powered-optic-model-generation equations: (a) initial model shape, (b) final mirror-model shape, and (c) final lens-model shape.

deformation representations to be applied in the same optical analysis. Alternatively, a single interpolated rectangular array may be used for codes that accept only one surface deformation description per surface.

Fig. 5.19 compares the residual surface error after subtraction of all Zernike terms through hexafoil for the original FEA data and the interpolated array data. The finite element results in Fig. 5.19(a) are well represented by the 401×401 grid array using shape function interpolation shown in Fig. 5.19(b).

5.1.5 Generation of powered optic models

5.1.5.1 On-axis slumping

It is often easiest to construct a finite element model of a lens or mirror as a flat optic, and then use a program or spreadsheet to modify the node coordinate values to obtain the final shape. Eqs. (5.13) and (5.14) are two example transformation relations that give the new coordinate values in terms of the flat optic coordinate values. The equations are expressed in polar coordinate variables and the variable definitions are given in Fig. 5.20. These equations may be employed in a variety of ways including specialized features within finite element preprocessors, formula features within spreadsheet tools, and implementations in user-developed software.

Fig. 5.20(a) represents a flat model of a lens or mirror that is to be transformed into either the mirror model shown in Fig. 5.20(b) or the lens model shown in Fig. 5.20(c). The transformation from Fig. 5.20(a) to Fig. 5.20(b) is as follows:

$$\begin{aligned} r' &= \frac{r_1}{r_a} r \\ Z' &= Z \frac{t_1}{t_0} + R - \frac{|R|}{R} \sqrt{R^2 - r'^2}, \end{aligned} \quad (5.27)$$

whereas the transformation from Fig. 5.20(a) to Fig. 5.20(c) is as follows:

$$\begin{aligned} r' &= \begin{cases} \frac{r_2}{r_b} r & 0 \leq r \leq r_b \\ r_2 + (r_1 - r_2) \frac{(r - r_b)}{(r_a - r_b)} & r_b \leq r \leq r_a \end{cases} \\ Z' &= \begin{cases} Z \frac{t_1}{t_0} + \left(R_2 - \frac{|R_2|}{R_2} \sqrt{R_2^2 - r'^2} \right) \left(\frac{t_0 - Z}{t_0} \right) + \left(R_1 - \frac{|R_1|}{R_1} \sqrt{R_1^2 - r'^2} \right) \left(\frac{Z}{t_0} \right) & 0 \leq r' \leq r_2 \\ \left(R_2 - \frac{|R_2|}{R_2} \sqrt{R_2^2 - r_2^2} \right) \left(\frac{t_0 - Z}{t_0} \right) + \left(t_1 + R_1 - \frac{|R_1|}{R_1} \sqrt{R_1^2 - r_2^2} \right) \left(\frac{Z}{t_0} \right) & r_2 \leq r' \leq r_1. \end{cases} \end{aligned} \quad (5.28)$$

Notice that the above methods can be applied to any of the optic displacement models discussed in this section.

5.1.5.2 Off-axis slumping

The mathematics of generating off-axis segments of an aspheric primary mirror are quite complicated. The complexity arises in that the calculation of surface sag expressed in the local segment-coordinate system involves coordinate transformations and a root-finding procedure as discussed below. Consider a segmented mirror with three rings of segments, as shown in Fig. 5.21. The segments within each ring have the same geometry but vary from ring to ring, as indicated by the alphabetic labels.

The three unique segments labeled A, B, and C are shown in Figs. 5.22 and 5.23 with local coordinate systems whose z axes are locally normal to the part centers.

The aspheric sag measured parallel to the parent optical axis is shown in Fig. 5.24 with a shaded contour plot over all three segments.

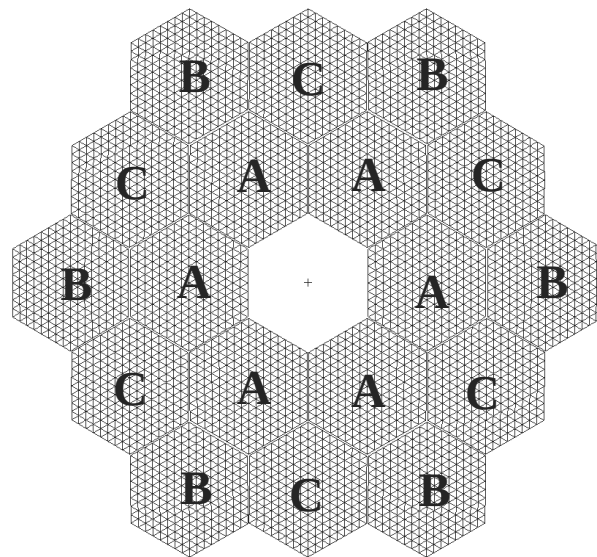


Figure 5.21 Segmented primary mirror.

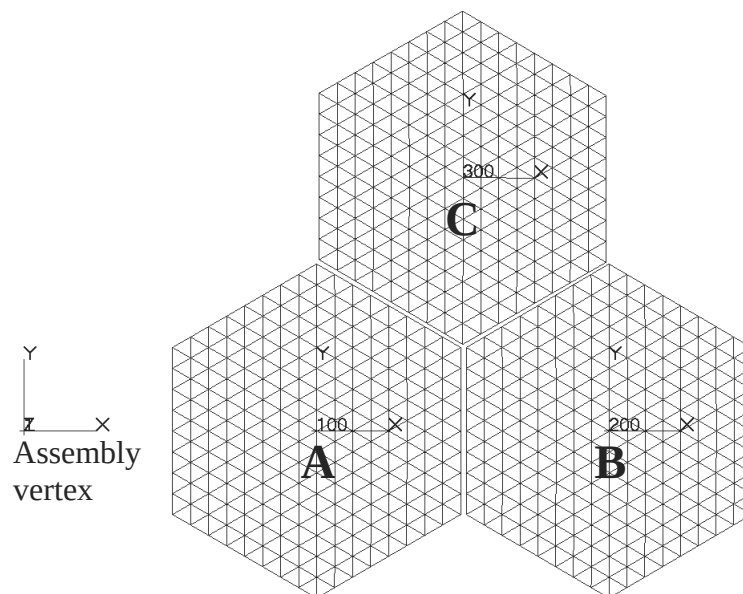


Figure 5.22 Top view of the primary mirror's three unique segments.

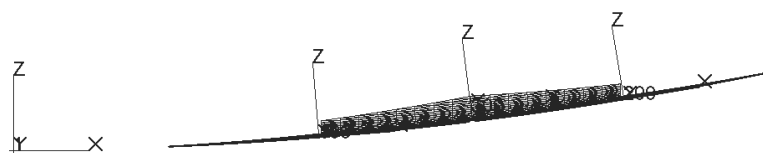


Figure 5.23 Side view of the primary mirror's three unique segments.

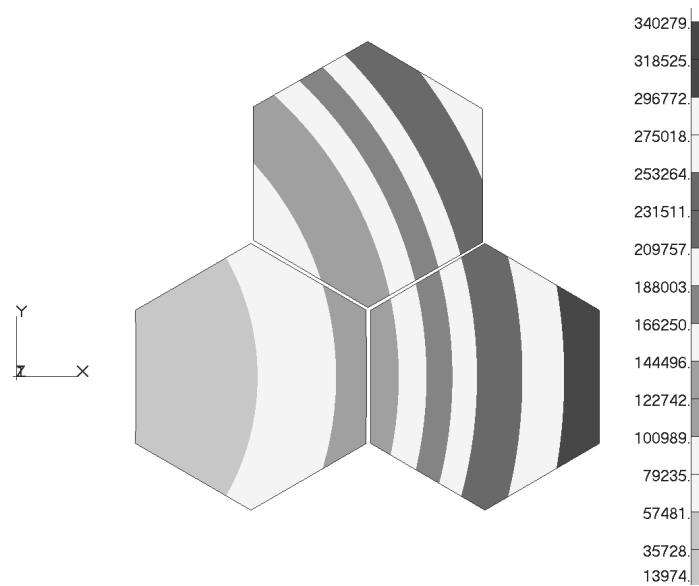


Figure 5.24 Global surface sag in parent coordinate system.

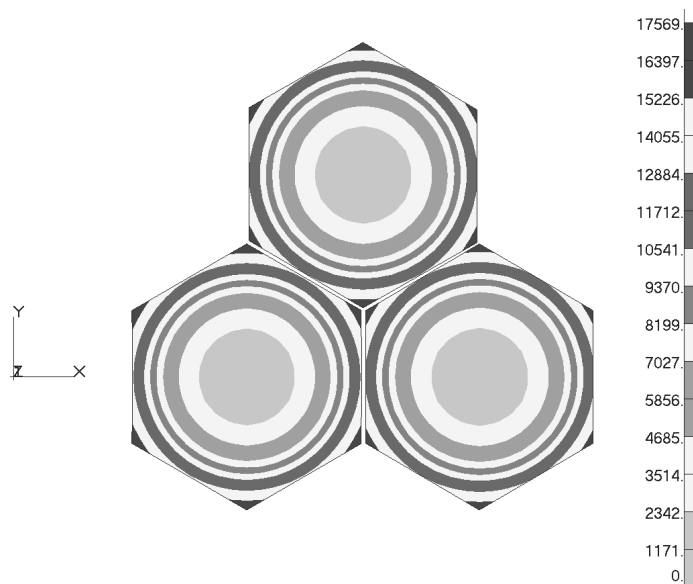


Figure 5.25 Local surface sag in segment system.

The local segment sag measured from a plane tangent to the parent asphere at each segment's center is shown in Fig. 5.25. It is difficult to see the difference between segments in this plot since the power dominates the local prescriptions. However, after power has been removed, Fig. 5.26 shows the differences in segment surface geometry.

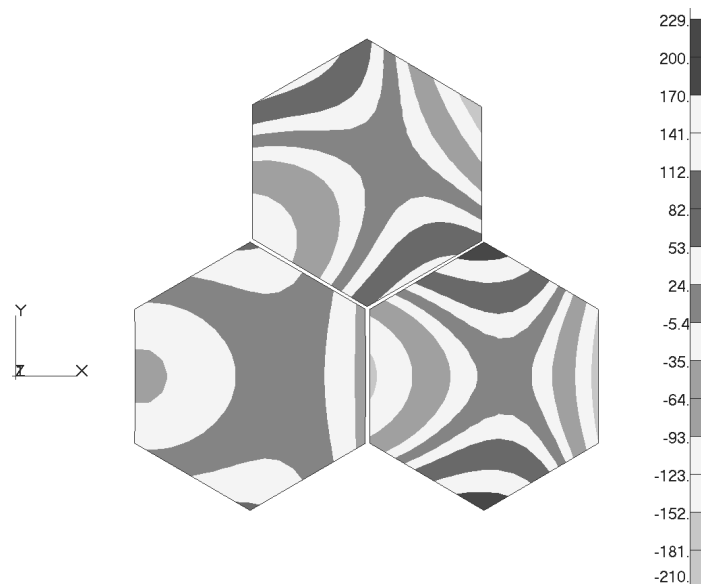


Figure 5.26 Local surface sag after power removed.

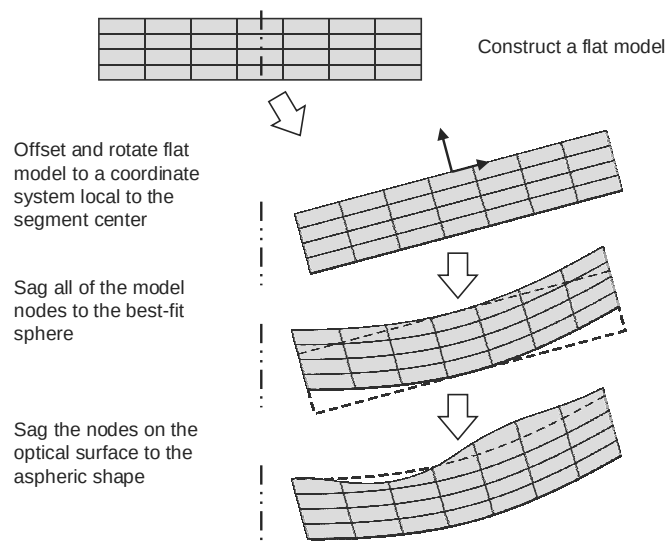


Figure 5.27 Process of slumping an off-axis segment model to an aspheric prescription.

The finite element models of these segments may be accurately constructed in a manner similar to a way in which the surfaces may be fabricated. In the fabrication process, the segment blanks may be initially figured with flat surfaces and then slumped to the geometry defined by the best-fit sphere. During the subsequent polishing cycles, the surface is then finished to the true aspheric geometry. This same process can be applied to the creation of the finite element model, as shown in Fig. 5.27. The process begins by creating a flat finite element

model. All of the nodes of the flat model are then slumped to the best-fit sphere in a direction along the axis of the segment-centered coordinate system using the procedure presented in the previous section. Finally, the locations of only the optical surface nodes can then be adjusted to the exact aspheric geometry. This final adjustment in nodes requires finding the sag of the optical surface in the segment-centered coordinate system, a process that requires some numerical root-finding techniques discussed in the next subsection.

The amount of adjustment of the surface nodes is usually very small, so there may be a negligible effect on deformation results due to mechanical loads. However, thermo-elastic deformations are significantly affected by such small changes in shape representation.

5.1.5.3 Calculation of local segment sag

It is often of interest to calculate the sag of a figured segment expressed in the z axis of the local segment coordinate system, such as those shown in Fig. 5.23. The reader may note that the z axes of the local segment coordinate systems are not parallel to the z axis of the parent vertex coordinate system centered on the segment array. This requires an iterative root-finding process in order to compute the sag expressed in the local segment coordinate system, the steps of which are as follows:

- Create a local segment coordinate system. For a given segment center with offset (x_0, y_0) find the corresponding segment center sag z_0 . Use (x_0, y_0, z_0) as the origin of the local segment coordinate system. Calculate the local normal to the vertex sag at this origin point to use as the direction of the z axis of the local segment coordinate system.
- For any point on the segment tangent plane (x_s, y_s) measured in the local segment coordinate system, the local segment sag z_s that locates the point (x_s, y_s, z_s) on the parent array surface is found. In this iterative root-finding process, x_s and y_s remain fixed while z_s is varied.
- The local segment locations may be used to define the segment model in the local segment coordinate system.
- The local segment sag may be used to obtain a polynomial fit of the segment surface geometry expressed in the local segment coordinate system. This expression of the surface geometry is quite useful in fabrication and testing of the segment.

5.1.6 Symmetry in optic models

5.1.6.1 Creating symmetric models

The most common and often quickest way to create a detailed finite element model is to simply automesh the full CAD geometry. The resulting mesh, however, will most likely not possess the symmetry that exists in the mechanical design. If the mesh is asymmetric, the resulting deformations may be asymmetric to some degree. A common practice in the validation of a finite element model is

to apply an isothermal load to a kinematically supported model with all materials set to a single value of CTE. The resulting Zernike fit to the response in most optical models should be a combination of primarily power and other spherical terms with all nonaxisymmetric terms zero. For this stress-free analysis, the size of the nonaxisymmetric terms can be an indication of the geometric asymmetry in the model. In addition, if constraining boundary conditions that display the same symmetry as the mechanical design are applied to the same model, then these results may be used to understand the elastic asymmetry in the model. However, an asymmetric mesh layout can yield asymmetric results and make it more difficult to find the presence of other modeling errors that would be detected with a symmetric mesh.

If it is desired to automesh the CAD geometry yet preserve as much symmetry as possible, then the following procedure may be followed:

- Step 1: Break the CAD geometry into the smallest symmetric subsection possible.
- Step 2: Mesh the subsection.
- Step 3: Reflect and rotate the mesh as required to obtain a full model.

5.1.6.2 Example creation of a symmetric model

The CAD geometry of a mirror with three mounting tabs is shown in in Fig. 5.28. An automeshed finite element model is to be constructed for the purpose of predicting the sensitivity to moment loads as shown. The geometry and loading exhibit threefold symmetry and, therefore, threefold symmetry is expected in the results.

The generation of a symmetric automeshed model begins by extracting a one-sixth slice of the original geometry by cutting along symmetry planes. The geometry resulting from this process is shown in Fig. 5.29. The process continues by meshing the one-sixth subsection as shown in Fig. 5.30.



Figure 5.28 Full CAD geometry of a mirror with threefold symmetry.

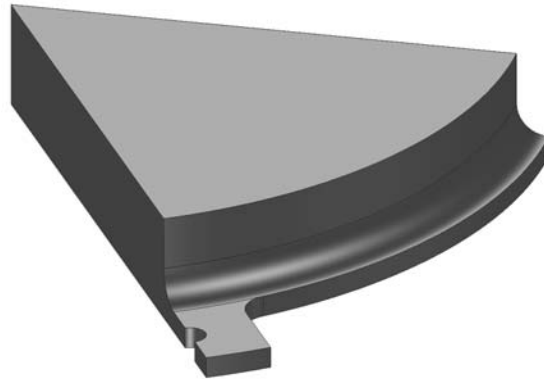


Figure 5.29 One-sixth subsection of the CAD geometry.

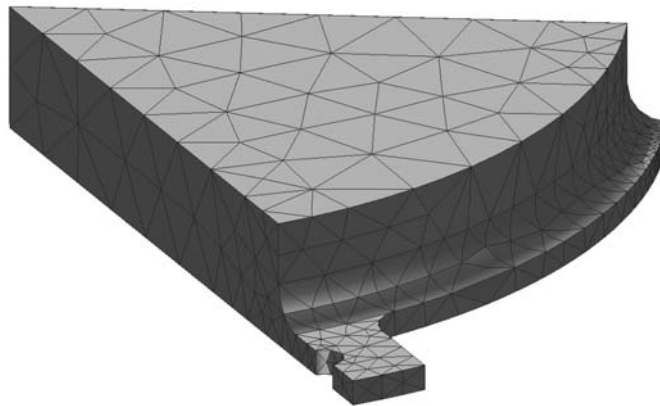


Figure 5.30 Finite element mesh on one-sixth subsection.

The final full model is created by reflecting the elements of the one-sixth section shown in Fig. 5.30 and then rotating all elements twice by an angle of 120 deg. An additional step to equivalence duplicate nodes at the symmetry planes is often required with most finite element pre-processing software packages. A free edge or free face check should be performed to verify that the mesh is fully equivalenced as intended. The final symmetric finite element model is shown in Fig. 5.31(a), while a full model meshed to the original CAD geometry is shown in Fig 5.31(b).

Table 5.5 shows a comparison of the Zernike polynomial fits of deformations predicted by each model shown in Fig. 5.31. The results show that the asymmetric mesh generates measureable asymmetric behavior as illustrated by the nonzero primary astigmatism and other terms that do not display threefold symmetry.

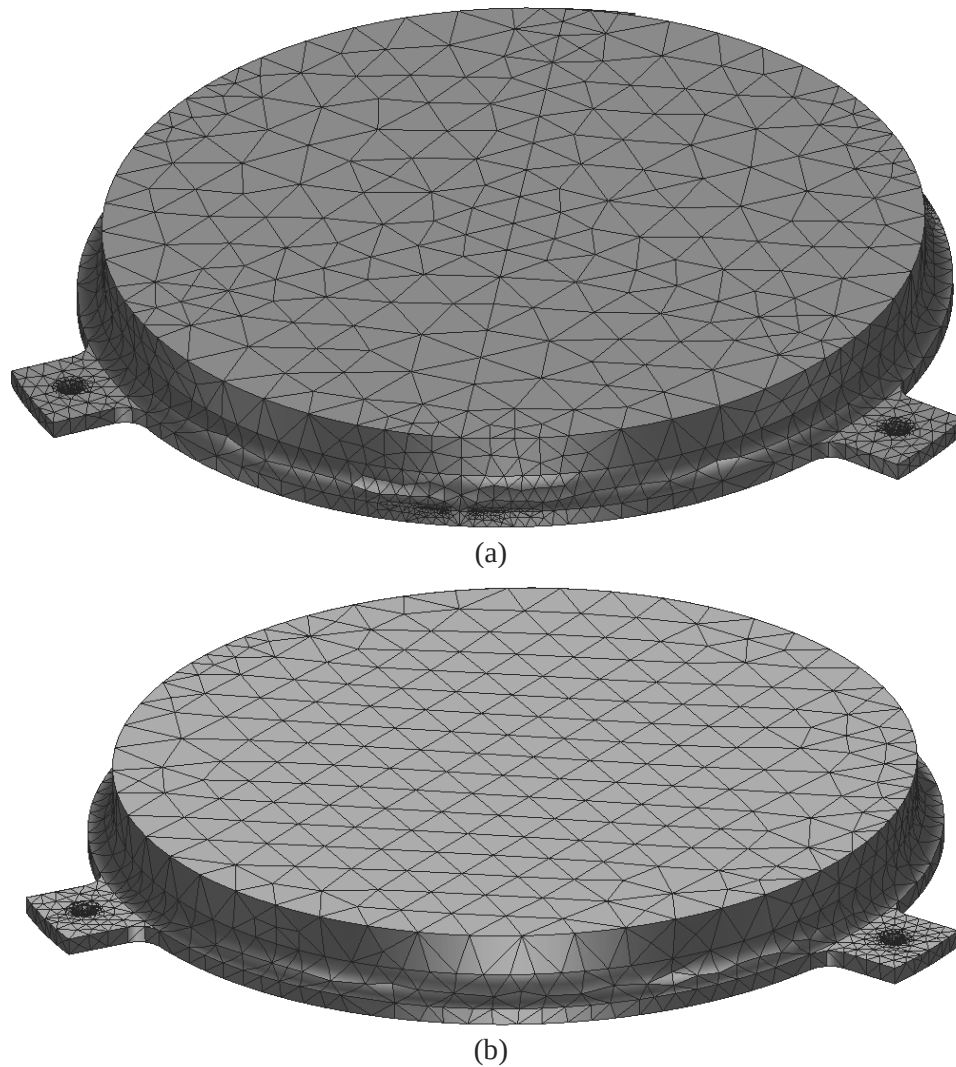


Figure 5.31 Finite element models of full mirror geometry: (a) generated from one-sixth subsection and (b) generated from full original CAD geometry.

5.1.6.3 Example of symmetry verification check

As discussed above, the presence of symmetry can be a useful tool in the process of analysis model validation. However, automeshing techniques can introduce asymmetries that consequently hamper the use of symmetry for model validation. Consider a solid circular mirror fabricated of fused silica whose outer diameter is 2.0 m, thickness is 0.1 m, and surface radius of curvature is 3.0 m. A simple support is applied to the outer edge as the optic is subjected to an isothermal

Table 5.5 Comparison of Zernike polynomial fits of surface deformation predicted by models shown in Fig. 5.33.

			Asymmetric Mesh	Symmetric Mesh
0	0	Bias	-3.48	-3.71
1	1	Tilt	0.82	0.00
2	0	Power (Defocus)	2160.05	2158.40
2	2	Pri Astigmatism	7.59	0.01
3	1	Pri Coma	0.60	0.00
3	3	Pri Trefoil	2889.64	2888.04
4	0	Pri Spherical	-5.00	-6.61
4	2	Sec Astigmatism	0.60	0.00
4	4	Pri Tetrafoil	1.92	0.00
5	1	Sec Coma	0.91	0.00
5	3	Sec Trefoil	164.73	161.34
5	5	Pri Pentafoil	0.72	0.00
6	0	Sec Spherical	-6.33	-9.95
6	2	Ter Astigmatism	0.85	0.00
6	4	Sec Tetrafoil	0.18	0.00
6	6	Pri Hexafoil	541.82	544.27

strain of 5.8 ppm. The mirror is modeled with three different meshes of plate elements as follows:

- Model 1: A polar mesh of 360 4-noded elements, 3-noded elements, and 361 nodes. Radial node spacing is 0.1 m.
- Model 2: An automatically generated mesh of 751 3-noded elements and 408 nodes. Edge node spacing is 0.1 m.
- Model 3: An automatically generated mesh of 386 4-noded elements and 419 nodes. Edge node spacing is 0.1 m.

Table 5.6 shows the axisymmetry in the surface geometry and surface deformation results in Model 1 and the nonaxisymmetry in Models 2 and 3. In order to quantify the axisymmetry in the surface deformation results, analyses were performed with two different representations of each of the three models. In the first representation, Z location values of the nodes were truncated to four significant digits, yielding a positional error of $\pm 50.0 \mu\text{m}$. In the second case the nodal Z locations were truncated to nine significant digits, yielding a positional error of $\pm 0.5 \text{ nm}$. The maximum nonaxisymmetric term and the residual surface RMS error with all Zernike terms through hexafoil subtracted was computed for the analysis with each of the three models. The result summary is shown in Table 5.7.

Table 5.6 Comparison of symmetric mesh layout with nonsymmetric mesh layouts.

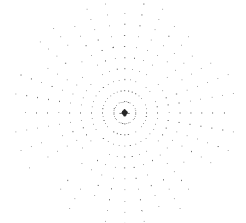
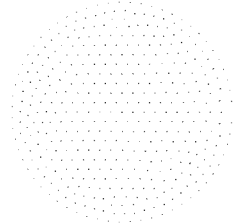
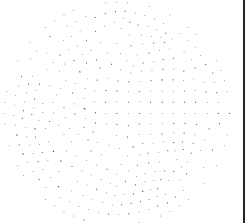
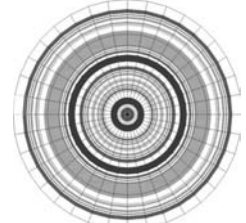
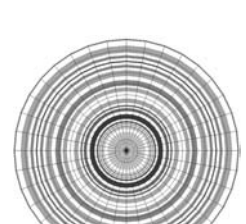
	Model 1	Model 2	Model 3
Plots of mesh			
Z nodal-position errors			
Residual error of surface deformation after subtraction of Zernikes through hexafoil			

Table 5.7 Surface deformation comparison summary in microns for expected axisymmetric results.

	Z position error $\pm 50.0 \mu\text{m}$			Z position error $\pm 0.5 \text{ nm}$		
	Model 1 Polar	Model 2 Auto-Tri	Model 3 Auto-Quad	Model 1 Polar	Model 2 Auto-Tri	Model 3 Auto-Quad
Maximum Nonaxisym. Term	8.10E-16	6.70E-05	1.80E-04	8.00E-16	1.50E-08	1.80E-04
Residual RMS	1.10E-04	1.50E-04	4.30E-04	1.30E-07	1.30E-07	4.10E-04

As shown in Table 5.7, the maximum nonaxisymmetric Zernike coefficient for Model 1, the axisymmetric polar mesh, is essentially zero, whereas both automeshed models yield nonzero, nonaxisymmetric coefficients. As the Z position error is decreased, the auto-tri mesh exhibits a decreasing maximum nonaxisymmetric term while the maximum nonaxisymmetric term for the auto-quad mesh remains constant. The residual RMS is similar for both the polar mesh

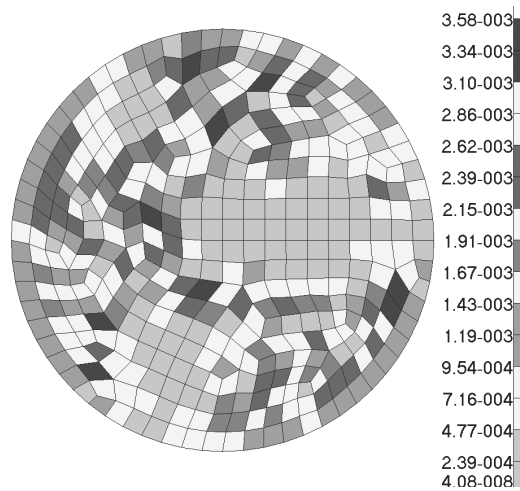


Figure 5.32 Warping in the four-noded quadrilateral element mesh of Model 3.

and the auto-tri mesh. In this example, the deformations were represented by seven significant digits, which prevents the residual RMS from getting smaller than $1.3\text{E}-7$ microns.

The poor performance of the auto-quad mesh is attributed to the warping of the four-noded quadrilateral elements. The only way that a four-noded quadrilateral-element mesh can represent the geometry of a sphere without warped elements is with a polar mesh layout, as is used in Model 1. A plot of the warping in the auto-quad mesh is shown in Fig. 5.32. When a four-noded quadrilateral element is warped, the stiffness matrix is generated for an average plane through the four corners. This causes forces with offsets that result in moments at those corners. The warping effect is the reason that the residual RMS and the large nonaxisymmetric coefficients do not decrease with higher-precision node location.

Warping is only a problem with four-noded quadrilateral elements. Triangles with three nodes cannot warp. Higher-order shell elements allow curvature of the element within their stiffness formulation. The four-noded quadrilateral faces of solid elements, such as an eight-noded hexahedron, allow warping of a face because these elements do not derive their stiffness matrix on an average plane of the faces.

When generating shell models of curved optics, the analyst should try to avoid warping four-noded quadrilateral elements. Model checks, such as those shown in this example, are required to verify the model. Switching to three-noded triangles can provide more accurate results than quadrilateral elements in many cases.

5.2 Analysis of Surface Effects

The causes of surface effects that induce deformations of optics are many, including thermoelastic mismatching between an optic and its coating, coating-

cure shrinkage, coating-moisture absorption, and the Twyman effect. The Twyman effect is the thin compression layer caused by polishing a glass surface. This compression layer will cause deformation of an optic in a fashion similar to thermoelastically generated deformations. The Twyman effect can be reversed by abraiding the polished surface, thereby breaking the compressive stresses created by the polishing process. All of these surface effects can cause optical-surface deformation. Behaviors associated with stresses in the surface coatings can additionally lead to cracking in the coating.

In this section, it is assumed that the coating layer is very thin compared to the optic substrate. To determine the coating stress, a flat test coupon is coated in the same manner as the full optic. The test sample is then tested optically to determine the radius of curvature induced by the coating. The coating stress is determined from Stoney's equation, where the subscript *O* refers to the optic substrate or test sample, and the subscript *C* refers to the coating layer⁴:

$$\sigma_c = \frac{E_o t_o^2}{6 t_c (1 - \nu_o) R_o C}. \quad (5.29)$$

Note that the coating modulus is ignored because the coating thickness is very small compared to the substrate.

All of the surface effects mentioned above can be simulated with a thermoelastic analysis. In the cases of coating shrinkage, moisture absorption, and the Twyman effect, effective-thermoelastic strains α_c^* are computed from Table 5.7. These effective-thermoelastic strains can be applied to the model by using α_c^* as the CTE for the coating and a unit temperature change. Details for each modeling method are given below.

5.2.1 Composite-plate model

Some finite element codes have composite property features with which the user may specify the material and thickness of each layer of a composite-layer stack. This feature can be used to predict surface deformation effects in plate models representing an optic and its coating. The optic and its coating are modeled with one layer of plate elements, which is given a composite-property description as illustrated in Fig. 5.33. The property values assigned to each layer of the

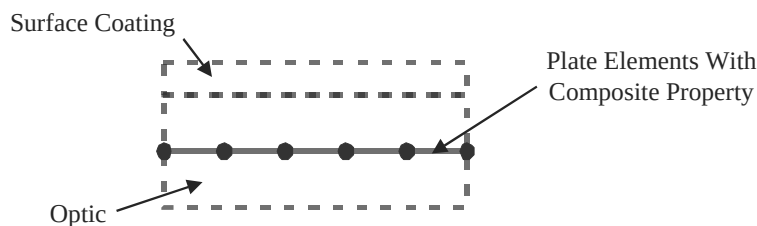
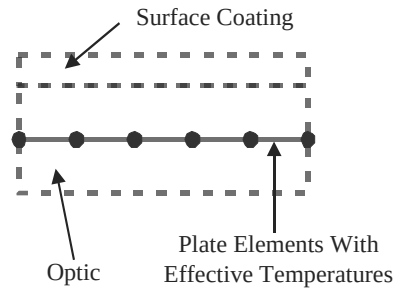


Figure 5.33 Composite-plate model for surface-effect deformation prediction.

Table 5.8 Composite-layer property values for surface-effect analysis.

COATING EFFECT	OPTIC LAYER				SURFACE LAYER				TEMPERATURE LOAD
	E	ν	CTE	T	E	ν	CTE	T	
Thermoelastic	E_o	ν_o	α_o	t_o	E_c	ν_c	α_c	t_c	ΔT
Moisture absorp.	E_o	ν_o	0	t_o	E_c	ν_c	α_c^*	t_c	Unity
Cure shrinkage	E_o	ν_o	0	t_o	E_c	ν_c	α_c^*	t_c	Unity
Twyman effect	E_o	ν_o	0	t_o	E_o	ν_o	α_c^*	Arbitrary	Unity

**Figure 5.34** Homogeneous-plate model for surface-effect deformation prediction.

composite description and the corresponding temperature load are shown in Table 5.8.

One advantage of the composite plate model is that the stresses in the surface layer and the interlaminar shear stresses may be recovered. In cases where the surface layer represents a surface coating, this may be useful information if cracking of the coating is suspected.

5.2.2 Homogeneous-plate model

Since many finite element codes do not feature composite-property descriptions, results for surface deformation effects in plate models can be obtained with a homogeneous-plate model and effective thermoelastic loads. Such a model is illustrated in Fig. 5.34. The properties of the homogeneous-plate model are simply unmodified values one would use for any other type of analysis. The effective thermoelastic loads, however, include a bulk temperature shift and a thermal gradient to include the surface effect. Eqs. [5.30(a)] and [5.30(b)] are used to compute the loads for thermoelastic analyses. Eq. [5.30(a)] gives the bulk temperature shift $\overline{\Delta T}$ and temperature gradient T' , which should be applied to the homogeneous model. The equivalent coated and uncoated surface temperatures ΔT_1 and ΔT_2 for codes that require such a format are given in Eq. [5.30(b)]:

$$\Delta T^* = \frac{E_c \alpha_c \Delta T}{E_o \alpha_o}, \quad \overline{\Delta T} = \frac{\Delta T t_o + \Delta T^* t_c}{t_o}, \quad T' = \frac{6 \Delta T^* t_c}{t_o^2}, \quad [5.30(a)]$$

Table 5.9 Effective $\alpha\Delta T$ for surface effects.

<u>COATING EFFECT</u>	<u>EFFECTIVE $\alpha\Delta T$ OF COATING (α_c^*)</u>
Moisture Absorption Growth	$CME \Delta M$
Cure Shrinkage	$-\frac{\sigma_c(1-\nu_c)}{E_c}$
Twyman Effect ⁵	$\frac{4C}{3t_c}$

CME is the coefficient of moisture expansion of the coating, and ΔM is the moisture change value in units consistent with CME.
 σ_c is the stress in the coating deposited on a rigid substrate.
 ν_c and E_c are the Poisson's ratio and Young's modulus, respectively, of the coating.
 Assume $\nu_c = 0$ for a worst case condition if ν_c is not known.
 C is the Twyman constant.
 t_c is an arbitrary small thickness that must also be used in the finite element model if applicable.

$$\Delta T_1 = \overline{\Delta T} - \frac{3\Delta T^* t_c}{t_o}, \quad \Delta T_2 = \overline{\Delta T} + \frac{3\Delta T^* t_c}{t_o}. \quad [5.30(b)]$$

Symbols used in Eqs. [5.30(a)] and [5.30(b)] are defined as follows:

$\overline{\Delta T}$ = effective bulk temperature shift
 T' = effective thermal gradient
 ΔT_1 = effective temperature of uncoated surface
 ΔT_2 = effective temperature of coated surface
 E_c = Young's modulus of coating
 E_o = Young's modulus of optic
 α_c = CTE of coating
 α_o = CTE of optic
 ΔT = temperature change
 t_c = thickness of coating
 t_o = thickness of optic

These equations assume that the plate elements are defined such that a positive value of T' will generate a thermoelastic load with the highest temperature on the coated side. It is also assumed that t_c is much smaller than t_o .

For analogous analyses, such as those listed in Table 5.9, the following equations are used:

$$\Delta T^* = \frac{E_c \alpha_c^*}{E_o \alpha_o}, \quad \overline{\Delta T} = \frac{\Delta T^* t_c}{t_o}, \quad T' = \frac{6\overline{\Delta T}}{t_o} \quad [5.31(a)]$$

and

$$\Delta T_1 = \overline{\Delta T} - 3\Delta T^*, \quad \Delta T_2 = \overline{\Delta T} + 3\Delta T^*, \quad [5.31(b)]$$

where α_c^* is one of the effective $\alpha\Delta T$ values found in Table 5.9, and α_o is an arbitrary CTE, which must also be used in the optic's finite element model material description. Notice that for simulation of the Twyman effect, t_c is an arbitrary value if the Twyman constant is used.

This model lacks the material properties of the coating layer. Therefore, stresses in the coating layer are not correctly predicted by the homogeneous-plate model.

5.2.3 Three-dimensional model

For optics that require 3D solid models, such as thick lenses, surface effects can be included by a mesh of membrane elements on the optic's surface as shown in Fig. 5.35. The definition of properties for the solid and surface meshes are similar to the methods used for the composite plate description discussed in Section 5.2.1.1. Table 5.10 gives the correct property values for the solid-element mesh of the optic and the membrane-element mesh of the surface, where α_c^* is found in Table 5.9. Notice that the stresses predicted by the membrane elements can be recovered to compute the stress in the coating.

5.2.4 Example: coating-cure shrinkage

A reflective coating is deposited onto the optical surface of a solid 1.0-in-thick flat mirror fabricated of a glass whose Young's modulus is 13.2 Msi and whose Poisson's ratio is 0.272. The Young's modulus of the coating is 1500 psi, and its thickness is 0.0001 in. The coating stress is 2100 psi. The change in surface figure is desired due to the cure shrinkage of the coating layer. The results from the three methods of analysis (composite plate, homogeneous plate, and 3D solid) are to be used to compute the surface error for comparison.

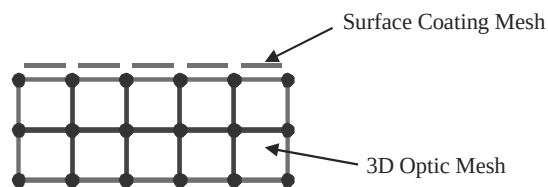


Figure 5.35 3D model for surface-effect deformation prediction.

Table 5.10 3D-model property values for surface-effect analysis.

<u>COATING EFFECT</u>	<u>OPTIC MESH</u>			<u>SURFACE MESH</u>				<u>TEMPERATURE LOAD</u>
	<u>E</u>	<u>ν</u>	<u>CTE</u>	<u>E</u>	<u>ν</u>	<u>CTE</u>	<u>T</u>	
Thermo-elastic	E_o	ν_o	α_o	E_c	ν_c	α_c	t_c	ΔT
Moisture absorp.	E_o	ν_o	0	E_c	ν_c	α_c^*	t_c	Unity
Cure shrinkage	E_o	ν_o	0	E_c	ν_c	α_c^*	t_c	Unity
Twyman effect	E_o	ν_o	0	E_o	ν_o	α_c^*	Arbitrary	Unity

From Table 5.8, we can compute the effective thermoelastic load $\alpha\Delta T = \alpha_c^*$ to apply to the coating:

$$\alpha_c^* = -\frac{\sigma_c(1-\nu_c)}{E_c} = -\frac{(2100 \text{ psi})(1-0.272)}{(1500 \text{ psi})} = -1.0192. \quad (5.32)$$

This effective thermo-elastic load is applied by setting the CTE of the coating to -1.0192 , setting the CTE of the optic to 0.0 , and applying a unit increase in temperature to the model in a thermo-elastic analysis.

5.2.4.1 Composite-plate model

The composite-plate-model property description defines a 1.0-in layer with a Young's modulus of 13.2 Msi, and a CTE of 0.0 laminated to a second layer 0.0001-in thick with a Young's modulus of 1500 psi, and a CTE of -1.0192 . The optic mesh is supported by kinematic constraints, and a unit temperature drop is applied to the model as an isothermal thermoelastic load.

5.2.4.2 Homogeneous-plate model

The homogeneous-plate model simply consists of a plate mesh whose property is a constant 1.0-in thickness, and whose material properties are those of the glass. The element normals are defined so that a positive thermal gradient is consistent with a higher temperature on the coated surface as compared to the uncoated surface. The effective temperatures, which include the effect of the coating, are computed from

$$\Delta T^* = \frac{E_c \alpha_c^*}{E_o \alpha_o} = \frac{(1500 \text{ psi})(-1.0192)}{(13.2 \times 10^6 \text{ psi})(6.80 \times 10^{-6} / ^\circ\text{C})} = -17.03^\circ\text{C},$$

$$\overline{\Delta T} = \frac{\Delta T^* t_c}{t_o} = \frac{(-17.03^\circ\text{C})(0.0001 \text{ in})}{(1.0 \text{ in.})} = -0.00170^\circ\text{C},$$

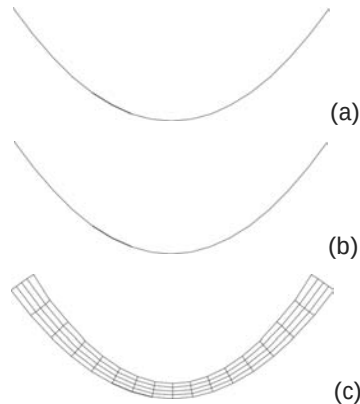
and

$$T' = \frac{6\overline{\Delta T}}{t_o} = \frac{6(-0.00170^\circ\text{C})}{1.0 \text{ in}} = -0.0102^\circ\text{C/in.} \quad (5.33)$$

The optic mesh is supported by kinematic constraints and the computed temperature loads are applied in a thermoelastic analysis.

Table 5.11 Result summary of coating-cure analyses.

<u>MODEL TYPE</u>	<u>TOTAL RMS</u>	<u>TOTAL P-V</u>	<u>P-V POWER</u>
Composite plate	70.3 nm	121.2 nm	121.2 nm
Homogeneous plate	70.3 nm	121.3 nm	121.3 nm
Three-dimensional	69.7 nm	121.1 nm	120.8 nm

**Figure 5.36** Exaggerated deformed shapes of optic after coating-cure shrinkage: (a) composite-plate model, (b) homogeneous-plate model, and (c) 3D model.

5.2.4.3 Three-dimensional model

A solid mesh of the 1.0-in-thick optic is created, and membrane elements are added to represent the coated surface. The material properties of the solid elements are those of the glass. The thickness and material properties of the membrane element are the same as those of the coating. However, the CTE of the solid elements is set to 0.0, while the CTE of the membrane elements is set to -1.0192 . The optic mesh is supported by kinematic constraints, and a unit temperature increase is applied in a thermoelastic analysis.

The analysis results of the three methods are summarized in Table 5.11. The results show an excellent correlation between all three model types. The 3D model shows some compliance associated with through-the-thickness deformation near the edges of the optic. Fig. 5.36 shows the exaggerated deformed shapes for the diametrical cross sections of each model type.

Both the composite-plate and 3D models predict a coating stress of 2885 psi. The homogeneous-plate model is unable to predict the coating stress.

5.2.5 Example: Twyman effect

A thin circular fused silica disk is polished on the top surface creating a layer of compressive stresses. Typical values of the compressive stress layer due to polishing were obtained from Ref. 6. The 5-deg wedge axisymmetric model

using composite properties is shown in Fig. 5.37(a). The resulting deformation is shown in Fig. 5.37(b).

The effective thermal strain can be computed from the equation given in Table 5.8 for cure shrinkage of a surface layer:

$$-\frac{\sigma_c(1-\nu_c)}{E_c} = -\frac{(-0.5076 \text{ Msi})(1-0.17)}{10.59 \text{ Msi}} = 0.03979. \quad (5.34)$$

The Twyman constant can be computed by equating the effective thermal strains associated with cure shrinkage of a surface layer and the Twyman effect:

$$\begin{aligned} -\frac{\sigma_c(1-\nu_c)}{E_c} &= \frac{4C}{3t_c} \Rightarrow C = \frac{3t_c}{4} \left(-\frac{\sigma_c(1-\nu_c)}{E_c} \right) \\ C &= \frac{3(1.0 \times 10^{-6} \text{ in})}{4} \left(-\frac{(-0.5076 \text{ Msi})(1-0.17)}{10.59 \text{ Msi}} \right) = 2.984 \times 10^{-8} \text{ in}. \end{aligned} \quad (5.35)$$

The radius of curvature of the substrate may be computed by using Stoney's equation given by Eq. (5.29):

$$\begin{aligned} \sigma_c &= \frac{E_o t_o^2}{6t_c(1-\nu_o)RoC} \Rightarrow RoC = \frac{E_o t_o^2}{6t_c(1-\nu_o)\sigma_c} \\ RoC &= \frac{E_o t_o^2}{6t_c(1-\nu_o)\sigma_c} = \frac{(10.59 \text{ Msi})(0.04 \text{ in})^2}{6(1.0 \times 10^{-6} \text{ in})(1-0.17)(-0.5076 \text{ Msi})} = -6702.9 \text{ in}. \end{aligned} \quad (5.36)$$

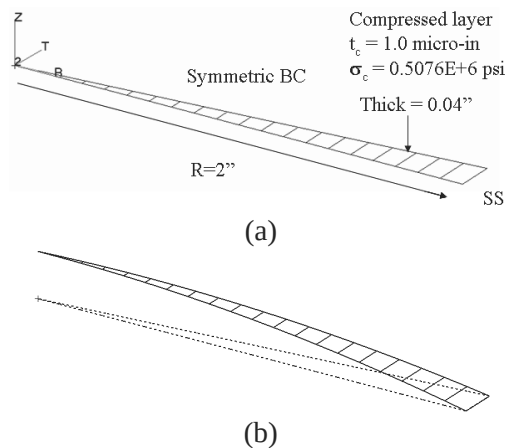


Figure 5.37 The model parameters relevant to the deformation analysis are shown in Table 5.12.

Table 5.12 Finite element model parameters.

	English Units	Metric Units
Substrate Diameter	4.0 in	0.1016 m
Substrate Thickness	0.04 in	0.001016 m
Substrate Modulus	10.59 Msi	73 GPa
Poisson Ratio of Substrate	0.17	0.17
Thickness of Stressed Layer	1.00×10^{-6} in	2.54×10^{-8} m
Compressive Stress in Stressed Layer	0.5076 Msi	3.5 GPa

Table 5.13 Comparison of FE results and Stoney equation.

	FE Model	Stoney Equation
Radius of Curvature	6702.0	6702.9
Power in Waves HeNe	5.99λ	5.99λ

The power of the deformed surface may be approximated by half of the sag:

$$\begin{aligned}
 \text{Power} &= \frac{RoC - \frac{|RoC|}{RoC} \sqrt{RoC^2 - \text{radius}^2}}{2} \\
 &= -6702.9 \text{ in} - \frac{|-6702.9 \text{ in}|}{-6702.9 \text{ in}} \sqrt{(-6702.9 \text{ in})^2 - (4.0 \text{ in})^2} \quad (5.37) \\
 &= -1.4919 \times 10^{-4} \text{ in} = 5.99 \lambda_{\text{HeNe}}.
 \end{aligned}$$

References

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◀Chapter 6▶

Modeling of Optical Mounts

This chapter presents techniques relevant to finite element modeling of optical mounts and other support conditions. The treatment given to developing models of local mounting hardware such as adhesive bonds and flexures is aimed to give the reader various options and suggested practices for representing such hardware accurately and as simply as possible. Employment of idealized mounting configurations is presented to assist the analyst with simplified representations to improve the simplicity of models used for early design-trade studies. The discussion of the modeling of test supports illustrates how the reader may predict errors induced by how the optic is supported while being tested. Lastly, modeling the process of assembly is presented as a tool to understand the impact of locked in strains induced by the processes of integrating multiple optical subsystems and components together. As in Chapter 5, a key concept in developing models of such mounting hardware is understanding how the results of the analysis will be used and allowing such understanding to guide the specific modeling techniques employed. For example, the modeling techniques used to represent mounting flexures can be very different for the analysis goals of predicting the deformation optical surfaces versus predicting the stress levels in the flexures themselves.

6.1 Displacement Models of Adhesive Bonds

Adhesives commonly used to bond optics to their mounts are not trivial items to model. Characteristics such as near incompressibility and extremely small thicknesses pose difficulties to developing accurate numerical models for these bonds. However, low stiffness, high cure shrinkage, and high thermo-elastic growth characteristics of adhesives must be well represented in optomechanical models in order to obtain useful predictions. Treatment of the effective modulus through-the-thickness of thin, nearly incompressible bonds has been the topic of several sources in the published literature.^{1–3} Application of these techniques can be generalized to the full elastic description of Hooke's law and multiple geometries.⁴

6.1.1 Elastic behavior of adhesives

The objective of the foregoing discussion is to illustrate two important aspects when modeling nearly incompressible bonds. The first is that it is essential that an accurate value of Poisson's ratio be included in the model since the stiffness of nearly incompressible bonds is highly sensitive to this parameter. The second aspect is that the finite element mesh must be capable of predicting the free-face deformations, such as those illustrated by Fig. 6.1.

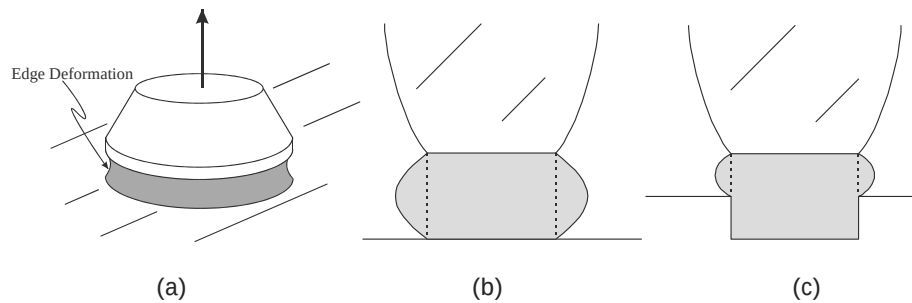


Figure 6.1 Exaggerated illustrations of incompressible bond behavior: (a) “hockey-puck” type bond, (b) ring bond of a lens, and (c) partially constrained ring bond of a lens.

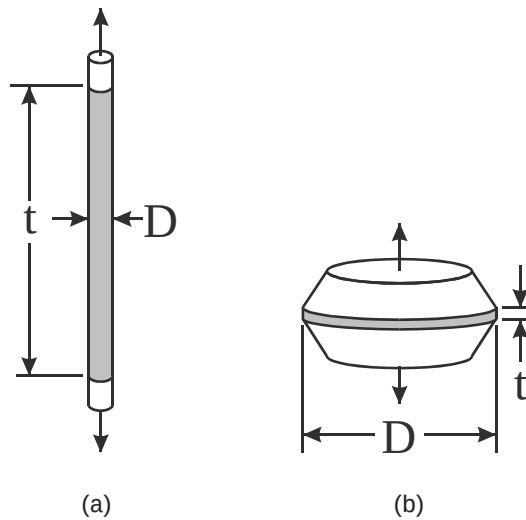


Figure 6.2 Uniaxial test sample and thin-layer test sample.

The design of bonds using nearly incompressible materials must not ignore the effects of restraining volume-changing strains. Bond designs that restrain volume changes will behave stiffer with nearly incompressible materials than designs that allow “bulging and necking.” This leads to very different behaviors in the bond designs shown in Fig. 6.1(b) and 6.1(c), for example. Bond geometries often prohibit simple hand calculations, and detailed bond analysis is required to properly characterize the stiffness accurate predictions.

To familiarize the reader with the relevant aspects of adhesive-bond behavior, we will first introduce two extreme cases of adhesive test samples. These cases, shown in Fig. 6.2, are the uniaxial test sample and the thin-layer test sample. The material’s near incompressibility and the difference in geometries cause these two samples to behave with very different stress-to-strain ratios. While the uniaxial test sample freely allows the lateral strains required to allow straining in the loaded direction, the thin-layer test sample strongly resists such lateral strains. Therefore, the thin-layer test sample appears to behave with a higher stress-to-strain ratio as the allowed lateral straining occurs only near the

free surface of the bond. This behavior causes difficulties in modeling optomechanical bonds that have elastic characteristics similar to the thin-layer test sample. In order to correctly represent the compliance of the bond, the local deformations near the free edge must be included. This section illustrates methods by which such behavior can be accurately represented in a finite element model.

We can bound the stress-to-strain ratios of these two test samples by making assumptions about the stresses and strains in each case and applying them to the stress-to-strain relationships in Eqs. (1.2) and (1.3). For each of the two test samples, a load is applied in the manner shown in Fig. 6.2, and the strain along the load direction e_z is calculated. From Hooke's law, the expression for this strain in terms of the stresses is

$$e_z = -\frac{\nu}{E}\sigma_x - \frac{\nu}{E}\sigma_y + \frac{1}{E}\sigma_z. \quad (6.1)$$

For the uniaxial test sample shown in Fig. 6.2(a), we may assume that σ_x and σ_y are zero. This gives

$$\frac{\sigma_z}{e_z} = E, \quad (6.2)$$

which is the familiar uniaxial stress-strain relationship.

From Hooke's law, the expression of the test load stress in terms of the strains is

$$\sigma_z = \frac{\nu E}{(1+\nu)(1-2\nu)}e_x + \frac{\nu E}{(1+\nu)(1-2\nu)}e_y + \frac{(1-\nu)E}{(1+\nu)(1-2\nu)}e_z. \quad (6.3)$$

For the thin-layer test shown in Fig. 6.2(b), we assume that e_x and e_y are 0. This gives

$$\frac{\sigma_z}{e_z} = \frac{(1-\nu)E}{(1+\nu)(1-2\nu)} = M, \quad (6.4)$$

which is defined as the *maximum modulus*, M . Notice from Eq. (6.4) that the maximum modulus is increasingly dependent on Poisson's ratios greater than about 0.45 and is undefined at a Poisson's ratio of 0.5. This dependence on Poisson's ratio is shown in Fig. 6.3 and Table 6.1. Notice that for Poisson's ratios greater than 0.49, each additional "9" adds an order of magnitude to the maximum modulus. Recall that if Poisson's ratio equals 0.5, then the material is incompressible.

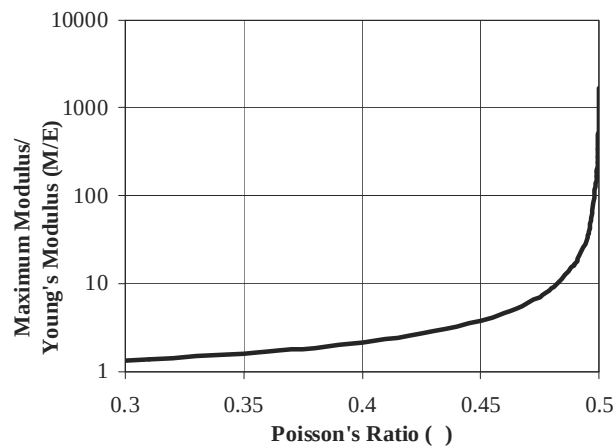


Figure 6.3 Plot of maximum modulus M divided by Young's modulus E vs. Poisson's ratio ν .

Table 6.1 Maximum modulus to Young's modulus ratio vs. Poisson's ratio.

POISSON'S	
<u>RATIO</u>	<u>M/E</u>
0.45	4.8
0.49	17.1
0.499	167.1
0.4999	1667.1
0.49999	16667.1

The spectrum of cases bounded by the uniaxial and thin-layer extremes discussed previously is large. These test samples each have a unique diameter-to-thickness ratio D/t , where D is the diameter of the test sample perpendicular to the applied load, and t is the thickness parallel to the applied load. With a series of detailed finite element analyses, the ratio of applied stress to axial modulus E is shown in Fig. 6.4(a), while the comparison to the maximum modulus M is shown in Fig. 6.4(b).

These computed stress-to-strain ratios can be compared to the stress-to-strain ratios for each of the extreme cases as computed with Eqs. (6.2) and (6.4). The comparison to the uniaxial modulus E is shown in Fig. 6.4(a), while the comparison to the maximum modulus M is shown in Fig. 6.4(b).

As would be expected, the comparison to the uniaxial modulus is closer for more uniaxial cases than for thin-layer cases. In other words, $(\sigma/\epsilon)/E$ in Fig. 6.4(a) approaches unity for cases with low diameter-to-thickness ratios. Likewise, $(\sigma/\epsilon)/M$ in Fig. 6.4(b) approaches unity for cases with higher diameter-to-thickness ratios. Between the two extremes are cases that possess complex

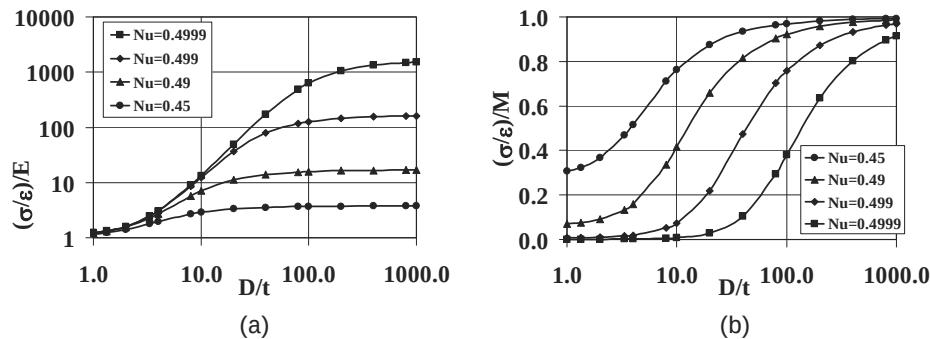


Figure 6.4 (a) Plot of stress-to-strain ratio σ/ϵ divided by Young's modulus E vs. diameter-to-thickness ratio D/t and (b) plot of stress-to-strain ratio σ/ϵ divided by maximum modulus M vs. diameter-to-thickness ratio D/t .

strain states. These complex strain states are characterized by the radial-edge deformation that can contribute to a large percentage of the compliance of the bond (see Fig. 6.1).

In addition to the dependence on a diameter-to-thickness ratio, the overall stiffness varies with Poisson's ratio. Higher values of Poisson's ratio show less agreement with either of the two extremes for a given diameter-to-thickness ratio, because higher Poisson's ratios weaken the validity of the assumptions used to generate Eqs. (6.2) and (6.4). Therefore, larger values of Poisson's ratio yield a wider range of diameter-to-thickness ratios that do not behave like either of the two extreme cases presented above.

6.1.2 Detailed 3D solid model

One obvious method of modeling adhesive bonds is to use solid elements with enough resolution to represent their nearly incompressible behavior. Four elements or more should be used along any free surface to represent the deformation effects illustrated in Fig. 6.4. Enough elements should be used in the plane of the bond to represent the decay in the edge deformation as well. The Young's modulus E and bulk modulus B can be obtained from tests. Poisson's ratio can then be calculated from the Young's modulus and bulk modulus:

$$\nu = \frac{1}{2} - \frac{E}{6B}. \quad (6.5)$$

The shear modulus can then be computed as

$$G = \frac{E}{2(1 + \nu)}. \quad (6.6)$$

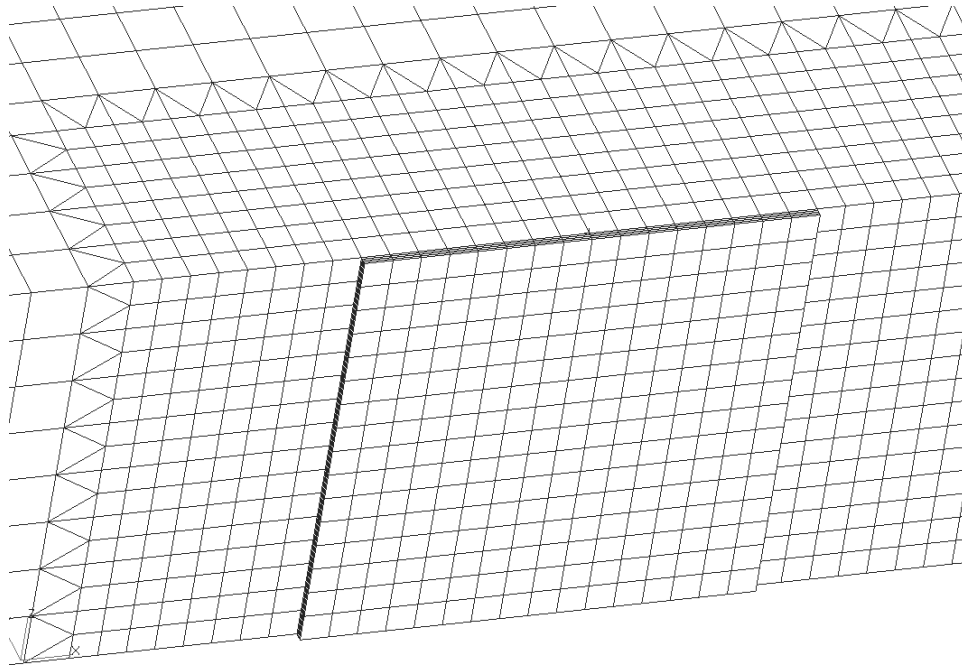


Figure 6.5 Congruent mesh model.

6.1.2.1 Congruent mesh models

In order to get accurate predictions of the behavior of high Poisson bonds, the finite element model must have enough resolution through the thickness and near the edge to capture the local edge bulging effect. A single rectangular-edge bond on an optic can be represented with the model shown in Fig. 6.5, where the mesh of the bond has four layers of elements through the thickness. Since the edge can represent the bulging deformation of the bond, the material properties from a uniaxial test of the bond material are used. The mesh detail in the bond must be carried into the optic before transitioning to a coarser mesh, possibly creating very large FE models.

6.1.2.2 Glued contact models

Some finite element programs have a feature called “glued contact.” In this approach, two mating meshes need not be congruent. The bond mesh may contain a high mesh resolution while mating with the coarser mesh of an optic as shown in Fig. 6.6. Internally, the FE program forces compatibility across the interface. This approach can provide high-quality stress results in the optic at the bond interface.⁵

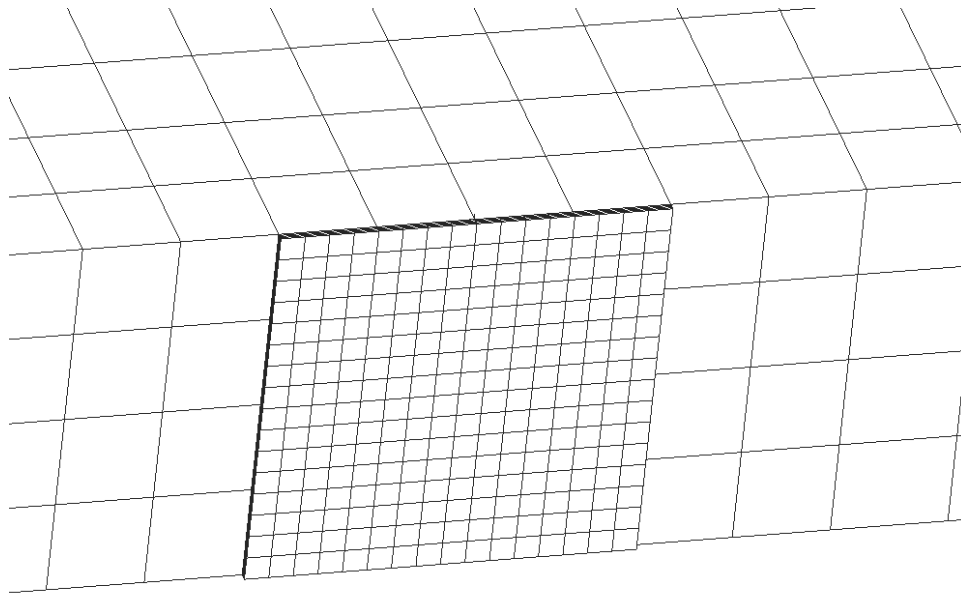


Figure 6.6 Glued contact mesh model.

6.1.3 Equivalent-stiffness bond models

It is of interest to obtain a method of using coarse meshes of adhesive bonds without sacrificing an accurate representation of the stiffness of the bond. The method involves using effective properties with a 3D solid model of the adhesive bond that is coarse enough such that the free-face deformation is not represented at all. The effective properties, however, are chosen such that the compliance of the bond due to the free-face deformation is included in the coarse model's stiffness. The motivation for such a simplified model is that the fidelity required in the detailed 3D solid model in order to properly represent the stiffness of the bond is often too fine to be practically integrated with the adjacent displacement models of the bonded parts. In addition, when several elements are used through the thickness of the bond, unacceptable aspect ratios can result.

When employing effective properties of bonds in simplified solid models, the user must be aware of their limitation. The effective properties to be used with coarse-bond models are simply intended to match the overall stiffness of the bond. Since regions of the bond closer to the free faces will display more compliance in the actual hardware, there may be a significant variation in stiffness throughout the bond. Such variation in stiffness may be important to the behavior of the hardware and must be included in the model in such cases. Use of the effective properties, however, prevents representation of such a distribution of stiffness. Therefore, in ring-bond designs like that shown in Fig. 6.7(a), a detailed model of the bond might be required. However, designs like that shown in Fig. 6.7(b) have been found to be accurately represented by coarse solid models and effective properties.

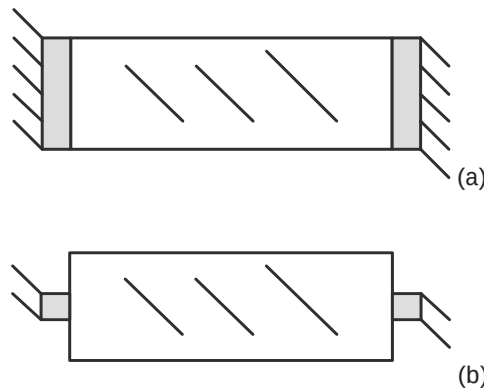


Figure 6.7 Cross-section plots of two example ring bond designs: (a) full width bond and (b) partial width bond.

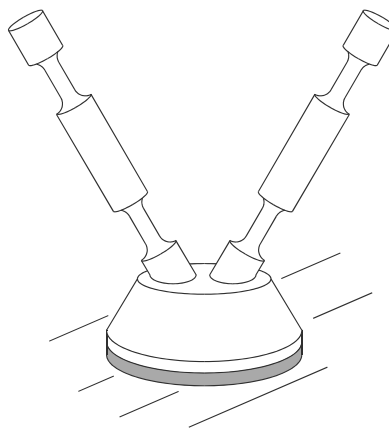


Figure 6.8 Example of a “hockey-puck” bond.

Calculation of the effective properties involves modeling the adhesive bond in a detailed test model and computing the bond’s overall stiffness. From these stiffness predictions, an effective Hooke’s law matrix relating stress to strain can be computed for use as effective properties in a coarse model. Fortunately, these effective properties are functions of the uniaxial test material properties and a small number of geometric parameters. Therefore, effective property curves can be generated as functions of these parameters to create “look-up” tables. The relevant geometric parameters vary, however, for different applications.

6.1.3.1 Effective properties for hockey-puck-type bonds

A “hockey-puck” bond is a thin, relatively flat bond such as that illustrated in Fig. 6.8. The bond may be circular or a more complex shape. The method for computing the effective properties of hockey-puck-type bonds is as follows:

- ◁1▷ Compute the diameter-to-thickness ratio D/t . For noncircular geometries, an effective diameter D_{eff} is suggested by Lindley³ and is computed as follows:

$$D_{\text{eff}} = \frac{4A}{C}, \quad (6.7)$$

where A is the plane-view area of the bond, and C is the circumference of the plane-view area. Compute the diameter-to-thickness ratio using D_{eff} for the diameter.

- ◁2▷ Compute the maximum modulus M , given by Eq. (6.4), with values of E and ν .
- ◁3▷ With ν and D/t , find the correction factors k_{33} and k_{31} from Table 6.2.
- ◁4▷ Use one of the modeling methods described in the text below.

Figs. 6.9 and 6.10 show plots of k_{33} and k_{31} vs. D/t ratio for the values of Poisson's ratio shown in Table 6.2. However, it is advised that values be taken by interpolation from Table 6.2 rather than graphically from Figs. 6.9 and 6.10.

There are several methods of using the correction factors, k_{33} and k_{31} , to obtain effective properties. One method is to mesh the adhesive bond with solid elements and use only one element through the thickness. A mesh fidelity in the plane of the bond can be chosen to reasonably match the mesh of the models

Table 6.2 Correction factors for “hockey puck” bonds with various combinations of D/t ratio and Poisson's ratio.

	$\nu = 0.45$		$\nu = 0.49$		$\nu = 0.499$		$\nu = 0.4999$	
D/t Ratio	k_{33}	k_{31}	k_{33}	k_{31}	k_{33}	k_{31}	k_{33}	k_{31}
1	0.3069	0.1973	0.0710	0.1918	0.0073	0.1908	0.0007	0.1907
2	0.3665	0.3862	0.0900	0.3761	0.0095	0.3741	0.0010	0.3739
5	0.5804	0.7443	0.2014	0.7555	0.0244	0.7604	0.0025	0.7609
10	0.7624	0.8908	0.4172	0.9141	0.0739	0.9250	0.0080	0.9263
20	0.8746	0.9507	0.6579	0.9682	0.2198	0.9788	0.0295	0.9803
50	0.9458	0.9814	0.8505	0.9895	0.5580	0.9953	0.1508	0.9966
100	0.9700	0.9908	0.9209	0.9950	0.7574	0.9981	0.3797	0.9990
200	0.9822	0.9954	0.9573	0.9976	0.8715	0.9991	0.6342	0.9997
500	0.9897	0.9981	0.9794	0.9990	0.9440	0.9997	0.8394	0.9999
1000	0.9927	0.9992	0.9869	0.9995	0.9689	0.9998	0.9151	0.9999

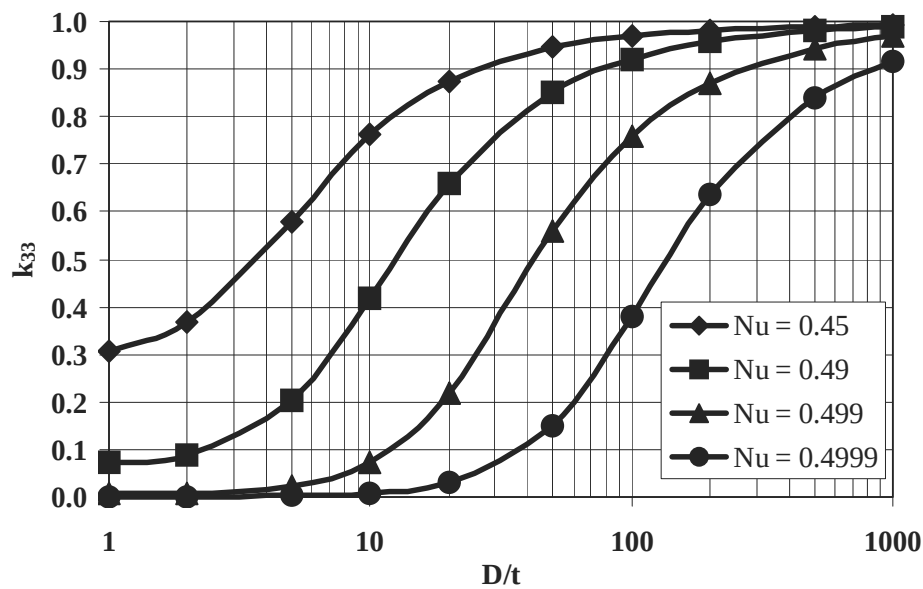


Figure 6.9 Plots of k_{33} vs. D/t for various values of Poisson's ratio.

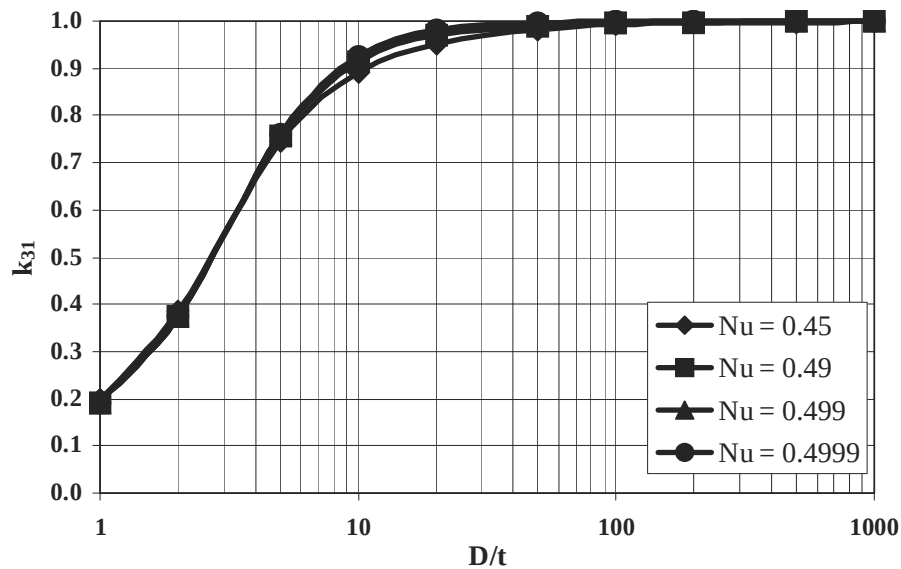


Figure 6.10 Plots of k_{31} vs. D/t for various values of Poisson's ratio.

being connected. An effective form of Hooke's law for the coarse adhesive-bond model is defined in Eq. (6.8), which assumes that the "3" direction is through the thickness of the bond, while the "1" and "2" directions are in the plane of the

bond. The analyst should be careful to orient the material coordinate system of the adhesive mesh such that the material description in Eq. (6.8) is aligned correctly with respect to the through-the-thickness direction. The rows and columns of the Hooke's law matrix may be rearranged to facilitate this.

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \tau_{12} \\ \tau_{23} \\ \tau_{31} \end{Bmatrix} = \begin{bmatrix} M & \frac{\nu M}{(1-\nu)} & \frac{k_{31}k_{33}\nu M}{(1-\nu)} & 0 & 0 & 0 \\ \frac{\nu M}{(1-\nu)} & M & \frac{k_{31}k_{33}\nu M}{(1-\nu)} & 0 & 0 & 0 \\ \frac{k_{31}k_{33}\nu M}{(1-\nu)} & \frac{k_{31}k_{33}\nu M}{(1-\nu)} & k_{33}M & 0 & 0 & 0 \\ 0 & 0 & 0 & G & 0 & 0 \\ 0 & 0 & 0 & 0 & G & 0 \\ 0 & 0 & 0 & 0 & 0 & G \end{bmatrix} \begin{Bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{33} \\ \gamma_{12} \\ \gamma_{23} \\ \gamma_{31} \end{Bmatrix}. \quad (6.8)$$

Alternatively, a beam element may be used to represent a hockey puck bond. Such an approach might be used for single-point optic models or for very coarse models where a single node is to represent the bonded area. The mesh of the adhesive bond would be a single-beam element whose axis is oriented in the through-the-thickness direction. The geometric properties to be used for such a model are identical to the usual calculation of beam properties, where the cross-sectional area is the plane-view area of the bond, and the Young's modulus E should be replaced by $k_{33}M$. In addition, an effective CTE must be employed for the beam model because the beam model lacks the elastic coupling to the strains in the plane of the bond. This effective CTE is not required in the solid element employment of the bond model as the coupling between the strains is present in such implementations. Effective CTEs for hockey puck bonds are shown in Table 6.3 for various D/t ratios and Poisson's ratios. A plot of this data is shown in Fig. 6.11.

Table 6.3 CTE correction factors for beam models of hockey puck bonds with various combinations of D/t ratio and Poisson's ratio.

D/t Ratio	$\nu = 0.4500$	$\nu = 0.4900$	$\nu = 0.4985$	$\nu = 0.4999$
1	1.3228	1.3686	1.3794	1.3812
2	1.6319	1.7227	1.7439	1.7475
5	2.2180	2.4517	2.5111	2.5147
10	2.4576	2.7564	2.8374	2.8518
20	2.5557	2.8605	2.9442	2.9599
50	2.6059	2.9013	2.9775	2.9826
100	2.6213	2.9120	2.9836	2.9973
200	2.6288	2.9169	2.9860	2.9985
500	2.6333	2.9197	2.9873	2.9914
1000	2.6351	2.9206	2.9876	2.9991

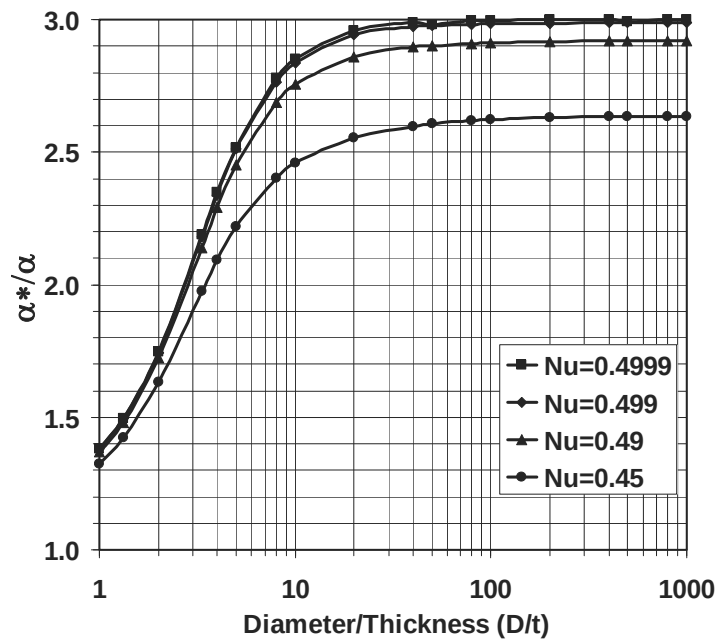


Figure 6.11 Plots of effective CTE vs. D/t in hockey puck bonds for various values of Poisson's ratio.

A third method of modeling a hockey puck bond with the effective material properties is to use six scalar elastic elements or springs. The six spring constants can be computed from

$$k_x = k_y = \frac{KGA}{t} \quad [6.9(a)]$$

$$k_z = \frac{k_{33}MA}{t} \quad [6.9(b)]$$

$$k_{\theta x} = k_{\theta y} = \frac{k_{33}MI}{t} \quad [6.9(c)]$$

$$k_{\theta z} = \frac{GJ}{t}, \quad [6.9(d)]$$

where $k_x = k_y$ are the in-plane shear stiffnesses of the bond, G is the shear modulus of the adhesive, A is the cross-sectional area of the bond, t is the thickness of the bond, K is the effective shear factor corresponding to the cross-section of the bond, k_z is the through-the-thickness stiffness of the bond, $k_{\theta x} = k_{\theta y}$ are the rotational stiffnesses of the bond about the axes in the plane of the bond, I is the bending moment of inertia of the cross-section of the bond, $k_{\theta z}$ is the torsional stiffness of the bond about the through-the-thickness direction, and J is the torsional constant of the cross-section of the bond.

6.1.3.2 Example: modeling of a hockey-puck-type bond

The bond in Fig. 6.8 is to be included in a finite element analysis of a mirror. The diameter of the bond is 8.0 cm, while its thickness is 1.3 mm. The Young's modulus and bulk modulus of the adhesive were measured to be 3.45 and 575 MPa, respectively.

We first compute the diameter-to-thickness ratio D/t as

$$\frac{D}{t} = \frac{80 \text{ mm}}{1.3 \text{ mm}} \approx 60. \quad (6.10)$$

The Poisson's ratio ν is computed from Eq. (6.5) as

$$\nu = \frac{1}{2} - \frac{E}{6B} = \frac{1}{2} - \frac{(3.45 \text{ MPa})}{6(575 \text{ MPa})} = 0.499, \quad (6.11)$$

and the shear modulus G is computed from Eq. (6.6) as

$$G = \frac{E}{2(1+\nu)} = \frac{(3.45 \text{ MPa})}{2[1+(0.499)]} = 1.15 \text{ MPa}. \quad (6.12)$$

The maximum modulus M given by Eq. (6.4) is calculated as

$$M = \frac{(1-\nu)E}{(1+\nu)(1-2\nu)} = \frac{(1-0.499)3.45 \times 10^6 \text{ MPa}}{(1+0.499)[1-2(0.499)]} = 5.765 \times 10^8 \text{ MPa}. \quad (6.13)$$

With Table 6.1 for $\nu = 0.499$ and $D/t = 60$, k_{33} can be interpolated as

$$k_{33} = \frac{60-50}{100-50}(0.7574-0.5580) + 0.5580 = 0.5979. \quad (6.14)$$

Similarly, k_{31} can be interpolated as

$$k_{31} = \frac{60-50}{100-50}(0.9981-0.9953) + 0.9953 = 0.9959. \quad (6.15)$$

Because the adhesive bonds are shaped by the spherical form of the back surface of the optic, the material coordinate system is chosen to be a spherical system centered at the pads' center of curvature. Because the through-the-thickness direction is in the radial direction of this spherical coordinate system, the terms in Eq. (6.8) must be reorganized to the following form:

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \tau_{12} \\ \tau_{23} \\ \tau_{31} \end{Bmatrix} = \begin{bmatrix} k_{33}M & \frac{k_{31}k_{33}vM}{(1-v)} & \frac{k_{31}k_{33}vM}{(1-v)} & 0 & 0 & 0 \\ \frac{k_{31}k_{33}vM}{(1-v)} & M & \frac{vM}{(1-v)} & 0 & 0 & 0 \\ \frac{k_{31}k_{33}vM}{(1-v)} & \frac{vM}{(1-v)} & M & 0 & 0 & 0 \\ 0 & 0 & 0 & G & 0 & 0 \\ 0 & 0 & 0 & 0 & G & 0 \\ 0 & 0 & 0 & 0 & 0 & G \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{12} \\ \gamma_{23} \\ \gamma_{31} \end{Bmatrix}. \quad (6.16)$$

Substitution of values gives a material matrix as follows:

$$\begin{bmatrix} 3.447 \times 10^8 & 3.419 \times 10^8 & 3.419 \times 10^8 & 0 & 0 & 0 \\ 3.419 \times 10^8 & 5.765 \times 10^8 & 5.742 \times 10^8 & 0 & 0 & 0 \\ 3.419 \times 10^8 & 5.742 \times 10^8 & 5.765 \times 10^8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.15 \times 10^6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.15 \times 10^6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.15 \times 10^6 \end{bmatrix} \text{MPa.} \quad (6.17)$$

The effective properties computed above for the RTV bond are compared to an epoxy bond of the same dimensions in Table 6.4. Although the Young's modulus, 3.45 MPa, of the RTV is approximately 1/7 that of the epoxy, 25.3 MPa, the effective stiffness of the RTV bond through the thickness, 1332.8

Table 6.4 Comparison of effective stiffnesses of RTV and epoxy bonds of the same geometry.

Properties	Units	RTV	Epoxy	Ep/RTV
Bond Thickness	m	0.0013	0.0013	1
Bond Diameter	m	0.08	0.08	1
Young's Modulus	MPa	3.45	25.3	7.33
Poisson's Ratio	---	0.499	0.43	0.86
Shear Modulus	MPa	1.15	8.85	7.69
Effective Through-the-Thickness Stiffness	MN/m	1332.8	264.6	0.20
Effective Shear Stiffness	MN/m	4.0	30.8	7.69
Effective Bending Stiffness	MN-m/rad	0.5331	0.1058	0.20
Effective Torsion Stiffness	MN-m/rad	0.0036	0.0274	7.69

MN/m, is approximately 5 times greater than that of the epoxy, 264.6 MN/m. The bond bending stiffness is derived from through-the-thickness compression, causing the RTV bond bending stiffness to be approximately 5 times greater as well. The shear and torsion strains of the bond do not involve volume change, and, therefore, their ratio follows that of the shear moduli of the two materials. This shows the significant effect of the high Poisson's ratio and constraining geometry on the effective stiffness of nearly incompressible bonds.

6.1.3.3 Effective properties for ring bonds

Fig. 6.12 shows an example of a ring bond. The form of the effective properties for this type of bond is shown in Eq. (6.18). While the “1” direction is in the radial direction through the thickness of the bond, the “2” and “3” directions are in the hoop and axial directions, respectively.

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \tau_{12} \\ \tau_{23} \\ \tau_{31} \end{Bmatrix} = \begin{bmatrix} k_{11}M & \frac{k_{12}k_{11}\nu M}{(1-\nu)} & \frac{k_{13}k_{11}\nu M}{(1-\nu)} & 0 & 0 & 0 \\ \frac{k_{12}k_{11}\nu M}{(1-\nu)} & k_{11}M & \frac{k_{13}k_{11}\nu M}{(1-\nu)} & 0 & 0 & 0 \\ \frac{k_{13}k_{11}\nu M}{(1-\nu)} & \frac{k_{13}k_{11}\nu M}{(1-\nu)} & k_{33}M & 0 & 0 & 0 \\ 0 & 0 & 0 & G & 0 & 0 \\ 0 & 0 & 0 & 0 & G & 0 \\ 0 & 0 & 0 & 0 & 0 & G \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{12} \\ \gamma_{23} \\ \gamma_{31} \end{Bmatrix}. \quad (6.18)$$

The correction factors k_{22} , k_{12} , k_{13} , and k_{33} are tabulated in Table 6.5 for various b/t ratios and Poisson's ratios. The effective properties for ring bonds are insensitive to the ratio of the radius of the ring bond to its thickness (R/t) for

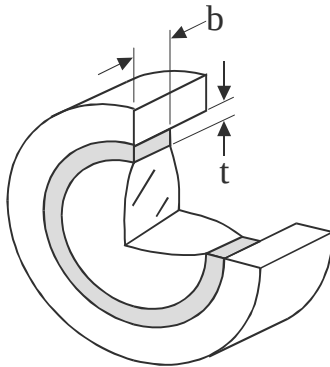


Figure 6.12 Example of a ring bond design with thickness t and width b .

Table 6.5 Correction factors for ring bonds with various combinations of b/t ratio and Poisson's ratio.

B/T RATIO	$\nu = 0.45$				$\nu = 0.49$			
	K_{11}	K_{12}	K_{13}	K_{33}	K_{11}	K_{12}	K_{13}	K_{33}
1	0.4036	0.6717	0.2704	0.1433	0.1018	0.6399	0.2652	0.0355
2	0.5101	0.7866	0.5257	0.3750	0.1484	0.7657	0.5219	0.1099
5	0.7521	0.9267	0.8372	0.8518	0.3665	0.9295	0.8560	0.4523
10	0.8760	0.9685	0.9301	0.9907	0.6295	0.9760	0.9510	0.8248
20	0.9383	0.9854	0.9675	0.9994	0.8126	0.9906	0.9808	0.9871
50	0.9756	0.9944	0.9876	0.9952	0.9252	0.9967	0.9933	1.0002
100	0.9883	0.9974	0.9941	0.9904	0.9628	0.9984	0.9968	0.9955
200	0.9954	0.9990	0.9977	0.9941	0.9820	0.9993	0.9985	0.9897
500	0.9991	0.9998	0.9996	0.9987	0.9949	0.9998	0.9996	0.9951
1000	0.9997	0.9999	0.9999	0.9996	0.9985	0.9999	0.9999	0.9985

B/T RATIO	$\nu = 0.499$				$\nu = 0.499$			
	K_{11}	K_{12}	K_{13}	K_{33}	K_{11}	K_{12}	K_{13}	K_{33}
1	0.0108	0.6328	0.2641	0.0037	0.0011	0.6321	0.2640	0.0004
2	0.0165	0.7611	0.5213	0.0123	0.0017	0.7607	0.5213	0.0012
5	0.0554	0.9316	0.8630	0.0703	0.0058	0.9319	0.8637	0.0074
10	0.1691	0.9803	0.9605	0.2386	0.0205	0.9809	0.9617	0.0292
20	0.4169	0.9944	0.9888	0.5922	0.0742	0.9950	0.9900	0.1086
50	0.7460	0.9986	0.9973	0.9616	0.3155	0.9991	0.9983	0.4580
100	0.8730	0.9994	0.9988	0.9999	0.6005	0.9997	0.9995	0.8299
200	0.9365	0.9997	0.9995	0.9994	0.7977	0.9999	0.9998	0.9883
500	0.9749	0.9999	0.9998	0.9899	0.9192	1.0000	0.9999	1.0001
1000	0.9891	1.0000	0.9999	0.9915	0.9597	1.0000	1.0000	0.9948

ratios above 10. Therefore, these effective properties may also be used for a very long straight bond in which the “1” direction is through the thickness of the bond, the “2” direction is in the long dimension, and the “3” direction is along the width of the bond.

6.2 Displacement Models of Flexures and Mounts

6.2.1 Classification of structures and mounts

6.2.1.1 Classification of structures

Consider the 2D planar truss structures with pinned joints shown in Fig. 6.13. These structures have pinned joints, so there are no moments at the individual nodes.

Unstable: In Fig. 6.13(a), the structure is an unstable 4-bar linkage and will not support the load shown. A static finite element solution will be mathematically singular due to the internal mechanism.

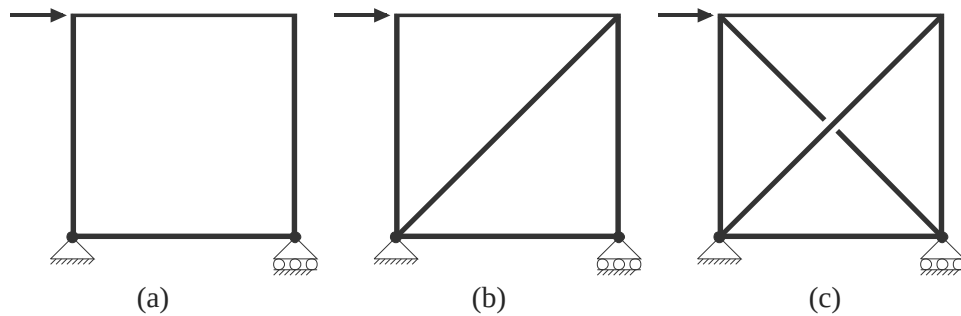


Figure 6.13 Classification of structures: (a) unstable, (b) statically determinate, and (c) statically indeterminate.

Statically Determinate: In Fig. 6.13(b), the eight unknowns are five internal member forces and three reactions. Summing forces in two directions at each of the four nodes yields eight equations to find the eight unknowns. This structure is described as statically determinate because it can be solved from a static summation of forces without knowledge of the elastic properties of the truss members. Thus, design changes of member areas have no impact on the force distribution. Furthermore, temperature changes of a statically determinate structure fabricated of mixed materials will not generate forces within the elements. If the top horizontal member represented a glass optic while the other members represented metal support structure components, the optic will be stress free during temperature changes, support motion, or member length imperfections.

Statically Indeterminate: In Fig. 6.13(c), an extra member with unknown force has been added, but the number of equilibrium equations remains at eight. Additional information regarding the elastic stiffness of each member, that is, member area, modulus of elasticity, and length, is required to find the static response of the system due to applied loads. The force distribution will change in all members if the cross-sectional area of one member changes. Temperature changes of a structure made of mixed materials will cause nonzero element forces. If the top horizontal member represents a glass optic while the other members represent metal support structure, the optic will be subjected to stress due to temperature changes, support motion, or member length imperfections.

6.2.1.2 Classification of mounts

Mounts can be classified in terms of how they are linked to their surroundings. All mounts can be grouped into one of the following types.

Unstable refers to mounting schemes that fail to react to at least one rigid-body motion of the mounted structure, as illustrated in Fig. 6.14(a). In a static finite

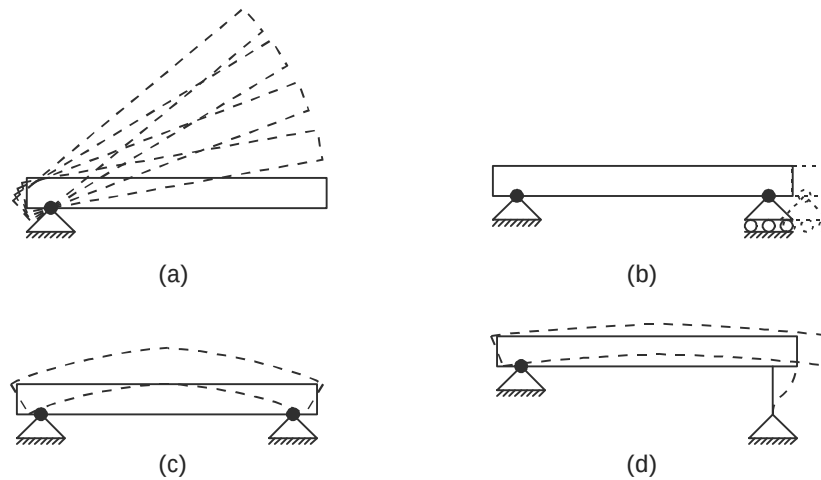


Figure 6.14 Classification of mounts: (a) unconstrained, (b) perfectly kinematic, (c) redundant, and (d) pseudo-kinematic.

element solution, the stiffness matrix representing an unstable structure is singular. Therefore, attempts to obtain a static solution of displacements to any applied loading will result in division by zero.

Kinematic or *statically determinate* refers to a mounting scheme that reacts to all rigid-body motions of the mounted structure with no redundancy, as illustrated in Fig. 6.14(b). The reactions to the structure at the kinematic points of contact may be determined without regard to the knowledge of the stiffness of the structure or of the surroundings to which the structure is mounted. A kinematically mounted structure is isolated from elastic deformations of its surroundings, although it may undergo rigid-body motion as its surroundings deform and move.

Redundant or *statically indeterminate* refers to a mounting scheme that elastically couples a structure to its surroundings, as illustrated in Fig. 6.14(c). Such a structure will elastically deform when its surroundings are elastically deformed, and such deformations are dependent on the stiffness of the structure and the surroundings.

Pseudo-kinematic is a term referring to the special case of weakly redundant mounting. Pseudo-kinematic mounts are attempts to approximate a kinematic mounting scheme, as illustrated in Fig. 6.14(d). The redundancies are minimized by designing flexures or other hardware that exhibit relatively large stiffness only in directions where kinematic constraints would be applied.

6.2.1.3 Mounts in 3D space

An optic must be constrained in six DOF in 3D space to

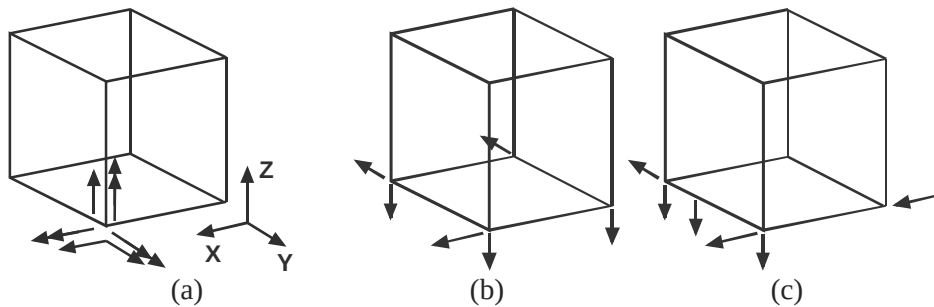


Figure 6.15 Various mount configurations: (a) kinematic constraint located at a single point, (b) kinematic constraint distributed over four points and (c) unstable constraint over four points.

prevent three translations and three rotations. There are many possibilities, but some are more desirable than others depending on the application. The cube shown in Fig. 6.15 is a simple structure useful for illustrating several key points concerning mounting in 3D space. Each cube shown has three DOF constrained. In Fig. 6.15(a), the displacements and three rotations are constrained at a single point. This is stable, but moment constraints are usually weak, and all forces are concentrated at a single point, causing high stresses. In Fig. 6.15(b), there are six translational constraints, three in the xy plane and three along the z axis. This arrangement creates a stable and comparatively stiff design. Because the locations have a relatively wide footprint, moment loads applied to the structure are balanced by couples with the smallest force components possible. In Fig. 6.15(c), there are also six translational constraints, three in the xy plane and three along the z axis. However, the arrangement in Fig. 6.15(c) is unstable in rotation about the y axis because the forces aligned along the z axis are colinear. Rotation about the z axis is also unstable because two forces in the x direction are colinear. Therefore, not all arrangements of six constraints provide a stable system.

6.2.2 Modeling of kinematic mounts

Although perfectly kinematic mounts are not achievable in practice, it is often useful to idealize a mounting interface as kinematic for an analysis. Such an approach allows an analyst to simplify the model used for an early design trade study or to bound the displacement prediction by eliminating the redundant mounting effects. Kinematic mounts are modeled with either constraints or rigid elements. Constraints are used if the surroundings are not included in the model, while rigid elements are employed when interfacing the models of two components.

Both methods of modeling kinematic mounts must be defined so that the directions they constrain or link are correctly represented. Fig. 6.16 shows three mounting schemes using the same set of three mounting point locations but using different constrained directions. Each of these kinematic mounts will behave differently; therefore, it is important to correctly represent the intended scheme.

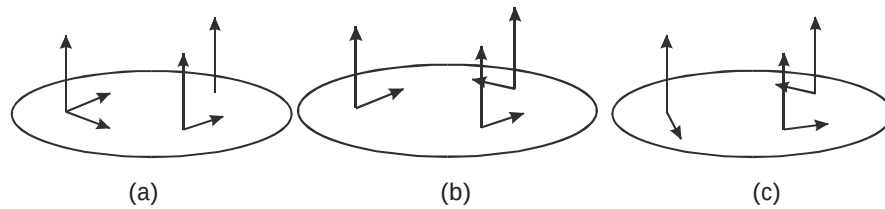


Figure 6.16 Example kinematic mounting schemes: (a) cone, groove, and flat, (b) three grooves, (c) three grooves in a modified configuration.

In Fig. 6.16(a), the mount has no plane of symmetry, even if the optic is axisymmetric. Thus, a temperature change in Fig. 6.16(a) will cause the optic to decenter. In Fig. 6.16(b), the mount has one plane of symmetry, whereas the mount in Fig. 6.16(c) has three planes of symmetry. The geometry in Fig. 6.16(c) is preferred because temperature changes cause no decenter of the optic. Each finite element code uses its own method of defining the direction in which constraints will act; thus, it is important that the analyst follow the method properly.

In addition to the defined directions of the kinematic constraints, the defined locations of the nodes to which the constraints are applied are equally important. Fig. 6.17 shows an optic mounted with kinematic mounts located at its midplane in one case and at its backplane in a second case. As the illustration shows, the locations of the kinematic-mount points affect the resulting deformed shape; therefore, they must be properly represented. If the construction of the model is such that finite element nodes are not defined where the kinematic mount points are located, then rigid elements may be used to link the mount points to the model, allowing the constraints to be applied in the proper location, as shown in Fig. 6.17(b).

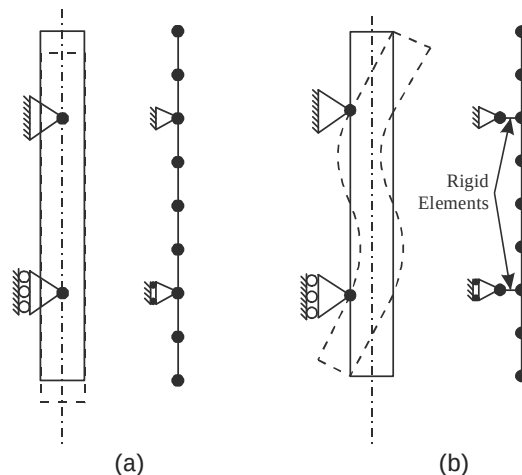


Figure 6.17 Effect of kinematic constraint location on a laterally loaded mirror: (a) kinematic constraints located on the neutral plane, and (b) kinematic constraints located off the neutral plane.

6.2.3 Modeling of flexure mounts

The flexure mounts may be modeled to include their redundant stiffness characteristics if the goal of an analysis warrants such detail. Inclusion of such stiffnesses will show, for example, the transmitted moments of a flexure-mounted optic whose metering structure undergoes elastic deformation.

6.2.3.1 Arrangement of strut supports

A common approach to mount larger optics and assemblies is the use of support struts such as those shown in Fig. 6.18. Support struts generally attempt to approximate a kinematic mount in combination with other struts by providing high axial stiffness with minimum bending and shear stiffness. Strut supports may be pinned end beams, which are true one-DOF stiffness elements, but they suffer from gapping and friction at the ball joints. More commonly, strut supports have flexures at either end that simulate ball joints by reducing the moment stiffness. The flexures do not have the gapping or friction of ball joints but can transmit some moment loads to the optic. In this section, the discussion applies to either strut concept.

Fig. 6.18 shows an optical component kinematically mounted on support struts. Since two forces that intersect at a point can be resolved into any other two forces at the same point, the strut forces F_A and F_B shown in the figure can be resolved in F_x , a force parallel to optic midplane, and F_{z1} , a force normal to the midplane. The force F_x is offset from the midplane, causing a moment that will bend the optic. To minimize distortion of the optic, the strut line-of-action should intersect at the optic midplane, as in Fig. 6.19(a). As shown by the corresponding free-body diagram in Fig. 6.19(b), this mounting configuration causes no moment on the optic for laterally applied loads. The imaginary intersection point is called the “virtual intersection” of the struts. The configuration in Fig. 6.19(c) is often used when strength requirements drive a design that uses more mount points on the optic than is required for equilibrium. This is generally done to spread the load out over as many mount points as possible in order to reduce the stress levels in the optic and minimize self-weight deflections. However, such configurations use virtual intersections that are not at the midplane of the optic and will, therefore, exhibit increased surface figure errors due to moments induced by lateral loads, as shown by the corresponding free-body diagram in Fig. 6.19(d).

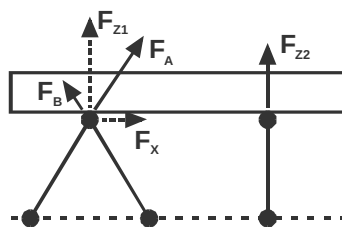


Figure 6.18 Strut support in 2D space.

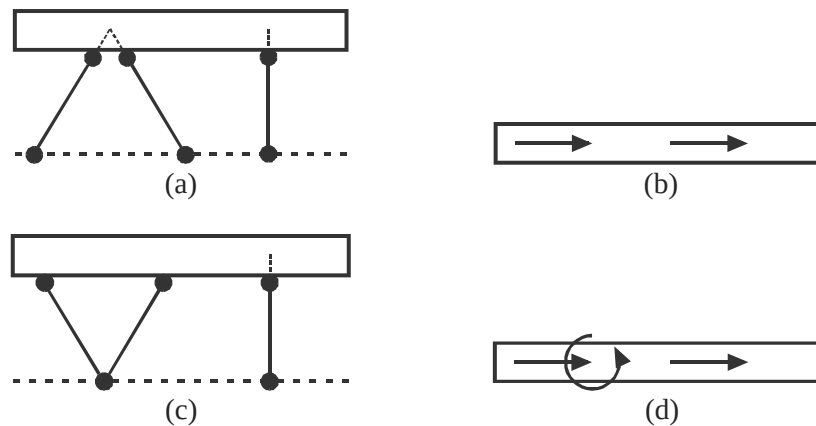


Figure 6.19 Stable strut configurations in 2D space and the associated free-body diagrams of the mounted optic.

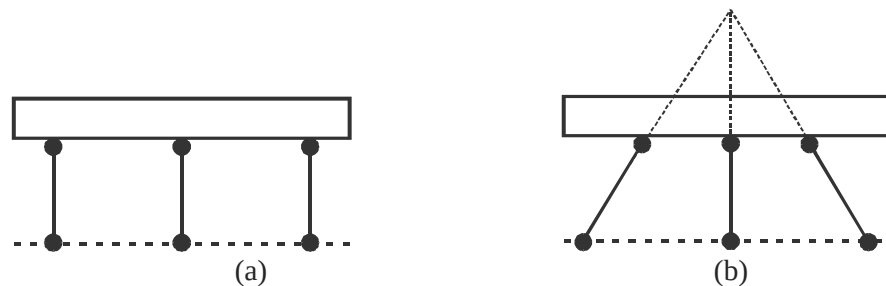


Figure 6.20 Unstable strut configurations in 2D space.

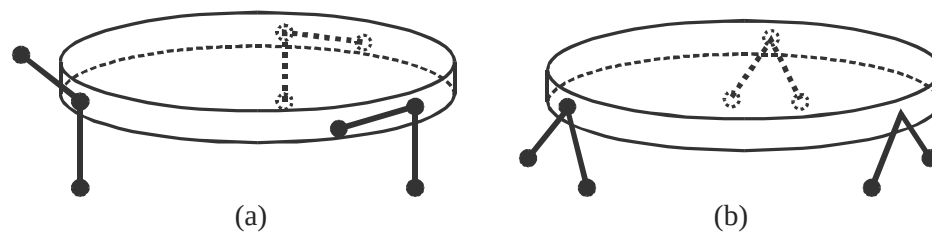


Figure 6.21 Strut configurations in 3D space.

Fig. 6.20 shows two strut configurations that are unstable: in Fig. 6.20(a), the optic can freely translate laterally in a four-bar linkage mode; in Fig. 6.20(b), the optic can rotate about the strut virtual intersection.

In 3D space, the two strut configurations in Fig. 6.21 are equivalent from the optics point of view. Two forces that intersect at a point can be resolved into any other two forces. The diagonal struts in Fig. 6.21(b) provide Z and Θ constraint just as in Fig. 6.21(a).

The most common configuration of support struts for large mirrors is shown in Fig. 6.22. Three bipod struts are located at 0.65 to 0.70 of the outside radius of

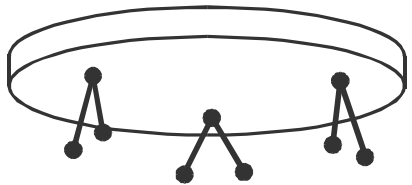


Figure 6.22 Typical strut configuration for large mirrors.

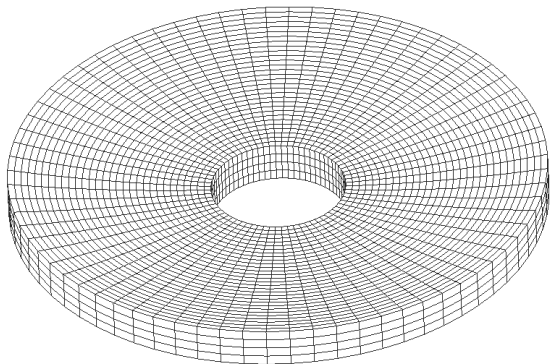


Figure 6.23 Mirror model with central hole.

Table 6.6 Mirror design parameters.

Solid Mirror Parameters Material = Fused Silica Outside Diameter = 1.0m Inside Diameter = 0.25m Overall height = 0.075m Radius of Curvature = 3m	Lightweight Mirror Parameters Same geometry as solid mirror Faceplate thickness = 0.002m Core thickness = 0.00133m Cell size B = 0.133m Solidity ratio α = 0.010

the mirror with the strut virtual intersection at the mirror’s CG plane. Bipod spread angles typically vary from 60 to 90 deg, depending on the ratio of axial to lateral loads or the stiffness required.

6.2.3.2 Optimum radial location of mounts

For large mirrors mounted on three bipod flexures, the optimum radial location can be found to minimize the gravity-induced surface RMS error. Consider the finite element model of a mirror shown in Fig. 6.23 mounted on a three-point kinematic mount at a variable radial location. Two mirror design concepts were considered: a solid mirror and a lightweight mirror with the same overall geometry. Mount location optimization of both mirror design concepts were performed with and without a central hole resulting in a mount location design trade for four mirror designs. Defining parameters of each mirror design are given in Table 6.6.

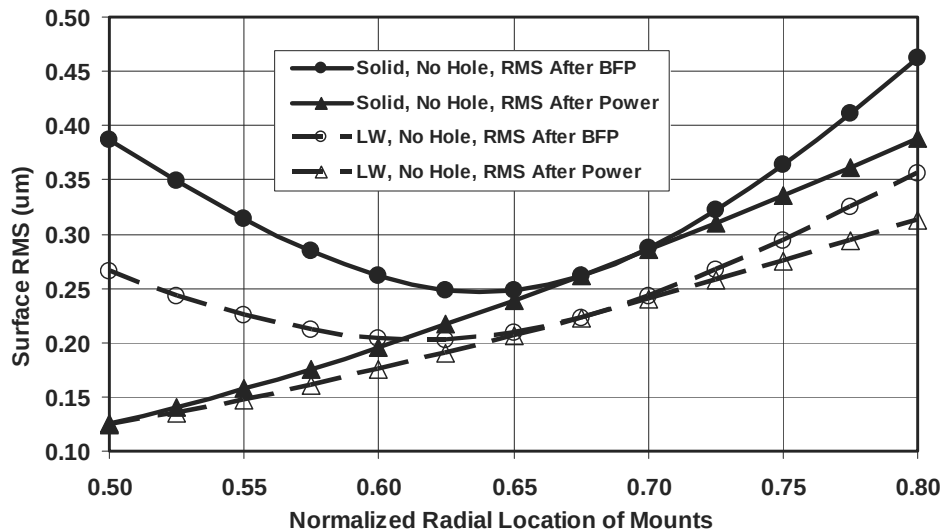


Figure 6.24 Surface RMS error versus normalized radial mount location for a solid mirror and a lightweight mirror with no central hole.

A gravity load in the axial direction is applied to all four mirror models for varying radial mount locations. Curves of surface RMS error of the resulting surface sag displacement versus normalized radial mount location are given in Fig. 6.24. The normalized radial mount location is expressed as the ratio of the radial mount location to the mirror radius, $R_0 = 0.5$ m. Fig. 6.24 shows the surface RMS error vs. normalized radial mount location for the two mirrors, with no central hole loaded by gravity. Curves of surface RMS error after the best-fit plane is removed and surface RMS error after best-fit plane and power are removed are presented for each mirror with no central hole.

From the curves in Fig. 6.24 it can be seen that a radial mount location of $0.60R_0$ to $0.65R_0$ provides the minimum surface RMS error after the best-fit plane is removed. It can also be seen that a radial mount location of $0.67R_0$ to $0.70R_0$ nulls the power term since the curves with power included and the curves without power included intersect in this region.

Fig. 6.25 shows the surface RMS error after best-fit plane is removed vs. the normalized radial mount location for all four mirrors loaded by gravity. The curves illustrate the effect of the central hole on both the solid mirror design and the lightweight mirror design. With a central hole, the minimum surface RMS error after best-fit plane removed shifts toward the outside. For the example presented here, the minimum surface RMS error is exhibited at radial mount locations of $0.67R_0$ and $0.65R_0$ for the solid and lightweight mirrors, respectively, with a central hole present in each design. This may be compared to the optimum radial mount locations of $0.63R_0$ and $0.62R_0$ for the solid and lightweight mirrors, respectively, with no central hole present in the design.

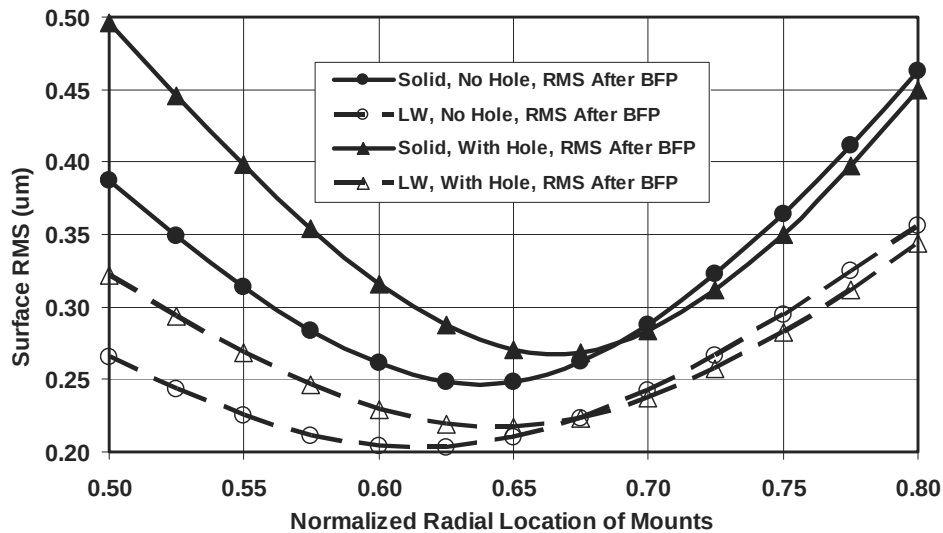


Figure 6.25 Surface RMS error after best-fit plane is removed vs. normalized radial mount location for a solid mirror and lightweight mirror with a central hole.

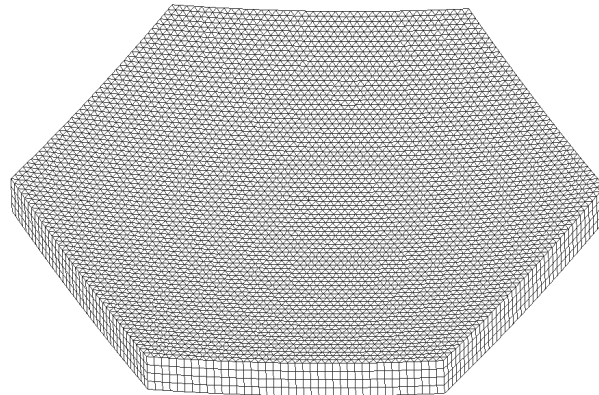


Figure 6.26 Finite element model of a hexagonal lightweight mirror.

To examine the effect of the plan view shape of a mirror design on the optimum radial mount location, consider a hexagonal mirror design whose finite element model is shown in Fig. 6.26. The mirror has a center-to-point dimension of $R_0 = 0.5$ m, and the same core and faceplate design as the circular mirror design described above. A similar mount location trade as those discussed above was performed for this hexagonal mirror, and a comparison of the results with the circular lightweighted mirror with no hole discussed above is given in Fig. 6.27. The radial mount location yielding the minimum surface RMS error after best-fit plane is removed is $0.55R_0$, where R_0 is the radius from the center to a point on the plane-view hexagon.

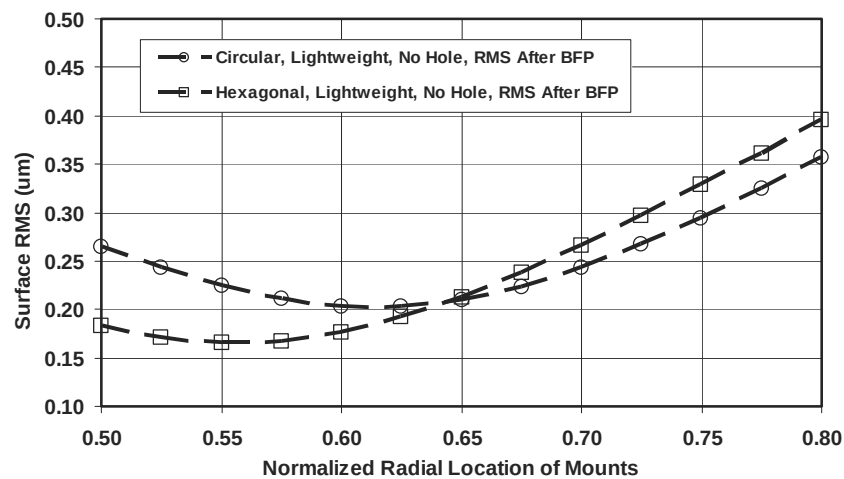


Figure 6.27 Comparison of radial mount location trade studies for hexagonal and circularly shaped lightweight mirrors.

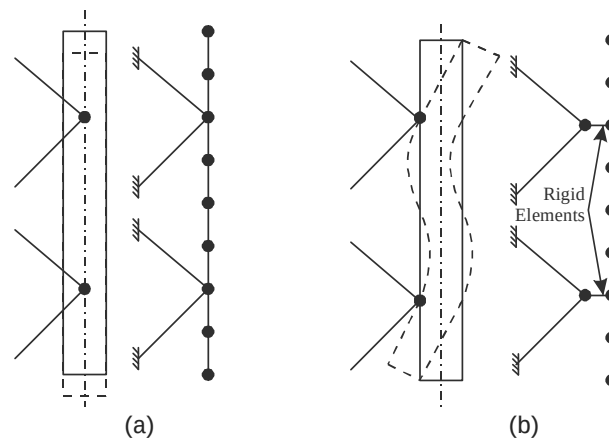


Figure 6.28 Effect of bipod flexure strut-intersection point (SIP) location on a laterally loaded mirror: (a) SIP located on the neutral plane, and (b) SIP located off the neutral plane.

6.2.3.3 Modeling of beam flexures

Beam flexures exhibit significant stiffness only along their axes. The bending and transverse shear stiffnesses are relatively small. In situations where beam flexures are used in pairs, as shown in Fig. 6.28, the location of the strut intersection points (SIP) are very important to the behavior of the mounted optic. In addition, the orientation of the bipod pair can be an important factor. The importance of properly modeling the location of the SIP and the orientation of the bipod pair is analogous to the importance of properly modeling the locations and reaction directions of kinematic mounts discussed above.

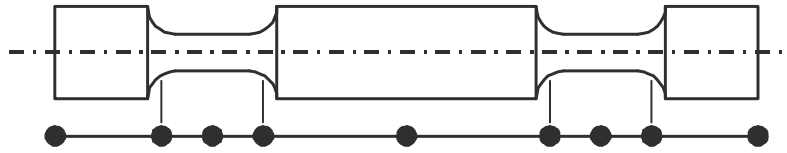


Figure 6.29 Example beam-flexure design and corresponding finite element mesh. The modeled active flexure lengths include half of each fillet length.

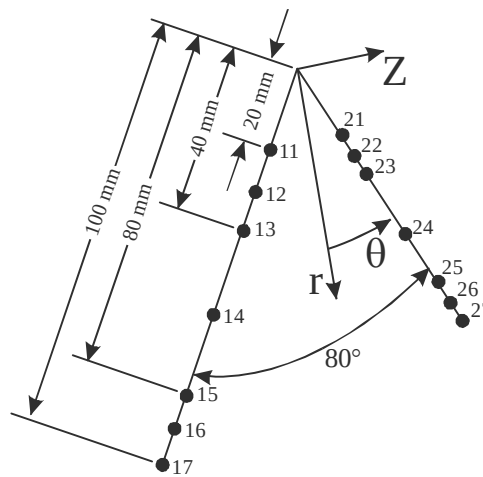


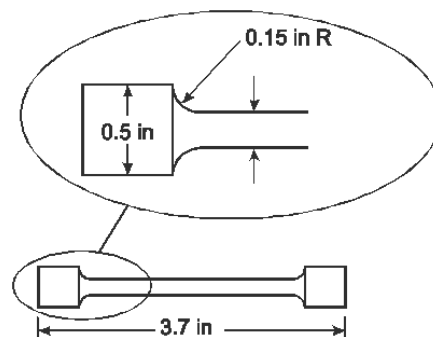
Figure 6.30 Finite element model of beam flexure bipod showing the defining coordinate system to allow easy changes to the model. See Table 6.7 for a corresponding list of coordinate locations.

In order to correctly represent the bending and transverse shear stiffnesses of beam flexures it is important to choose the proper active flexure length to use in the model. An active flexure length is any length of reduced thickness or diameter as shown in Fig. 6.29. The length of the active flexures in the hardware is often not well defined due to the fillets at each end. Therefore, effective lengths must be chosen to represent the active flexures and their fillets. Figure 6.29 shows an example beam-flexure design and the corresponding finite element beam model. Proper representation of the active flexure length in most designs can be achieved by including half of each of the fillet lengths in the flexure portions of the mesh. The nominal beam properties of the active flexure are then assigned to this effective length.

It is helpful to organize the flexure models so they can be easily modified in a text editor when performing design-trade studies on beam-flexure bipods. If the finite element nodes of a beam-flexure bipod are defined in a cylindrical coordinate system as shown in Fig. 6.30, then the analyst can change the dimensions along the length of the flexure by editing the radial coordinate locations. Furthermore, the spread angle of the bipod flexures can be altered

Table 6.7 Cylindrical coordinates of flexure bipod model nodes shown in Fig. 6.30.

<u>NODE ID</u>	<u>R (MM)</u>	<u>θ (°)</u>	<u>Z (MM)</u>
11	20.0	-40.0	0.0
12	30.0	-40.0	0.0
13	40.0	-40.0	0.0
14	60.0	-40.0	0.0
15	80.0	-40.0	0.0
16	90.0	-40.0	0.0
17	100.0	-40.0	0.0
21	20.0	40.0	0.0
22	30.0	40.0	0.0
23	40.0	40.0	0.0
23	60.0	40.0	0.0
25	80.0	40.0	0.0
26	90.0	40.0	0.0
27	100.0	40.0	0.0

**Figure 6.31** Dimensions of example beam-flexure design.

simply by changing the azimuthal coordinate locations. Also, the lean angle of the bipod-pair plane is defined solely by the orientation of the z axis of the cylindrical coordinate system. Table 6.7 shows the coordinate locations of the numbered nodes in Fig. 6.30 as an example. Changes to this model description are more easily made in a text editor with column select-and-replace features than in a graphical preprocessor.

6.2.3.4 Example: modeling of bipod flexures

Finite element models of the beam flexures used to mount a mirror are to be included in an analysis that predicts the optical-surface deformation due to enforced motion of the flexure ends. Such motion may be associated with thermoelastic expansion of the metering structure to which the flexures are bonded, or due to locked-in strain during assembly of the flexures to the metering structure. The flexures, whose dimensions are shown in Fig. 6.31, are to be



Figure 6.32 Finite element models of example beam flexures: (a) beam model and (b) solid model.

Table 6.8 Comparison of surface deformation results computed with different flexure models.

	BAR-ELEMENT FLEXURES			SOLID-ELEMENT FLEXURES		
	RESIDUAL RESIDUAL			RESIDUAL RESIDUAL		
	MAG. (NM)	RMS (NM)	P-V (NM)	MAG. (NM)	RMS (NM)	P-V (NM)
Input		1065.8	609.2		1056.6	598.8
Bias	1052.9	149.5	609.2	1043.9	146.9	598.8
Power	239.4	59.5	324.9	235.3	58.5	319.3
Pri Trefoil	151.3	25.9	149.8	148.8	25.4	147.2
Pri Spherical	-14.8	24.7	149.8	-14.5	24.3	147.2
Sec Trefoil	32.7	23.1	146.6	32.2	22.7	144.1
Sec Spherical	-29.2	20.4	123.9	-28.7	20.1	121.8
Pri Hexafoil	34.5	18.2	113.3	33.9	17.9	111.4
Ter Trefoil	58.7	10.0	71.8	57.7	9.8	70.6
Ter Spherical	5.2	9.8	70.9	5.1	9.7	69.7
Sec Hexafoil	11.7	9.4	66.9	11.5	9.2	65.8

fabricated of titanium; the mirror is identical to that used in Example 5.1.4.4. The enforced displacements are 0.01 in and are applied to each flexure base in the direction normal to the plane defined by the flexure bipod. The flexures are modeled with solid elements in one analysis and with beam elements in another analysis to provide a means of comparing the two representations of the flexures.

The finite element meshes of the two model types are shown in Fig. 6.32. While the bar-element model includes 26 nodes and 24 elements per bipod, the solid-element model contains 118,530 nodes and 115,200 elements per bipod. A comparison of the results from each analysis is given in Table 6.8.

Notice that the results correlate very well, which illustrates that the bar-element model is equally as capable of describing the stiffness of the bipod flexures as the solid-element model. This excellent correlation results from the fact that the assumptions made in using the bar element models to represent the bipod flexures were very valid assumptions for this problem. Furthermore, the bar element model provides a more-effective tool for predicting certain results,

such as the moments transmitted into the mirror. However, the reader should not dismiss the use of high-fidelity models when they are necessary. The key point to be conveyed by this example is that by understanding the capabilities of each modeling method and by anticipating the mechanical behavior of the system to be analyzed, the most cost-effective approach can be chosen to accomplish the goals of an analysis.

6.2.3.5 Design issues with bipod flexures

The following example will be used to show design issues encountered with the development of the design of bipod flexures. In this example a mirror is mounted on three bipod flexures located at 0.65R with geometry details given in Table 6.9. A plot of the finite element model associated with the design is shown in Fig. 6.33.

Three load cases were considered as follows:

- 1-g load in lateral direction (perpendicular to optical axis)
- 1-g load parallel to optical axis
- 10 °C isothermal temperature increase

Table 6.9 Dimensions of lightweight mirror and mount design.

Solid mirror: diameter = 40", thickness = 4"

Fused silica, weight = 396 lb = 180 kg

Lightweight mirror: diameter = 40", height = 4"

Faceplate and core thickness = 0.050"

Cell size = $B = 3.33"$, solidity ratio = 0.015

Fused silica, weight = 22 lb = 10 kg

Bipod struts: length = 5", spread angle = 90 deg

Titanium, diameter = varied

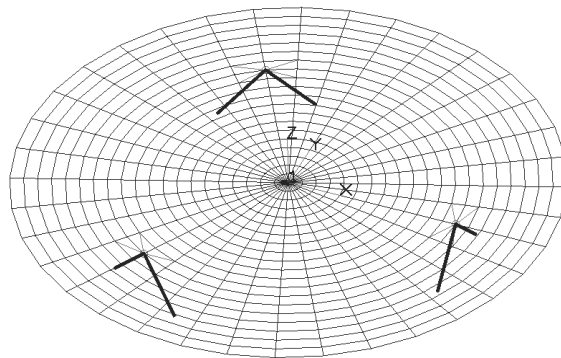


Figure 6.33 Mirror mounted on bipod flexures.

Analyses with the above load cases were run for various values of the strut flexure parameters, D , $D1$, and $D2$, as labeled in Fig. 6.34. In each analysis the surface RMS error with best-fit plane removed was calculated and expressed in HeNe waves. In addition, the buckling loads and stresses in the bipod struts were calculated to determine the maximum launch loads that each design can withstand. To determine allowable launch loads for each flexure design factors of safety were used for the buckling and stress predictions. For the buckling load limit a factor-of-safety of 4.0 was used. For stress in the flexure, a factor-of-safety of 2.0 on ultimate failure was used with a stress concentration factor of 2.0 for the strut fillet.

The results of the flexure design trade study with the solid mirror design and constant diameter strut is shown in Table 6.10 while the trade study results with the lightweight mirror and the same flexure design concept are shown in Table 6.11.

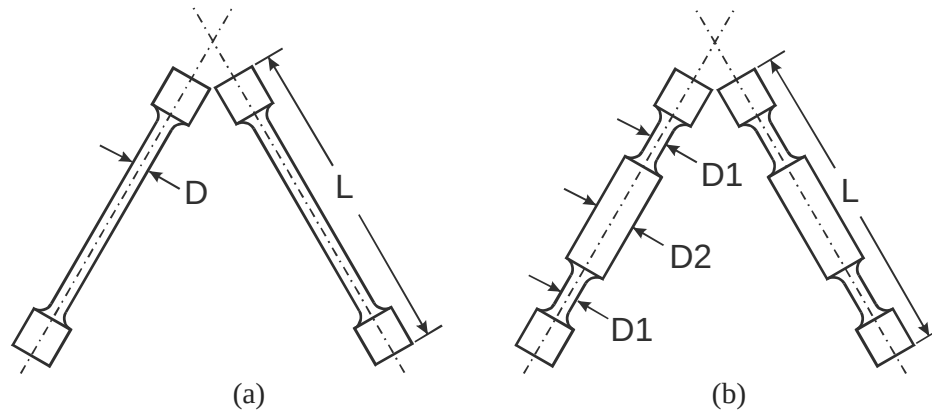


Figure 6.34 Bipod flexures of (a) constant diameter and (b) varied diameter.

Table 6.10 Trade study results for solid mirror design with constant diameter strut, as shown in Fig. 6.34(a).

D (IN)	SURFACE RMS ERROR (HeNe WAVES)			CRITICAL G LOAD FACTOR			
	1 G LATERAL	1 G AXIAL	+10 °C ISO- THERMAL	BUCKLE 1 G LATERAL	BUCKLE 1 G AXIAL	STRESS 1 G LATERAL	STRESS 1 G AXIAL
0.1	0.0002	0.2212	0.0000	0.20	0.34	1.8	3.2
0.2	0.0010	0.2212	0.0005	3.13	5.43	6.9	12.0
0.3	0.0022	0.2212	0.0024	15.8	27.0	14.8	25.4
0.4	0.0041	0.2212	0.0074	49.0	84.0	24.9	42.6
0.5	0.0066	0.2212	0.0179	118	200	36.9	62.5
0.6	0.0099	0.2212	0.0368	241	404	50.3	84.4

Table 6.11 Trade-study results for lightweight mirror design with constant diameter strut, as shown in Fig. 6.34(a).

D (IN)	SURFACE RMS ERROR (HeNe WAVES)			CRITICAL G LOAD FACTOR			
	1 G LATERAL	1 G AXIAL	+10 °C ISO- THERMAL	BUCKLE 1 G LATERAL	BUCKLE 1 G AXIAL	STRESS 1 G LATERAL	STRESS 1 G AXIAL
0.1	0.0001	0.2023	0.0004	3.6	6.3	33	57
0.2	0.0006	0.2023	0.0067	57.8	99.5	124	209
0.3	0.0013	0.2023	0.0339	288	495	261	435
0.4	0.0024	0.2022	0.1059	893	1523	435	714
0.5	0.0040	0.2022	0.2535	2130	3587	635	1026
0.6	0.0059	0.2021	0.5097	4300	7108	851	1333

Table 6.12 Trade-study results for lightweight mirror design with double-necked flexure design, as shown in Fig. 6.34(b).

D1 (IN)	D2 (IN)	SURFACE RMS ERROR (HeNe WAVES)			CRITICAL G LOAD FACTOR			
		1 G LATERAL	1 G AXIAL	+10 °C ISOTHERMAL	BUCKLE 1 G LATERAL	BUCKLE 1 G AXIAL	STRESS 1 G LATERAL	STRESS 1 G AXIAL
0.10	0.10	0.0001	0.2023	0.0004	3.6	6.3	33	57
0.20	0.20	0.0006	0.2023	0.0067	57.8	99.5	124	209
0.12	0.30	0.000004	0.2023	0.0011	47	73	47	78

Using a typical mass-acceleration curve for launch loads, a mirror of 180 kg would need to survive launch loads of 10–15 g. As indicated in Table 6.11, a strut diameter of 0.3 inches would be needed to meet strength and buckling requirements. This strut design would yield a surface RMS error of 0.0022 HeNe waves for the lateral loading and 0.0024 HeNe waves for the isothermal load.

According to the same mass-acceleration curve used for the analysis of the solid mirror, the lightweight mirror would experience launch accelerations of 35–40 g. As indicated in Table 6.11, a uniform strut diameter of 0.2 in. would be needed to meet strength and buckling requirements. This more flexible mirror would exhibit a surface RMS error of 0.0067 HeNe waves for the isothermal load. The results of a trade study similar to those discussed above but using the double-necked flexure design illustrated in Fig. 6.34(b) are shown in Table 6.12.

Results shown in Table 6.12 indicate that the double-necked-down flexure design can be used to improve optical performance. With a neckdown diameter of 0.12 inches to minimize moments and a central diameter of 0.30 inches to minimize buckling, the load-induced surface-figure errors decrease compared to the design using the constant-diameter flexure design discussed above. The surface RMS error induced by the 1-g lateral load decreases from 0.0006 HeNe

Table 6.13 Functional dependence of several mount characteristics on flexure diameter (D) and length (L).

MOUNT CHARACTERISTIC	FUNCTIONAL DEPENDENCE ON D AND L
Axial stiffness	D^2/L
Axial stress	$1/D^2$
Bending stiffness	D^4/L^3
Buckling load	D^4/L^2

Table 6.14 Desirable changes of flexure parameters for key performance metrics.

REQUIREMENT	D	L
Natural frequency	Larger	Smaller
Strength	Larger	Smaller
Buckling load	Larger	Smaller
Optical performance	Smaller	Larger

waves to less than 0.0001 HeNe waves, while surface RMS error induced by the 10 °C isothermal surface RMS drops from 0.0067 HeNe waves to 0.0011 HeNe waves.

The primary stiffness characteristic of the bipod flexures provides statically determinate support to the mirror through the axial stiffness of each flexure. This component of the mount stiffness is the desirable component as increasing the axial stiffness of the flexures improves the natural frequency, strength, and buckling performance at no expense to the optical performance. The secondary stiffness characteristic of the bipod flexures is the local bending stiffness, which causes moment loads to be imparted into the mirror. This secondary stiffness causes an impact on optical performance as the bending stiffness of the flexures are varied. The design trade discussed above results from the fact that the axial stiffness and bending stiffness of the rod flexures are both affected by the flexure diameter. Table 6.13 shows the functional dependence of several key mount characteristics on the flexure diameter D and length L . Table 6.14 shows the desirable changes of the same flexure parameters for several key performance metrics.

It should be noted that to realize the most benefit from a trade study such as that discussed above, the design of the flexures must be combined with the design of the mirror. For example, a stiffer mirror can resist larger bending moments imparted by the flexures, while the increased mass of such a heavier mirror also requires thicker flexures to satisfy strength, buckling, and natural frequency requirements. The coupled nature of such design trades may benefit from the use of automated design optimization techniques, such as those discussed in Chapter 11, for a broad and thorough evaluation of the design space.

6.2.3.6 Modeling of blade flexures

Blade flexures such as those shown in Fig. 6.35 are often not pseudo-kinematic. Redundant loads in the form of moments about the displayed x and z axes can be significant. They are best suited for situations where such redundant loads are not likely to be generated. Proper modeling of these flexures, however, should include such redundant stiffnesses.

Bar elements can be used to model blade flexures in some cases but should usually be limited to design trade study analyses. Bar elements give the analyst the advantage of easily verifying the absence of redundant loads by requesting the forces in the bar elements representing the flexure. However, while a bar element mesh may provide a first-order representation of the flexure stiffness, the bar stresses should not be considered accurate. Plate- and shell-element meshes of blade models will more correctly represent both the stiffness and stresses for final verification analyses.

If bar-element meshes are to be used to represent blade flexures, the analyst must be careful to calculate the properties correctly. If the flexures are sections of a cylindrical shell as shown in Fig. 6.36(a), the section properties of the hardware illustrated in Fig. 6.36(b) may not correctly represent the bending stiffness about the y axis. The curved geometry of the flexure cross-section can often add significant bending stiffness to the flexure. Eqs. [6.19(a)–(e)] give expressions developed by the authors except where noted for the beam properties of a flexure,

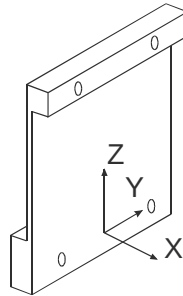


Figure 6.35 Example blade flexure.

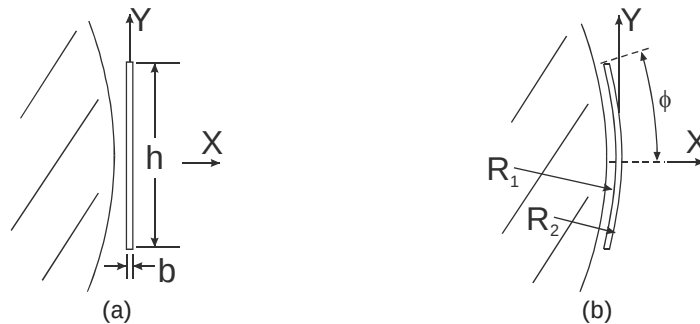


Figure 6.36 Two variations of blade flexures that have different bending stiffnesses about the y axis: (a) curved blade and (b) flat blade.

such as that shown in Fig. 6.36(b):

$$A = \phi(R_2^2 - R_1^2), \quad [6.19(a)]$$

$$C = \frac{2 \sin(\phi)}{3} \frac{(R_2^3 - R_1^3)}{\phi(R_2^2 - R_1^2)}, \quad [6.19(b)]$$

$$I_{XX} = \frac{1}{4}(R_2^4 - R_1^4) \left[\phi - \frac{1}{2} \sin(2\phi) \right], \quad [6.19(c)]$$

$$I_{YY} = \frac{1}{4}(R_2^4 - R_1^4) \left[\phi + \frac{1}{2} \sin(2\phi) \right] - \left[\frac{4 \sin^2(\phi)}{9} \frac{(R_2^3 - R_1^3)^2}{\phi(R_2^2 - R_1^2)} \right], \quad [6.19(d)]$$

$$J \approx \frac{\phi}{6}(R_1 + R_2)(R_1 - R_2)^3, \quad [6.19(e)]^6$$

where A is the cross-sectional area, ϕ , R_1 , and R_2 are as defined in Fig. 6.36(b), C is the distance from the center of curvature of the flexure to the centroid of the flexure cross-section, I_{XX} and I_{YY} are the moments of inertia at the centroid about the x and y axes, respectively, and J is the torsional constant. It should be noted that the expression for the torsional constant J is approximate based on an assumption that the thickness of the flexure is much smaller than the nominal radius of curvature. In addition, an expression for the location of the shear center relative to the centroid is not given.

6.3 Modeling of Test Supports

The purpose of performing a test-support deformation analysis is often to assess the surface-error contribution due to fabricating an optic to a desired prescription while in a test support that does not adequately represent the optic's in-use support. This error contribution, however, is as much a function of the optic in its in-use configuration as it is a function of the optic in its test support. Fig. 6.37 illustrates an optic that is tested on an air bag during the figuring process and subsequently supported in operation using an inclined configuration on its mounts. The error contribution of interest is the difference between the deformed optical surfaces of these two states. Since a linear finite element analysis assumes that the model begins in a stress-free and strain-free state, the deformation analysis of the optic in each state is the deformation change relative to a perfectly figured optic floating in a zero-gravity environment. To obtain the change in surface figure between two deformed states, a node-by-node difference in the finite element displacement results must be generated before deformed surface characterization is performed. Most finite element codes allow users to accomplish such a difference operation within the finite element analysis. However, a simple program or spreadsheet application can be used to difference the results of two analysis cases.

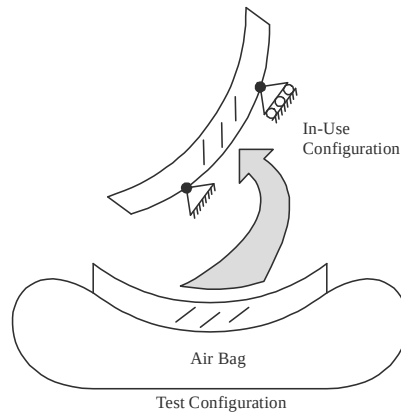


Figure 6.37 The optic is figured to the environment in which it is tested, and it can display different figures in its operational environments.

A test-support deformation analysis may be important if surface figuring methods such as ion figuring or small tool polishing are employed. The inverse of the shift in surface figure from the test-support configuration to the in-use configuration can be fabricated into the surface of the optic, thereby lessening the effects caused by testing the optic in an environment different from the in-use environment. This process is accomplished by generating an analytically computed prediction representing the deformation change caused by the test support relative to the operational configuration and adding this array to the interferogram results of each test measurement performed during fabrication of the optic. As each figuring pass is performed, the optical figure will converge to the desired prescription minus the anticipated deformation change. The surface figure error contribution associated with going from the test state to the in-use state would then be the error with which the analytical prediction was made.

In addition, various optical testing procedures require limits on the deviation of the optical surface from its intended shape. Such requirements may impose restrictions on how the test support should be designed to adequately support the optic or optical system so that accurate test results can be obtained. Therefore, analysis prediction of how optical systems deform in their test supports can be very important.

6.3.1 Modeling of air bags

Air bags are commonly used to simulate a 0-g environment during an optical test. Methods of modeling air bags stem from the fact that the pressure inside the air bag is either assumed constant or is a function of the hydraulic head h , as illustrated in Fig. 6.38. Therefore, for an axisymmetric optic supported by an air bag, the air bag can be represented by a uniform pressure applied normal to the supported face of the optic. This method, however, assumes that test engineers have inflated the air bag such that tangency is achieved at all points around the

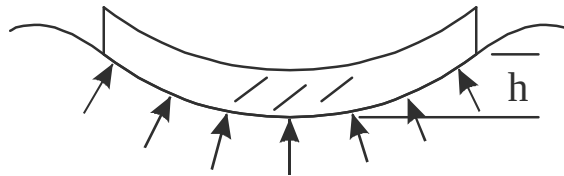


Figure 6.38 An air bag can be modeled as a representative pressure that reacts to the weight of the optic.

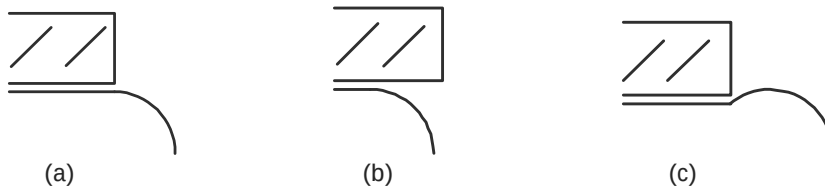


Figure 6.39 Three edge conditions of an air bag support: (a) tangency, (b) overinflated, and (c) underinflated.

edge of the optic, as shown in Fig. 6.39(a). If the air bag is underinflated or overinflated, then tangency will not be achieved, as shown in Figs. 6.39(b) and 6.39(c). Lack of tangency at any edge of the optic will result in edge loading dependent on the degree of nontangency. If the optic has a center hole, then a properly sized weight can be placed in the center hole so that the air bag is forced to become tangent at both the inner and outer edges. Therefore, the analyst is encouraged to communicate with test engineers who are responsible for the design and use of the test support hardware in order to understand what effects may need to be modeled.

If tangency can be assumed everywhere, the method for computing the proper pressure to apply is as follows:

- ◁1> With a finite element analysis, compute the model weight W and the net load in the direction of gravity F_p generated by a unit pressure load applied normal to the supported face of the optic.
- ◁2> Compute the pressure p that will identically balance the weight W by the following equation:

$$p = \frac{W}{F_p}. \quad (6.20)$$

- ◁3> Apply kinematic constraints, the pressure p , and the gravity load in a static finite element analysis. Request recovery of the reactions and verify that they are zero.

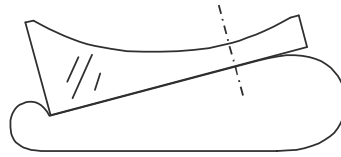


Figure 6.40 Exaggerated illustration of a nonaxisymmetric mirror supported by an air bag.

The rigid-body motions predicted by this analysis are arbitrary, but the elastic shape of the optical surface after its rigid-body motions are removed will be a reliable prediction.

If the optic is not axisymmetric, as shown in the highly exaggerated illustration in Fig. 6.40, then tangency cannot be achieved at all points around the edge of the optic. This lack of tangency can be modeled as a varying line load applied to the optic edge wherever tangency is lacking. The methods for computing the pressure and line load are as follows:

- ◁1▷ With a finite element analysis compute both the net vertical load and net moment generated by each of the following: the model weight, a unit pressure load applied normal to the supported face of the optic, and a line load $u(\theta)$ given by

$$u(\theta) = a \cos(\theta) + b, \quad (6.21)$$

where the constants a and b are chosen such that $u(0)$ is unity, and its values of zero are located at points where tangency is achieved. Fig. 6.41 shows an illustration of what this line load may look like. Table 6.15 defines the values that are calculated in this step.

- ◁2▷ The pressure p and line-load peak ω_0 , which balance the weight, can be found by the following:

$$p = \frac{-M_\omega W - F_\omega M_W}{-F_p M_\omega + F_\omega M_p}, \quad (6.22)$$

$$\omega_0 = \frac{M_p W + F_p M_W}{-F_p M_\omega + F_\omega M_p}. \quad (6.23)$$

The line-load function $\omega(\theta)$ then becomes

$$\omega(\theta) = \omega_0 u(\theta) = \omega_0 [a \cos(\theta) + b]. \quad (6.24)$$

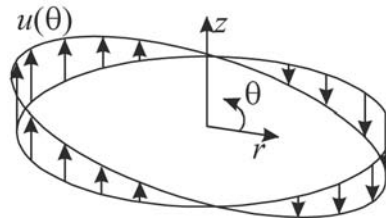


Figure 6.41 Varying line load representing the lack of tangency around the periphery of a nonaxisymmetric optic supported by an air bag.

Table 6.15 Net-load values computed for nonaxisymmetric optic supported by an air bag.

	<u>WEIGHT</u>	<u>UNIT PRESSURE</u>	<u>UNIT LINE LOAD</u>
Net vertical load	W	F_p	F_ω
Net moment	M_w	M_p	M_ω

- ◁3> Apply kinematic constraints, the pressure p , the line load $\omega(\theta)$, and the gravity load in a static finite element analysis. Request recovery of the reactions, and verify that they are zero. Reactions that are nonzero indicate an error in the application of the loads or an error in how they were computed.

An assumption inherent to the calculations shown above is that any areas of “lift off” as illustrated in Fig. 6.39(b) are small compared to the supported surface of the optic. Large “lift off” areas due to overinflation are not accurately represented as edge loads on the optic as they must be actually represented by a lack of support over the “lift off” area.

If the optic is nominally axisymmetric, but has a small offset of its center of mass due to manufacturing tolerances, then the above equations may be simplified. The tangency point is centered ($b = 0$ and $a = 1$); there is no moment due to pressure ($M_p = 0$), and the line load causes a pure moment with no net force ($F_\omega = 0$). The pressure to balance the weight is

$$p = \frac{W}{F_p}. \quad (6.25)$$

The line load to balance the offset center of gravity (CG) is

$$\omega_0 = \frac{M_w}{M_\omega}. \quad (6.26)$$

6.3.2 Example: test support deformation analysis of a nonaxisymmetric optic

An off-axis mirror whose cross-section is shown in Fig. 6.42 is to be tested on an air bag during fabrication but is to be mounted for operation in a 1-g environment on three back-surface points for operation. The surface error generated by transferring the optic from the air bag to its mounted in-use configuration is desired so that it may be included in the wavefront-error budget for the system. The mirror is fabricated of ULE whose material properties are given in Table 6.16.

A solid-element model of the mirror is shown in Fig. 6.43. Two analysis cases will be performed on this model to predict the change in surface error. The first case involves finding the deformation of the optic on the air bag relative to a 0-g environment. We must first calculate the proper unit edge load $u(\theta)$. The test engineers have specified that tangency will be achieved at two points shown in Fig. 6.44.

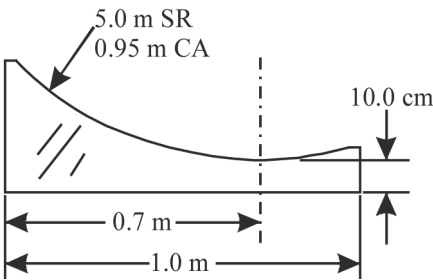


Figure 6.42 Dimensions of an off-axis mirror to be tested on an air bag during fabrication.

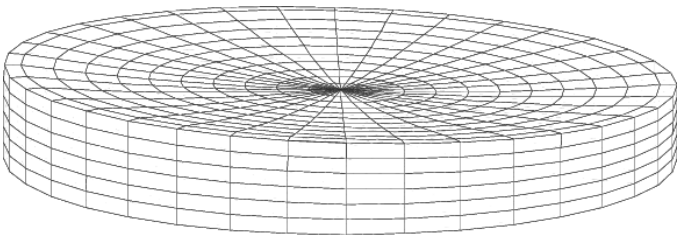


Figure 6.43 Finite element model of the off-axis mirror shown in Fig. 6.40.

Table 6.16 Material properties of ULE.

PROPERTY NAME	PROPERTY VALUE
Young’s Modulus	$6.757 \times 10^{10} \text{ }\mu\text{N/mm}^2$
Poisson’s Ratio	0.17
Mass Density	2.187 g/cm^3

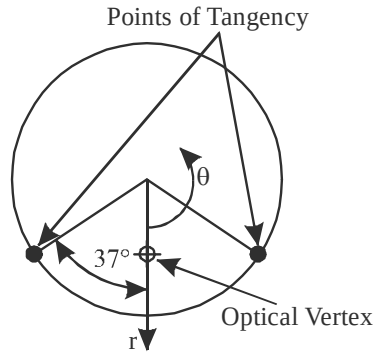


Figure 6.44 Locations of tangency between the air bag and the off-axis mirror.

Therefore, from Eq. (6.21), we write

$$u(0 \text{ deg}) = a \cos(0 \text{ deg}) + b = 1, \quad (6.27)$$

and

$$u(37 \text{ deg}) = a \cos(37 \text{ deg}) + b = 0. \quad (6.28)$$

The constants a and b are obtained by solving a set of two simultaneous equations:

$$\begin{bmatrix} \cos(0 \text{ deg}) & 1.0 \\ \cos(37 \text{ deg}) & 1.0 \end{bmatrix} \begin{Bmatrix} a \\ b \end{Bmatrix} = \begin{Bmatrix} 1.0 \\ 0.0 \end{Bmatrix}, \quad (6.29a)$$

$$\begin{bmatrix} 1.0 & 1.0 \\ 0.799 & 1.0 \end{bmatrix} \begin{Bmatrix} a \\ b \end{Bmatrix} = \begin{Bmatrix} 1.0 \\ 0.0 \end{Bmatrix}, \quad (6.29b)$$

$$\begin{Bmatrix} a \\ b \end{Bmatrix} = \begin{Bmatrix} 4.975 \\ -3.975 \end{Bmatrix}. \quad (6.29c)$$

Therefore, the unit line load becomes

$$u(\theta) = 4.975 \cos(\theta) - 3.975. \quad (6.30)$$

The net-vertical loads and moments about the optical-surface vertex are found by an initial finite element analysis with kinematic constraints for the application of gravity, a constant back pressure, and the unit line load found above. These net loads are shown in Table 6.17.

With the values in Table 6.17, and with Eqs. (6.22) and (6.23), we can compute the pressure p and the line-load peak ω_0 to balance the weight of the mirror:

Table 6.17 Net loads for weight, unit pressure, and unit line load.

	<u>WEIGHT</u>	<u>UNIT PRESSURE</u>	<u>UNIT LINE LOAD</u>
Net vertical load	$1.950 \times 10^9 \text{ N}$	$7.814 \times 10^5 \text{ N}$	$-1.247 \times 10^4 \text{ N}$
Net moment	$-4.163 \times 10^{10} \text{ N/mm}$	0.0 N/mm	$-3.883 \times 10^6 \text{ N/mm}$

$$\begin{aligned}
 p &= \frac{-M_{\omega}W - F_{\omega}M_W}{-F_pM_{\omega} + F_{\omega}M_p} \\
 &= \frac{-(-3.883 \times 10^6)(1.950 \times 10^9) - (-1.247 \times 10^4)(-4.163 \times 10^{10})}{-(7.814 \times 10^5)(-3.883 \times 10^6) + (-1.247 \times 10^4)(0.0)} \\
 &= 2,324.4 \frac{\text{N}}{\text{mm}^2}, \quad (6.31)
 \end{aligned}$$

$$\begin{aligned}
 \omega_0 &= \frac{M_pW + F_pM_W}{-F_pM_{\omega} + F_{\omega}M_p} \\
 &= \frac{(0.0)(1.950 \times 10^9) + (7.814 \times 10^5)(-4.163 \times 10^{10})}{-(7.814 \times 10^5)(-3.883 \times 10^6) + (-1.247 \times 10^4)(0.0)} \\
 &= -10,721.1 \frac{\text{N}}{\text{mm}}. \quad (6.32)
 \end{aligned}$$

The line load applied to the outer edge is redefined with Eq. (6.24) to become

$$\begin{aligned}
 \omega(\theta) &= \omega_0 u(\theta) = -10,721.1 [4.975 \cos(\theta) - 3.975] \\
 &= 53,337.5 \cos(\theta) - 42,616.4 \frac{\text{N}}{\text{mm}}. \quad (6.33)
 \end{aligned}$$

The displacements due to the gravity load, constant pressure p , and line load $\omega(\theta)$ are found with the kinematic-boundary conditions applied in a second finite element analysis. The air-bag loads balance the vertical load and moment from the weight to within $162 \mu\text{N}$ and $1936 \mu\text{N/mm}$, respectively. These imbalances are very small, indicating that the effective air-bag loads have been computed correctly.

The next step involves finding the deformation change between a zero gravity environment and the in-use mounted configuration. The mounts are idealized as kinematic mounts at the three mount locations. The displacements due to a gravity load applied along the optical axis are requested in a third analysis.

A node-by-node difference is performed to subtract the displacements associated with the air-bag case from the displacements of the in-use mounted case. The rigid-body motions of the surface are then extracted from the nodal displacement differences, and the residual RMS surface error is found.

The surface error results from the air-bag case, the in-use case, and the difference are summarized in Table 6.18. The resulting surface deformation may then be quantified by one of the surface deformation characterization methods of Chapter 3.

6.3.3 Modeling of V-block test supports

The modeling of a V-block test support, such as that shown in Fig. 6.45, can be performed by applying constraints to the optic along a line contact. If a frictionless surface is to be assumed, then the constraints must be oriented so that they only constrain displacements normal to the optic. For circular optics, the simplest way of assuring this is to define the constraints in the radial direction of a cylindrical coordinate system that is located on the axis of the optic. The analyst should be careful to construct the model of the optic so that a line of nodes is located at the line contact representing the V-block. In addition to the line constraints representing the V-block contacts, the analyst must also include enough additional constraints to remove the component rigid-body motions along and about the optical axis without adding fictitious redundant constraints. The predicted reactions associated with these constraints should be verified to be zero.

6.3.4 Modeling of sling and roller-chain test supports

A sling or roller-chain support, such as that shown in Fig. 6.46, can be modeled by a pressure load given by

Table 6.18 Surface error results.

	<u>SURFACE RMS (NM)</u>	<u>SURFACE P-V (NM)</u>
Air-bag loads	11 nm	47 nm
In-use loads	114 nm	499 nm
Difference	116 nm	511 nm

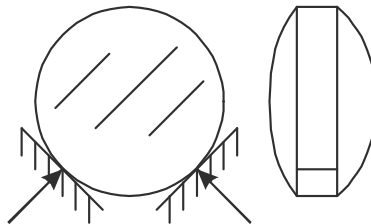


Figure 6.45 V-block supports are modeled by correctly oriented line constraints.

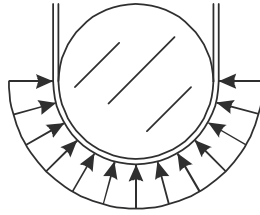


Figure 6.46 Sling supports are modeled by a constant pressure that reacts to the optic's weight.

$$p_0 = \frac{W}{Dt}, \quad (6.34)$$

where D is the diameter of the optic, and t is the width of the contact area between the sling or roller chain and the optic edge. As in other test-support modeling methods, enough constraints must be applied to precisely remove any singular rigid-body motions. The predicted reactions associated with these constraints should be verified to be zero.

6.3.5 Example: Comparison of three test supports

The gravity-induced deformations of solid circular mirror relative to its 0-g state is computed with the support of three test supports. The test supports are defined as follows:

- Test Support #1: Three-point support on back with the optical axis oriented vertically
- Test Support #2: V-block with the optical axis oriented horizontally
- Test Support #3: Sling with the optical axis oriented horizontally

Fig. 6.47 shows plots of surface error with best-fit plane removed for each test support listed above. The results with Test Support #1 in Fig. 6.47(a) has a



(a) RMS = 0.267 μm



(b) RMS = 0.053 μm



(c) RMS = 0.054 μm

Figure 6.47 Test induced surface error for mirror on three test support configurations described above: (a) Test Support #1, (b) Test Support #2, (c) Test Support #3.

SOLID MIRROR PROPERTIES

Material = fused silica
 Outside diameter = 1.0 m
 Inside diameter = 0.25 m
 Overall height = 0.075 m
 Radius of curvature = 3 m

much-larger surface RMS than those of the results with the other test supports, but the three-point support is simple to implement and very predictable. If properly centered, the deformed state induced by Test Support #1 in Fig. 6.47(a) has only axisymmetric, trefoil, and hexafoil terms, so all other terms observed during testing are associated with true figure errors. The test error predictions of Test Support #2 and Test Support #3, which are shown in Fig. 6.47(b) and Fig. 6.47(c), are dominated by power but include other polynomial terms as well.

6.4 Tolerance Analysis of Mounts

Many optical components are sensitive to redundant loads imparted by their mounts as the optics are integrated to their mounting hardware. Such induced loads are often functions of interface tolerances, bond shrinkages, or other variations that are not deterministically known. Such variables are instead characterized by statistical variations about a desired mean value. These random behaviors create the need for implementation of Monte Carlo analysis techniques.

6.4.1 Monte Carlo analysis

The response quantities U_{ij} (displacements, polynomial coefficients, surface RMS error, line-of-sight error, etc.) are determined by the following equations:

$$V_{ik}^* = V_{Nom_k} + \sigma_k \times \gamma_{ik}, \quad (6.35)$$

$$U_{ij} = U_{Nom_j} + \sum_k \frac{dU_{jk}^*}{dV_k} (V_{ik}^* - V_{Nom_k}), \quad (6.36)$$

where i is an index on the Monte Carlo analyses, k is an index on the variables, j is an index on the response quantities, V_{Nom_k} is the nominal or mean value for the k^{th} variable, σ_k is the uncertainty of the k^{th} variable, and γ_{ik} is a random number with a mean of 0.0 and an uncertainty of 1.0 with distribution specified as normal or uniform for the i^{th} Monte Carlo analysis and the k^{th} variable, and U_{Nom_j} is the nominal or mean value for the j^{th} response quantity. The partial derivative dU_{jk}^* / dV_k of the j^{th} response with respect to the k^{th} variable is

determined from the j^{th} response of the state used to define the k^{th} variable minus the j^{th} response of the specified nominal state. That is,

$$\frac{dU_{jk}^*}{dV_k} = \frac{U_{jk}^* - U_{\text{Nom}_j}}{V_k - V_{\text{Nom}_k}}, \quad (6.37)$$

where U_{jk}^* is the j^{th} response of the disturbance defining the k^{th} variable, and V_k is the variable value associated with U_{jk}^* .

In the Monte Carlo feature of SigFit, available response quantities in the Monte Carlo analysis feature include:

- best-fit rigid-body motion of optical surfaces
- residual surface RMS error
- polynomial coefficients
- line-of-sight error
- residual surface RMS error after adaptive control

The variations U_{jk}^* may be defined by subcase response data. Thus, any parameter or combination of parameters representable in a finite element model that when varied causes a change in response can be a Monte Carlo variable. Any format of input data allowable by SigFit may be used to characterize a Monte Carlo variable, including finite element results, rectangular grid arrays (e.g., interferogram arrays from test data), combinations of disturbances defined by polynomials, and general free-format vector data. These variables may also be linearly scaled and combined as desired. Furthermore, a finite element model is not required because SigFit can internally create a mesh for any common optical surface type.

6.4.2 Example: flatness/coplanarity tolerance of a mirror mount

The lightweight mirror in Fig. 6.48 is attached to a metering structure by bipod flexures and a mount plate at each bipod pair. Each mount plate is bolted to the metering structure, so any nonflatness or noncoplanarity of the attachment plates will cause bending of the mirror through the elastic isolation of the bipod flexures. As part of the design specification of the mirror mount plate and metering structure, flatness and coplanarity must be specified. Optomechanical analysis is used to relate mount flatness to the optical requirement on mirror surface RMS error. In a finite element model, individual load cases of unit flatness mismatch and noncoplanarity at each mount are applied. In this example, the mismatch at a single mount is represented as three load cases:

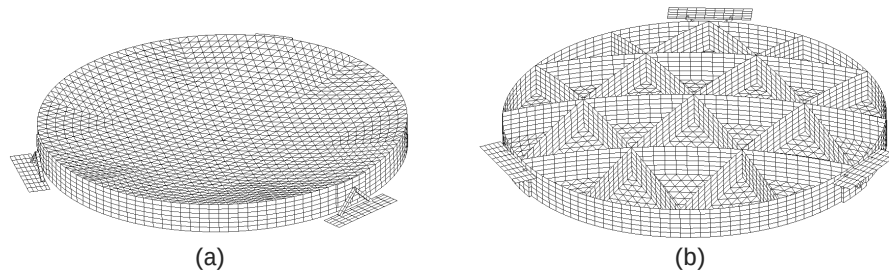


Figure 6.48: Lightweight mirror with three edge mounts: (a) top view and (b) bottom view.

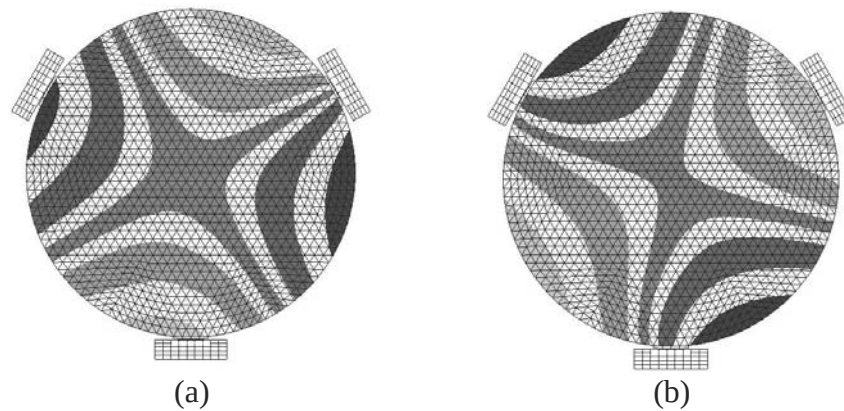


Figure 6.49: Surface deformations of variations with best-fit plane removed: (a) radial rotation of mount and (b) z offset of mount.

- Rotation of 0.0001 radian about the radial axis
- Rotation of 0.0001 radian about the circumferential axis
- Displacement of 0.0020 inch in the axial direction

For the Monte Carlo analysis, the random variables were assumed to be uniformly distributed over the ranges of ± 0.0001 radian for the flatness rotations and ± 0.0020 inch for the coplanarity translation. Monte Carlo results include mean, standard deviation, maximum value, minimum value, and user specified percentile. In Fig. 6.50, the cumulative probability vs. surface RMS error after subtraction of best-fit plane is shown. This plot shows the probability that a given surface RMS error after subtraction of best-fit plane will be exceeded due to the machining tolerances being considered.

In Table 6.19, the 95-percentile results are presented for the surface RMS error and the amplitude of the low-order Zernikes. Analysis results are presented for four cases, each including a different set of variables: all machining tolerances considered, only the radial rotation flatness variables, only the circumferential rotation flatness variables, and only the axial coplanarity variables. Results for each case were generated with 10,000 sets of Monte Carlo

realizations. The units used in the table are HeNe waves (0.6328 microns). The table shows that ± 0.0020 inches of coplanarity is much more significant than the ± 0.0001 -radian flatness errors. Also, the dominant response is primary astigmatism with very little power. This approach can be used to determine a realistic mechanical tolerance based on the optical response quantities.

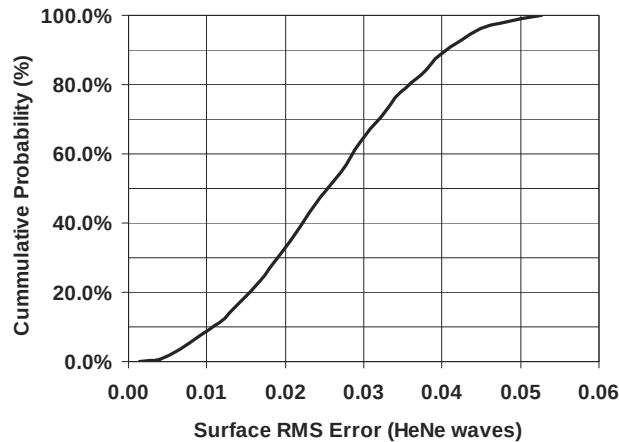


Figure 6.50 Cumulative probability vs. surface RMS error for a mount flatness/coplanarity variational study.

Table 6.19 Mount flatness/coplanarity results from Monte Carlo analyses.

			ALL MACHINING VARIABLES (WAVES)	RADIAL FLATNESS ROTATION (WAVES)	CIRCUM- FRENTIAL FLATNESS ROTATION (WAVES)	COPLANARITY TRANSLATION
SURFACE RMS ERROR			0.04475	0.00839	0.00159	0.04361
N	M	ZERNIKE TERM	AMPLITUDE	AMPLITUDE	AMPLITUDE	AMPLITUDE
2	0	Power (Defocus)	0.00041	0.00000	0.00041	0.00000
2	2	Pri Astigmatism-A	0.07939	0.01525	0.00284	0.07795
2	2	Pri Astigmatism-B	0.07996	0.01502	0.00288	0.07983
3	1	Pri Coma-A	0.00061	0.00007	0.00016	0.00058
3	1	Pri Coma-B	0.00060	0.00007	0.00016	0.00056
3	3	Pri Trefoil-A	0.00334	0.00338	0.00000	0.00000
3	3	Pri Trefoil-B	0.00034	0.00000	0.00034	0.00000
4	0	Pri Spherical	0.00007	0.00000	0.00007	0.00000
4	2	Sec Astigmatism-A	0.00466	0.00093	0.00020	0.00448
4	2	Sec Astigmatism-B	0.00469	0.00091	0.00021	0.00464
4	4	Pri Tetrafoil-A	0.00481	0.00063	0.00013	0.00479
4	4	Pri Tetrafoil-B	0.00491	0.00062	0.00013	0.00491

An alternative approach assumes the worst-case scenario to tolerance mount flatness. In this case, it is not obvious about which combination of mount rotations will cause the worst-case surface RMS after the best-fit plane is removed. The maximum RMS found from 10,000 Monte Carlo combinations was 0.0599 waves. An exhaustive envelope analysis of all possible extreme cases shows that the absolute maximum possible surface RMS error is 0.06472 waves. In many cases, the 95-percentile value (69.1% of the absolute maximum) would be an acceptable value with which to choose tolerances.

6.5 Analysis of Assembly Processes

6.5.1 Theory

In many applications, optical performance can be affected by deformations that are locked into an optical system during its assembly. Therefore, it is of interest in many situations to be able to predict how much deformation will result from a particular process of assembling an optical system. Fig. 6.51 shows an illustration of a simple assembly process in which a mirror is bonded to its mounts in a 1-g environment and subsequently placed in a 0-g operational environment. The process of bonding the mirror to its flexures while being supported by the assembly fixture locks in elastic strain, which remains in the unloaded final state.

Fig. 6.52 shows an illustration of the path of applied load. Notice that the change in deformation between states can be found through a single linear finite element analysis of the system to which changes in externally applied loads or internal connections are applied. The analysis of a whole process consisting of multiple assembly states is nonlinear and can be performed with a piecewise nonlinear analysis.⁷ Using this approach separate load steps may be defined to vary loads, model connections and boundary conditions. Furthermore, each load step must be applied to the deformed model of each previous load step.

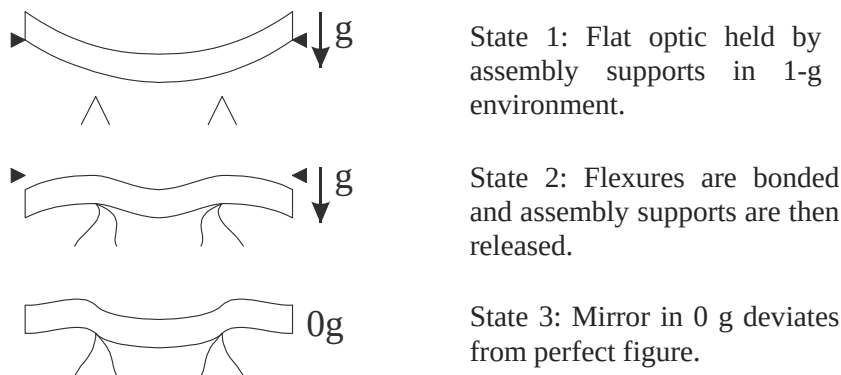


Figure 6.51 Assembly process for bonding an optic to flexure mounts.

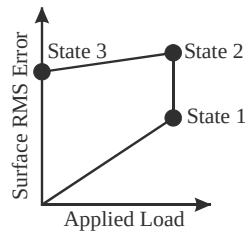


Figure 6.52 Load vs. surface-RMS error curve for assembly of an optical system. The path to State 1 is linear, starting at zero. The path from State 1 to State 2 keeps the same load and then ends with a different stiffness and deflection in State 2. The path from State 2 to State 3 is an unloading process ending with a state in which the deflection is not zero, representing locked-in strain.

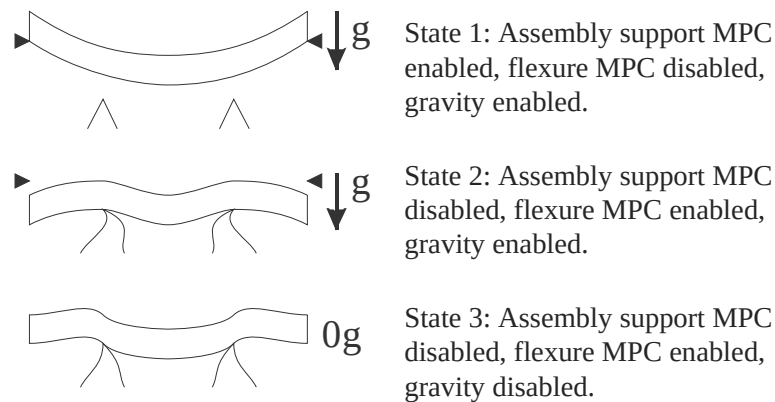


Figure 6.53 Analysis flow for simulation of assembly process for bonding an optic to flexure mounts.

Fig. 6.53 illustrates how this analysis would be executed in a finite element analysis. Three load steps would be defined in a nonlinear analysis. Multi-point constraints (MPC) are used to model the connections to the assembly support and flexures. These connections must have the capability of being active or inactive in each load step. The first load step activates the MPC between the optic and the assembly support, and applies the gravity load. The MPC between the optic and the flexures, however, is left inactive. The second load step activates the MPC between the optic and the flexures, and deactivates the MPC between the optic and the assembly support. The gravity load is left on. This simulates the process of bonding the optic to its mounts and transfers the weight of the optic to the flexures. The third load step keeps the MPC between the optic and the flexures active while removing the gravity load. This load step simulates the process of transferring the assembled system to a 0-g environment while having been bonded in a 1-g environment.

Although the analysis demonstrated in Fig. 6.53 is relatively simple, much more complex processes can be modeled by adding more steps and connection interfaces to the assembly analysis.

6.5.2 Example: assembly analysis of mirror mounting

The assembly process illustrated in Fig. 6.51 is to be performed on the mirror used in Section 5.1.4.4. The partial listing of the MSC/NASTRAN model below shows the analysis control section, the lines between CEND and BEGIN BULK, and some key entries in the bulk data section, the lines below BEGIN BULK. Comments are included in italics to clarify the purpose of important entries.

Fig. 6.54 shows that the deformed shape after gravity has been removed from the assembled system. Although the residual elastic deformation of the mounted mirror is negligible, 1200 nm of piston is the result of the initial deformation of the flexures before bonding.

```

SOL 106 NONLINEAR STATICS ANALYSIS
CEND
TITLE = 3D plate MIRROR
SUBTITLE = ASSEMBLY ANALYSIS
ECHO = NONE
NLPARM = 1      NONLINEAR PARAMETER REQUEST
DISP(PUNCH,PLOT) = ALL NODAL DISPLACEMENT REQUEST
SUBCASE 1      BEFORE ASSEMBLY SUBCASE - NO MIRROR TO FLEXURE MPC
  LOAD = 1      GRAVITY LOAD
  SPC = 1      FLEXURE BASE AND ASSEMBLY SUPPORT CONSTRAINED
SUBCASE 2      AFTER ASSEMBLY SUBCASE
  LOAD = 1      GRAVITY LOAD
  SPC = 2      ONLY FLEXURE BASE CONSTRAINED
  MPC = 2      CONNECTION BETWEEN FLEXURES AND MIRROR TURNED ON
SUBCASE 3      GRAVITY REMOVED - SHOWS LOCKED IN STRAIN EFFECTS
  SPC = 2      ONLY FLEXURE BASE CONSTRAINED
  MPC = 2      CONNECTION BETWEEN FLEXURES AND MIRROR KEPT ON
BEGIN BULK
NLPARM 1      NONLINEAR ANALYSIS PARAMETERS - USE ALL DEFAULTS
$ GRAVITY LOAD
GRAV 1 0 386.4 0.0 0.0 -1.0
$ FLEXURE BASE AND ASSEMBLY SUPPORT CONSTRAINTS FOR 1ST SUBCASE
SPC1 1 123456 300113 300213 300313 300413 300513 300613
SPC1 1 23 105618 105582 105600
$ FLEXURE BASE CONSTRAINTS FOR 2ND AND 3RD SUBCASES - NO ASSEMBLY
SUPPORT
SPC1 2 123456 300113 300213 300313 300413 300513 300613
MPC 2 300011 1 -1.0 300001 1 1.0
$ FLEXURE TO MIRROR CONNECTIONS FOR 2ND AND 3RD SUBCASES
MPC 2 300012 1 -1.0 300002 1 1.0
MPC 2 300013 1 -1.0 300003 1 1.0
.
.
MPC 2 300011 6 -1.0 300001 6 1.0
MPC 2 300012 6 -1.0 300002 6 1.0
MPC 2 300013 6 -1.0 300003 6 1.0

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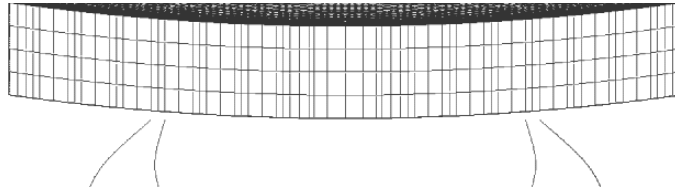


Figure 6.54 Highly exaggerated deformed shape of lightweight mirror in 0-g environment, which is mounted in 1-g environment.

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◀Chapter 7▶

Structural Dynamics and Optics

Optical systems must operate and survive in the presence of dynamic loads of various forms. Spaceborne instruments, for example, need to be designed to survive random vibration and acoustic loads during launch, shock loads from pyrotechnic devices during separation events, and operate in the presence of harmonic and random disturbances from the host satellite. Optical systems on airborne platforms are exposed to dynamic loads from gusts, engine disturbances, and air turbulence. Terrestrial sensors are subject to wind and seismic excitations. Additional sources of vibration for optical systems include dynamic forces from moving internal components. Managing dynamic disturbances and their effects on pointing stability, image quality, and structural integrity requires understanding and characterizing the dynamic behavior of the optical system. Mitigation strategies include modifications to the structural design along with the use of passive vibration and active stabilization techniques to reduce, attenuate, or correct the dynamic response to meet system requirements.

7.1 Natural Frequencies and Mode Shapes

All elastic structures have modes of vibration that are characterized by a mode shape and a natural frequency. For a single-degree-of-freedom (SDOF) system, shown in Fig. 7.1, the equation of motion is expressed as

$$m\ddot{u} + b\dot{u} + ku = 0 \quad (7.1)$$

where m is the mass, b is the damping, k is the stiffness, and u is the displacement.

The natural frequency f_n is a function of the mass and stiffness of the system as shown in Eq. (7.2) and has units of cycles/sec or Hz; the angular frequency, ω_n , has units of rad/sec. For lightly damped structures, typical of stiff optical systems, damping has little effect on the natural frequency and can be ignored. The corresponding mode shape of the SDOF system is the deflected shape of the mass on the spring.

$$f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k}{m}}. \quad (7.2)$$

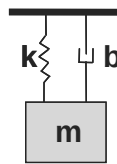


Figure 7.1 Single-degree-of-freedom mass-spring-damper system.

The natural frequency may be related to the static deflection Δ_{st} of the SDOF system under a 1-g load using the following relationship:

$$f_n = \frac{1}{2\pi} \sqrt{\frac{g}{\Delta_{st}}}. \quad (7.3)$$

This relationship is useful to estimate a natural frequency based on deflections due to gravity using analytical solutions^{1,2} for early design evaluations and/or to help anchor numerical simulations using finite element analyses.

7.1.1 Multi-degree-of-freedom systems

For complex structures, a multi-degree-of-freedom (MDOF) analysis is required to characterize the system natural frequencies and mode shapes. The equations of motion for a MDOF system are shown in Eq. (7.4), where m , b , and k are the mass, damping, and stiffness matrices, u is displacement, and p is the applied load:

$$[m]\{\ddot{u}\} + [b]\{\dot{u}\} + [k]\{u\} = \{p\}. \quad (7.4)$$

The natural frequencies for a MDOF system are computed using a real eigenvalue analysis assuming harmonic motion:

$$u = \Phi e^{i\omega t}, \quad (7.5)$$

$$\ddot{u} = -\omega^2 \Phi e^{i\omega t}. \quad (7.6)$$

The natural frequencies and mode shapes are obtained by solving the eigenvalue problem created by substituting the harmonic solution into Eq. (7.4):

$$(-\omega_{nj}^2 m + k) \Phi_j = 0, \quad (7.7)$$

where the eigenvector Φ_j is the mode shape for the j^{th} vibration mode, with an angular frequency ω_{nj} :

$$\omega_{nj} = 2\pi f_{nj}. \quad (7.8)$$

The natural frequencies of the system are important since these are the frequencies where the peak dynamic response of the structure occurs.

The corresponding mode shape for a given natural frequency is a normalized quantity whose magnitude has no meaning. However, the mode shape and corresponding response quantities, such as displacement, stress, and strain

energy, may be used to provide insight on how to modify the dynamic response of the system. In particular, plotting the strain energy density for the critical mode shapes using a finite element post-processor allows the analyst to identify possible regions to optimize. For example, to increase the natural frequency of a structure, the regions that have a high strain-energy density should be stiffened, and regions with low strain-energy density should be lightweighted.

7.2 Damping

Damping accounts for the energy dissipation in a structure and controls the peak response at resonance. Typical sources of damping in a structure include slippage between joints (e.g., friction or fretting), plasticity or viscoelastic behavior, material damping from internal friction, structural nonlinearities (plasticity, gaps), and air-flow effects.

Damping is often specified as viscous damping in the form of a damping ratio ζ , which is known as the “fraction of critical damping” or “percent of critical damping.” For example, a damping ratio of 0.01 corresponds to 1% of critical damping. Optical structures are typically lightly damped with damping ratios in the range of $\zeta = 0.001$ – 0.020 . The amount of damping for a given structure is difficult to predict and is best characterized via testing.³ The percent damping ratio may be converted to viscous damping b for use in dynamic response equations using the following expression:

$$\zeta = \text{damping ratio} = b/b_{cr}, \quad (7.9)$$

where the critical damping is expressed as $b_{cr} = 2m\omega_n$. Damping is also commonly expressed as amplification factor Q , or loss factor η :

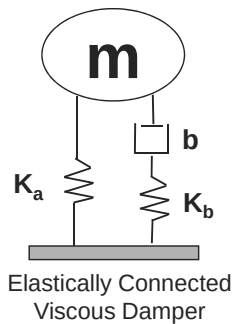
$$Q = 1/2\zeta = 1/\eta. \quad (7.10)$$

The relationship between the damped natural frequency f_d and undamped natural frequency f_n is expressed below:

$$f_d = f_n \sqrt{1 - \zeta^2}. \quad (7.11)$$

For lightly damped structures, the effects of damping are typically ignored in the calculation of natural frequencies. For example, for $\zeta = 0.01$ (1%), the damped natural frequency f_d equals 0.99995 of the undamped natural frequency f_n .

A case where the effects of damping on natural frequency are significant is for an optical system mounted to a highly damped vibration isolation system.



Example
 $K_a = K_b = 1000 \text{ lbs/in}$
 $m = 0.02588 \text{ lbs-s}^2/\text{in}$

b (lbs-s/in)	Freq (Hz)
0	31.3
1	31.7
2	33.4
3	36.4
4	39.1
5	40.8
6	41.8
7	42.4
8	42.8
10	43.4
20	43.9
Infinite	44.3

Figure 7.2 Effects of damping on the natural frequency of a simple elastically connected vibration isolator.

For example, the isolation characteristics of an elastically connected viscous damper, shown in Fig. 7.2, are a strong function of the amount of damping. The natural frequency is shown for various values of damping ranging from zero to infinity. For the case of zero damping, the natural frequency is a function of only the spring K_a . When the damping value is set to infinity, K_a and K_b combine to define the stiffness of the system. For other damping values, the natural frequency falls in between and is bounded by these two extremes. For a MDOF system with significant damping, a complex eigenvalue analysis is required to account for damping in the computation of the natural frequencies.

7.3 Frequency Response Analysis

Frequency response analysis (also known as harmonic response) computes the steady-state dynamic response of a system due to harmonic or steady-state oscillatory input. Frequency response analysis is discussed for both direct force excitation and base dynamic disturbances.

7.3.1 Force excitation

Direct dynamic forces may act on an optical system as illustrated in the SDOF system in Fig. 7.3. These dynamic forces include external inputs including aerodynamic and acoustic excitation, and internal disturbances such as reaction forces from scanning and steering mirrors, cryo-coolers, or other internal mechanisms.

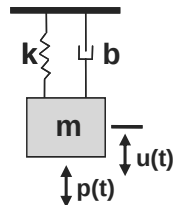


Figure 7.3 Force-excited single-degree-of-freedom system.

For a dynamic forcing function $p = p_0 e^{i\omega t}$ at a forcing frequency ω , the displacement at steady state is $u = u_0 e^{i\omega t}$ with a response amplitude of u_0 at the forcing frequency ω . The response amplitude can be determined by solving

$$[-m\omega^2 + ib\omega + k]u = p. \quad (7.12)$$

The structural response is computed directly for each frequency of excitation ω by solving the coupled matrices similarly to a static solution but using complex mathematics.

A second method to compute the frequency response of a structure is using modal methods. In this approach, the mode shapes of the structure are the physical coordinates of the system and are used to uncouple the equations of motion. The system behavior is computed as a summation of modal responses. This method provides benefits in the form of computational efficiencies and physical insight.

The modal equations are uncoupled using the orthogonality condition of eigenvectors,

$$\mathbf{K} = \Phi^T k \Phi, \quad \mathbf{M} = \Phi^T m \Phi, \quad \mathbf{P} = \Phi^T p, \quad (7.13)$$

where the matrices $\mathbf{K}$ and $\mathbf{M}$ are diagonal. Upon substitution in the harmonic response equation, an uncoupled system (e.g., diagonal coefficient matrices) results:

$$[-\omega^2 \mathbf{M} + ib\omega + \mathbf{K}]z = \Phi^T p = \mathbf{P}. \quad (7.14)$$

The solution of an uncoupled system may be written directly as

$$z_j = \frac{F_j}{(K_j - M_j \omega^2) + ib\omega}, \quad (7.15)$$

where z_j is the participation of mode j at the forcing frequency. Note that z is a complex number due to damping; thus, the response quantities may be out of phase with the forcing function. Any physical response (u, σ) can be computed as the combination of modal responses (Φ, S):

$$u = \sum z_j \Phi_j \quad \text{and} \quad \sigma = \sum z_j S_j. \quad (7.16)$$

Modal analysis is exact if all the modes are used but is approximate if a subset is used. Typically the response is dominated by the low-frequency structural modes, and computational efficiencies are realized in the analysis by eliminating the high-frequency modes in the response calculation.

Once the equations of motion have been solved, frequency response functions (FRF) plot the response quantities of interest as a function of the driving frequency. FRFs provide insight into the dynamic behavior of the system in the form of both magnitude and phase frequency response plots. The non-dimensional magnitude frequency response function for the force-excited SDOF system is expressed in Eq. (7.17) and plotted in Fig. 7.4 for several damping levels:

$$|H(r)| = \frac{u_0}{p_0/k} = \frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}. \quad (7.17)$$

The frequency ratio r is the ratio of the forcing frequency divided by the natural frequency:

$$r = \frac{\omega}{\omega_n} = \frac{f}{f_n}. \quad (7.18)$$

For small amounts of damping, the frequency response function reduces to:

$$|H(r)| = \frac{1}{1-r^2} = D. \quad (7.19)$$

The magnitude of $H(r)$ is also known as the dynamic magnification factor D or gain that multiplies the static response of the system. For frequency ratios r that are much less than one, the system oscillates at the driving frequency with an amplitude equal to the displacement produced by a static load p_0/k .

When r is approximately one, the condition of resonance occurs, and the response of the system is amplified and controlled by the amount of damping in the system. The peak response at resonance may be estimated by $D = 1/(2\zeta)$. This condition often presents design challenges for lightly damped structures.

For frequency ratios greater than one, the inertia of the structure dictates the response of the system. The response at the higher frequency ratios is less than the static response and decreases as the frequency ratio increases.

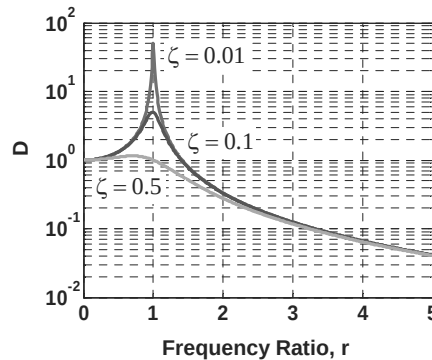


Figure 7.4 Non-dimensional frequency response function for a force-excited system.

7.3.2 Absolute motion due to base excitation

Optical systems experience base excitations from any dynamic disturbance that acts on their base or mounting structure. Sources of base excitation include spacecraft launch loads, aircraft engine noise and air turbulence, excitations from neighboring equipment with reciprocating motion, and seismic excitations. A base-excited single-degree-of-freedom system is shown in Fig. 7.5.

The non-dimensional, absolute-motion frequency-response function due to base excitation is expressed below and is shown in Fig. 7.6 for various levels of damping, where u represents the displacement of the mass and y the displacement of the base:

$$|H(r)| = \frac{u}{y} = \frac{\ddot{u}}{\ddot{y}} = \sqrt{\frac{1 + (2\zeta r)^2}{(1 - r^2)^2 + (2\zeta r)^2}}. \quad (7.20)$$

For small amounts of damping, the equation reduces to

$$|H(r)| = \frac{1}{1 - r^2} = D. \quad (7.21)$$

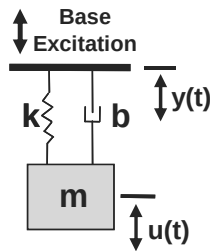


Figure 7.5 Base-excited single-degree-of-freedom system.

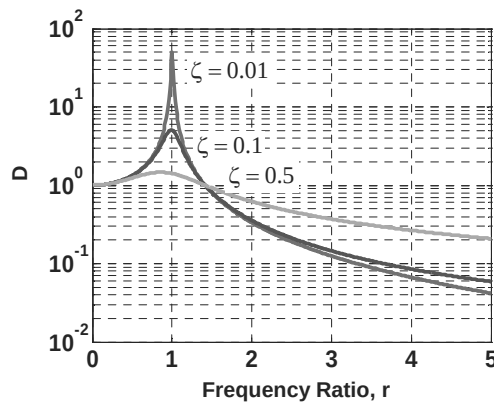


Figure 7.6 Non-dimensional frequency response function for absolute motion of a base-excited system.

For values of r much smaller than one, the magnitude of the response is equivalent to the static response and controlled by the stiffness of the system. For values of r near one, where the driving frequency is the same as the system natural frequency, the condition of resonance occurs, and the response is amplified and controlled by the amount of damping in the system. For large values of r , the response is attenuated and controlled by the inertia of the system. Notice that by increasing system damping, the frequency response is reduced near the condition of resonance, but it increases the response for frequency ratios above $\sqrt{2}$. This is an important consideration in the design of vibration isolation systems.

7.3.2.1 Absolute motion due to base excitation example

Absolute motion frequency response analysis is performed for an optical bench simply supported at four corners subject to base excitation, as shown in Fig. 7.7. Locations 1 and 2 represent locations of optical elements on the optical bench. The first five modes of the structure are shown in Fig. 7.8. The magnitude and phase frequency response functions describe the behavior of the optics, as shown in Fig. 7.9.

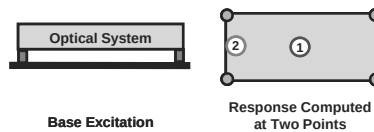


Figure 7.7 Optical bench mounted at the four corners.

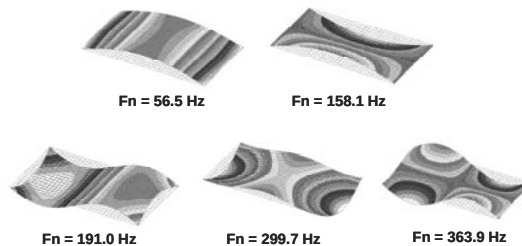


Figure 7.8 First five modes of the optical bench.

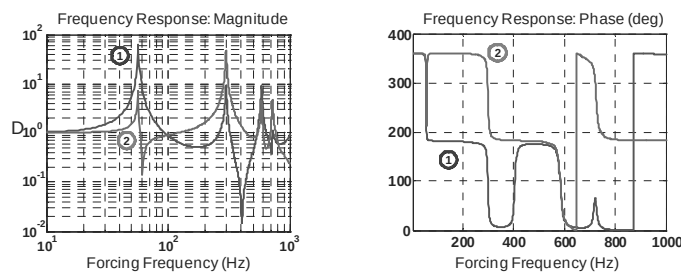


Figure 7.9 Magnitude and phase frequency response for two points on the optical bench.

The magnitude FRF reveals how the peak responses vary for each of the two optics, and the phase FRF reveals the phase relationship between the peak responses. In this example, the base excitation strongly couples with the first and fourth modes of the optical bench. At the frequency of the first and fourth modes, the magnitude of the peak responses of the two optics are not equal. In addition, the peak responses of the two optics are out of phase and do not occur at the same time. In performing complex dynamic simulations such as LOS jitter and wavefront error analyses (discussed in subsequent sections), this information is required for each optical element to predict the behavior of the optical system as a whole.

It is recommended when performing a FEA frequency response analysis to tailor the frequency-response step size in the region of the natural frequencies to capture the response in the narrow peaks that result from lightly damped systems.

7.3.3 Relative motion due to base excitation

The relative frequency response function provides the motion of the optic or the sensor relative to the base motion input. This type of analysis is common in meeting spacecraft dynamic design envelopes and ensuring neighboring systems do not interfere (i.e., contact), such as those on low-frequency isolation systems subject to dynamic disturbances.

The non-dimensional frequency response function for relative motion due to base excitation is expressed below and shown in Fig. 7.10 for various levels of damping:

$$|H(r)| = \frac{z}{y} = \frac{\ddot{z}}{\ddot{y}} = \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}, \quad (7.22)$$

where z is the relative motion between the base and the system, and is computed as

$$z(t) = u(t) - y(t). \quad (7.23)$$

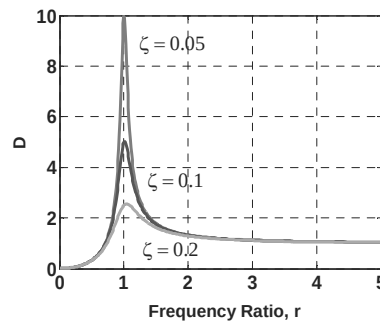


Figure 7.10 Non-dimensional frequency response function for relative motion of a base-excited system.

For small amounts of damping, the expression reduces to

$$|H(r)| = \frac{r^2}{1-r^2} = D. \quad (7.24)$$

For a frequency ratio r that is much less than one, the system and the base essentially move together; at resonance with a frequency ratio near one, there is an amplified system response; and for a frequency response ratio much greater than one, the system does not move, but the base moves beneath it.

7.3.4 Frequency response example

Optical systems are commonly mounted to vibration isolation tables to reduce the effects of external base vibration on the optics. Use of both absolute and relative frequency response equations will be utilized in predicting the relative motion of an optical element mounted on a vibration isolation table subject to a 10- μm , 5-Hz base excitation. The response of the 2-DOF system will be approximated by the solution of two SDOF systems since the mass of the optical element is much less than the isolation table.

The natural frequency of the isolation table is computed using the mass of the isolation table ($M_1 = 200 \text{ kg}$), and stiffness ($K_1 = 1974 \text{ N/m}$):

$$f_n = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2\pi} \sqrt{\frac{1974}{200}} = 0.50 \text{ Hz}. \quad (7.25)$$

Based on this natural frequency and driving frequency, the frequency ratio r is computed as

$$r = \frac{f}{f_n} = \frac{5}{0.5} = 10. \quad (7.26)$$

The ratio of the absolute motion of the isolation table to the base input is computed as the following with damping ($\zeta = 5\%$):

$$\left| \frac{u}{u_{base}} \right| = \frac{\sqrt{1 + (2\zeta r)^2}}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} = \frac{\sqrt{1 + [2(0.05)(10)]^2}}{\sqrt{(1-10^2)^2 + [2(0.05)(10)]^2}} = \frac{1.41}{99} = 0.0143. \quad (7.27)$$

For an input amplitude of 10 μm , the resulting isolation table displacement is 0.143 μm .

The relative motion of the optical mount to the vibration isolation table is computed next. The optic has a mass $M_2 = 1 \text{ kg}$, with a mount stiffness $K_2 = 395,000 \text{ N/m}$. The damping is assumed small and negligible. The natural frequency of the optical mount is computed below:

$$f_n = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2\pi} \sqrt{\frac{395000}{1}} = 100 \text{ Hz.} \quad (7.28)$$

This results in a frequency ratio of

$$r = \frac{f}{f_n} = \frac{5}{100} = 0.05. \quad (7.29)$$

and a relative frequency response of

$$\left| \frac{u_{rel}}{u_{base}} \right| = \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} = \frac{0.05^2}{\sqrt{(1-0.05^2)^2 + 0}} = 0.0025. \quad (7.30)$$

The relative motion of the optical mount to the isolation table may be computed as the fraction of relative motion multiplied by the isolation table input:

$$U_{rel} = 0.0025 U_{base} = 0.0025 * 0.143 = 0.00036 \text{ } \mu\text{m.} \quad (7.31)$$

7.4 Random Vibration

Optical systems must often meet performance and structural integrity requirements in the presence of random-vibration environments. Random-vibration disturbances are non-deterministic, and response levels cannot be predicted at any point in time. The use of probability of occurrence and statistics are required to describe the response in both the time and frequency domains. In this overview, the random-vibration time histories are assumed to be stationary and ergodic, which are typical design environments. Stationary means that the statistical properties (mean and standard deviation) of the random signal do not change with time. Ergodicity means that the statistical properties over a time sample are representative of the entire time history.

7.4.1 Random vibration in the time domain

Random variables are typically assumed to follow a Gaussian distribution. This allows the magnitude of the response to be statistically predicted as a percentage of time. A time history sample of a random variable Ω with a mean of zero and a root-mean-square (RMS) value of one over a 100-sec time period is shown in Fig. 7.11. The corresponding histogram describes the magnitude of Ω versus the percentage of time is shown on the right side of the figure. The RMS of the time history may be computed as follows:

$$\sigma = \sqrt{\frac{\Omega_1^2 + \Omega_2^2 + \dots + \Omega_n^2}{n}}, \quad (7.32)$$

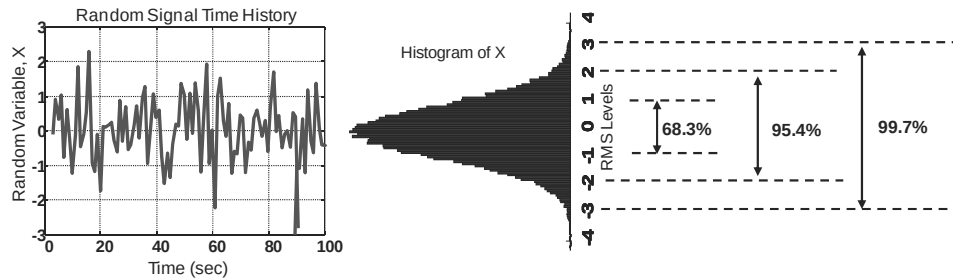


Figure 7.11 Random variable with zero mean and RMS equal to one.

where the 1σ value represents the RMS of the response.

For normally distributed Gaussian behavior, the peak response has the following statistical behavior:

- 1σ value $\Rightarrow$ peaks are less than 1σ for 68.3% of the time
- 2σ value $\Rightarrow$ peaks are less than 2σ for 95.4% of the time
- 3σ value $\Rightarrow$ peaks are less than 3σ for 99.7% of the time

7.4.2 Random vibration in the frequency domain

Random vibration levels are commonly described in the frequency domain using the power spectral density (PSD). This form provides the power (amplitude quantity squared) per unit frequency versus the frequency content of the time history. Common forms of the PSD for optical systems include launch load acceleration PSDs expressed in G^2/Hz or operational base-motion disturbance spectra expressed as $\mu\text{rad}^2/\text{Hz}$. The “power” term is generic—it can represent acceleration, velocity, displacement, pressure, force, etc.

The PSD is computed from a time history using signal processing theory (typically a discrete Fourier transform).⁴ For example, acceleration power spectral densities may be computed using this method from accelerometer data.

The random response of a system subject to a PSD input is computed as the input PSD multiplied by the square of the magnitude frequency response function, as expressed in Eq. (7.33). An example PSD response is computed as shown in Fig. 7.12:

$$PSD_{Resp} = PSD(f)_{Input} H(f)^2. \quad (7.33)$$

The RMS of the response quantity in the frequency domain is computed by taking the square root of the area A under the PSD response curve:

$$\text{RMS} = \sigma = \sqrt{A}. \quad (7.34)$$

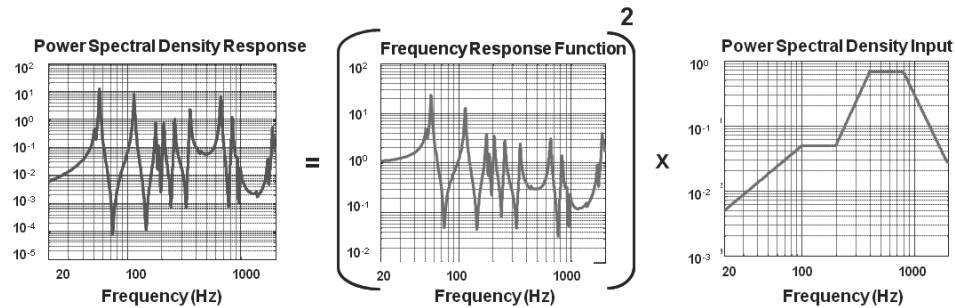


Figure 7.12 The PSD response is computed as the PSD input multiplied by the square of the frequency response function.

7.4.3 Random-vibration SDOF response

Based on a lightly damped SDOF system with white-noise random input (white noise is a constant PSD input over the frequency spectrum), first-order random response estimates may be generated for systems that have a dominant natural frequency. These values are useful for quick estimates of performance or for validating detailed simulations. Expressions for force-excited and base-excitation systems are discussed below.⁵

7.4.3.1 Random force excitation example

For a force-excited SDOF system, the RMS displacement subject to a random white-noise PSD forcing function may be computed using the following relationship:

$$Disp_{rms} = \sqrt{\frac{\frac{\pi}{2} \cdot f_n \cdot Q \cdot PSD_{Input}}{k^2}} \quad (7.35)$$

This relationship can be used to compute the response of a 50-Hz structure (stiffness, $k = 254.4$ lbs/in) subject to a random, white-noise force PSD of $0.2 \text{ lb}_f^2/\text{Hz}$. The percent critical damping ζ in the system is 0.5%. The amplification Q is computed as $Q = 1/2\zeta = 100$. The resulting RMS displacement is shown in Fig. 7.13.

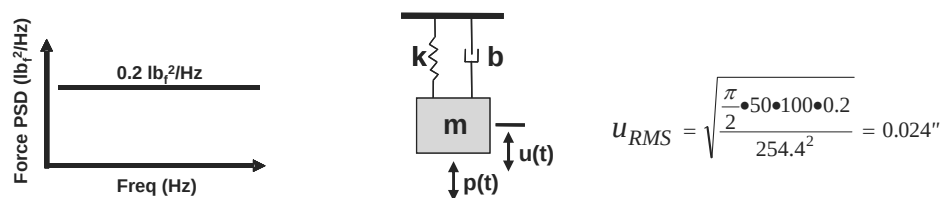


Figure 7.13 Displacement response of a SDOF system due to a random forcing function.

7.4.3.2 Random base excitation: absolute motion example

For a random base-excited SDOF system with a white-noise acceleration PSD input, the absolute acceleration is computed as

$$G_{rms} = \sqrt{\frac{\pi}{2} \cdot f_n \cdot Q \cdot PSD_{Input}}. \quad (7.36)$$

This relationship may be used to estimate the acceleration response of a primary mirror mounted on three bipods that is subject to a random white-noise base acceleration. The white-noise PSD acceleration input is $0.1 \text{ G}^2/\text{Hz}$. The fundamental frequency of the mirror on the bipods is 100 Hz , acting in the same direction as the input. The critical damping ratio is 1% , resulting in a Q of 50 . The acceleration response is computed in Fig. 7.14.

7.4.3.3 Base excitation: relative motion example

For a random base-excited SDOF system with a white-noise acceleration PSD input, the relative displacement is computed as

$$Z_{rms} = \sqrt{\frac{\frac{\pi}{2} \cdot f_n \cdot Q \cdot PSD_{Input}}{(2\pi f_n)^4}}. \quad (7.37)$$

where z is the difference between the motion of the mass and the base.

Using this expression, the relative motion of an optical system mounted on a 5-Hz vibration isolation system with an amplification of 10 subject to a base white-noise PSD excitation of $0.01 \text{ G}^2/\text{Hz}$ may be estimated to ensure adequate sway space. The calculation is shown in Fig. 7.15.

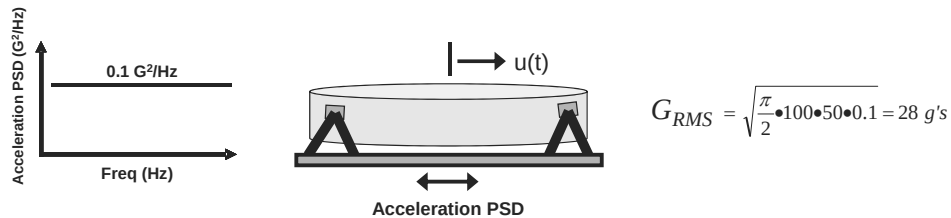


Figure 7.14 Acceleration response of the primary mirror due to random base excitation.

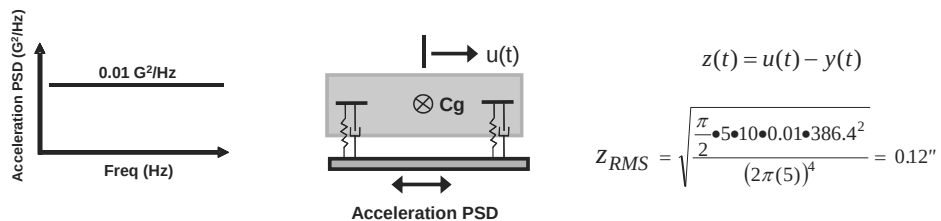


Figure 7.15 Relative motion of sensor subjected to random base acceleration PSD.

7.4.4 Random vibration design levels

Random vibration design accelerations are typically chosen as the 3σ levels that envelop 99.7% of the response quantities assuming a Gaussian distribution. However, the longer the time of exposure of a system to random vibration, the greater the chances the 3σ acceleration levels are exceeded. This is illustrated in the following example.

The peak random acceleration levels as a function of time are compared for three different time-history durations (1 sec, 10 sec, and 60 sec). Each of the time histories was computed based on the acceleration PSD shown in Fig. 7.16. The time histories for each of the three durations and the peak acceleration levels are shown in Fig. 7.17 and listed in Table 7.1, respectively. As expected, the peak acceleration increases as the duration of the time history increases. For the 60 s time history, the peak acceleration reaches over 5σ . Designing for 5σ levels for random vibration is an accepted practice and may be adopted as a more conservative approach based on the design philosophy of the program. More details are discussed in NASA-HDBK-7005.

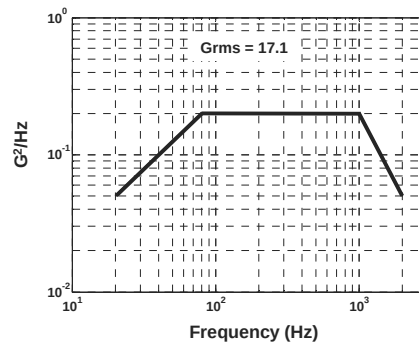


Figure 7.16 PSD acceleration spectrum with RMS value of 17.1 G_{rms} .

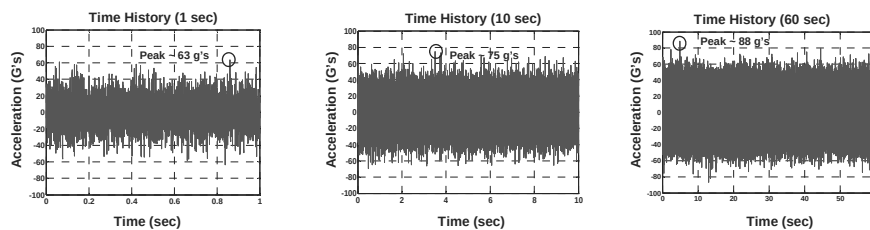


Figure 7.17 Random time histories of varying duration and peak accelerations.

Table 7.1 Peak accelerations versus time duration.

Time (s)	Max G's	RMS x
1	63	3.7
10	75	4.4
60	88	5.1

7.5 Vibro-Acoustic Analyses

Vibro-acoustic disturbances are a form of direct random excitation that act on optical systems in various service environments including airborne and spaceborne systems. Vibro-acoustic analyses account for the random fluctuating sound pressure distributions impinging on a structure that are inherently complex due to the nature in which sound gets transmitted, absorbed, and reflected within a structure. The patch or split loading method is an approximate technique that addresses the spatial coherence of diffuse acoustic waves using standard finite element analysis tools^{6,7} to represent partially correlated pressure distributions that act over a surface. More sophisticated techniques beyond the scope of this book include statistical energy analysis and combined finite element and boundary element techniques that account for the air–structure interaction.

7.5.1 Patch method

The patch method is illustrated using a flat-plate example subject to acoustic excitation, as shown in Fig. 7.18. The first step converts the sound pressure levels (SPL) in units of dB into a pressure spectral density (psi^2/Hz).⁸ This process is outlined in Fig. 7.19.

The patch method divides the pressure PSD into frequency ranges and applies uncorrelated, random pressure distributions over regions or patches of the

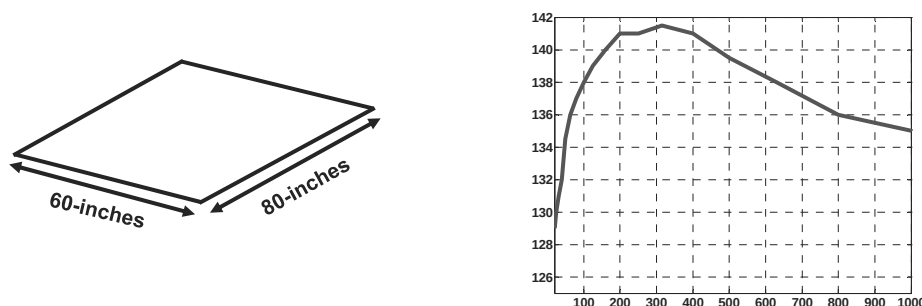


Figure 7.18 Acoustic pressure acting normal to the surface of a flat plate.

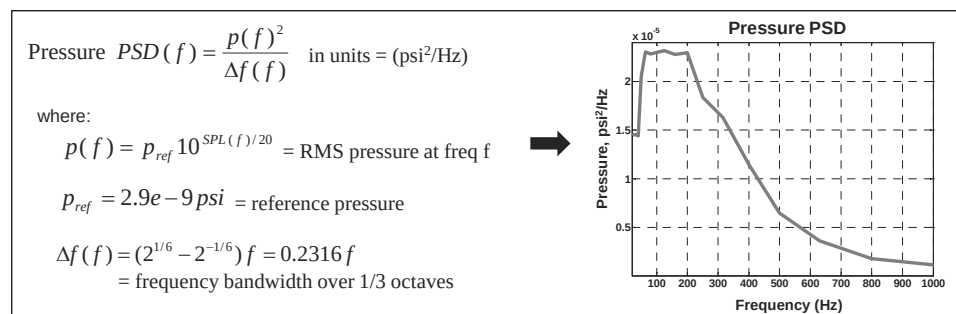


Figure 7.19 Converting sound pressure levels to PSD pressure levels.

structure. In the low-frequency region, uniform random pressure is applied over the structure. The structure is then broken into patches where it is assumed that uncorrelated random pressure distributions of higher frequency act over each patch. The structure can then be continually broken into smaller patches or regions in which the higher-frequency ranges are applied. The total response PSD is the sum of the responses applied to each of the patches.

The analysis assumes the pressure is correlated over the patch size whose dimensions or characteristic length D are equal to the acoustic half-wavelength λ_a of the center frequency of the band. The center frequency is computed as a function of the speed of sound c_s (assumed here to be 13200 in/s):

$$f_c = \frac{c_s}{\lambda_a} = \frac{c_s}{2D}. \quad (7.38)$$

The break frequency f_b defines the separation between the frequency ranges over which each random pressure acts for a given patch size:

$$f_b = \sqrt{f_{c1} f_{c2}}, \quad (7.39)$$

where f_{c1} and f_{c2} are the acoustic center frequencies based on the dimensions of the patch characteristic lengths using Eq. (7.38).

Using the plate shown in Fig. 7.18, the characteristic dimension is selected as the short side of the plate: 60 inches. The selection of the characteristic length is user-defined and somewhat arbitrary. This sets the acoustic half-wavelength to 60 inches, and the acoustic wavelength to 120 inches, resulting in f_{c1} equal to 110 Hz. Dividing the plate into four equal patches, with characteristic length of 30 inches, yields f_{c2} equal to 220 Hz. The break frequency is determined to be 156 Hz using Eq. (7.39). Thus, for the patch method acoustic analysis, uniform random pressure acts on the plate over 20–156 Hz, and uncorrelated random pressure acts over the four patches from 156–2000 Hz. This is illustrated in Fig. 7.20 and assumes that no further reduction of the patches is made.

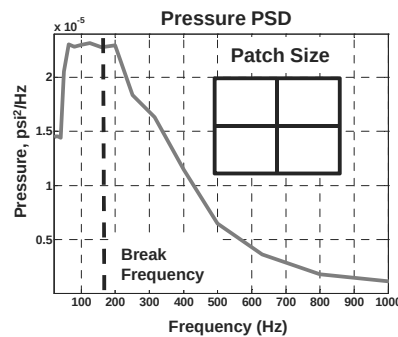


Figure 7.20 Break frequency and patches for random pressure analysis.

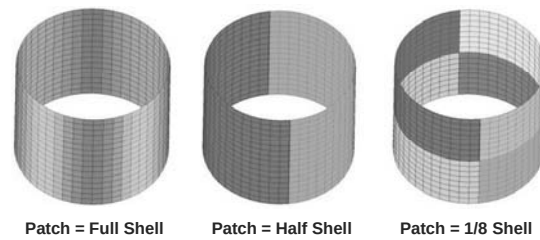


Figure 7.21 Patch sizes for a cylindrical structure.

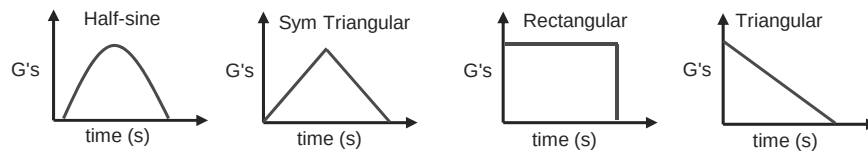


Figure 7.22 Common shock-time-history pulse shapes.

The process can be repeated for smaller sections with higher frequency bands. However, it is typically not carried out too far because the pressure spectral density typically drops off at higher frequencies and the displacement response drops with the inverse of the frequency squared.

The final step runs the finite element analysis with the random pressure distributions applied to all patches. For more complex structures, the characteristic length can vary for each section. For example, each surface may be divided into full, quarter, or eighth size patches. The frequency range can then be adjusted accordingly based on each individual patch size. An example of a cylinder and the patches selected is shown in Fig. 7.21.

Pressure distributions may act on both sides of a surface during acoustic loading. Scale factors may be used to multiply the load in this instance. If the pressure distributions are correlated, a scale factor of two is used in the analysis; if the pressure distributions are uncorrelated, a scale factor of $\sqrt{2}$ is used.

7.6 Shock Analyses

Optical systems are subject to shock environments (short duration impulse loading) incurred due to shipping, transportation, aircraft landing, wind gusts, pyrotechnic devices, and exposure to ordinances in the field. Simple shock pulses are shown in Fig. 7.22. Analytical solutions provide first-order response estimates for these shock inputs for a SDOF representation and may serve to validate numerical simulations.

For complex structures and detailed simulations, FEA may be used to perform shock-response spectrum analyses and transient time-domain simulations. Due to the nature of a shock load, these techniques approximate the true physical behavior. A shock pulse sends stress waves through the structure that propagate at high velocities. The shock wave encounters material interfaces,

joint locations, and structural discontinuities where the energy is transmitted, absorbed, or reflected that is difficult to capture using conventional FEA techniques. Empirical data provides methods to account for attenuation of shock inputs that may be coupled with FEA analysis for a more accurate representation. Testing may be performed to provide behavior of specific design configurations and used in conjunction with analyses to more accurately predict the behavior of a structure subject to shock loads.

7.6.1 Shock response spectrum analyses

A shock response spectrum (SRS) is a convenient means to describe the shock design environment. A shock response spectrum is created by driving a series of single-degree-of-freedom oscillators with a time history and plotting the peak acceleration response as a function of frequency, as shown in Fig. 7.23. Typically, several curves are created as a function of damping. Note that this curve should not be used to determine the maximum static acceleration for a complex system based on the natural frequency. A high-frequency shock wave dissipates quickly within a structure and typically does not have time to excite a complex mode shape of the structure such as it would in a SDOF system.

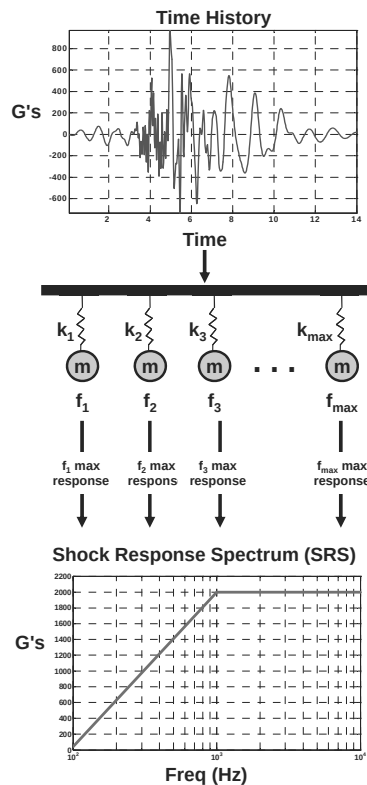


Figure 7.23 Creation of a shock response spectrum analysis curve for a given time history input.

Most FEA codes offer the ability to perform a shock response spectrum analysis using the SRS design curve. This is an approximate technique that is computationally efficient. Modal participation factors are computed for each mode as a function of the applied direction and the system response is determined by combining the maximum response of each of the modes of the structure. Because there is no knowledge of the phase information between the modes, a variety of approaches are available to combine the modal response quantities ranging from conservative to non-conservative.

7.6.2 Shock analysis in the time domain

A shock analysis may be performed in the time domain by using the time history of the shock load. If the time history is not provided, a time history may be created from a SRS curve. Numerically, this is performed using wavelet analysis or damped sinusoids.⁹ No single time history is unique in representing the SRS; however, the analyst can compare the response of multiple time histories to bound the response or gauge the sensitivity. Working in the time domain is computationally expensive and requires more data processing than the corresponding SRS method to obtain the peak responses.

7.6.3 Attenuation of shock loads

The magnitude of a shock impulse load is attenuated at bolted interfaces and by the distance the stress wave travels as it traverses a structure. This phenomenon is difficult to model using conventional FEA tools. However, attenuation factors based on empirical tests may be used during the analysis to reduce the shock levels at the base of critical components. For example, accounting for bolted joints and distance can reduce the SRS curve at the base of an optical component, such as a primary mirror, for a more accurate representation of the load. A single mechanically fastened bolted interface reduces the shock levels by 35–50% for up to a total of three bolted interfaces. In addition, the shock load is attenuated by distance traveled.^{6,10} This is illustrated in predicting the shock design levels at the mount of a primary mirror located 15 inches from the base shock input with one bolted interface between the base and the mount. The shock response spectrum at the primary mirror may be derated by 35% due to distance and an assumed 40% by the bolted interface, for a total attenuation of 61%. The resulting SRS curve as compared to the nominal base input is shown in Fig. 7.24.

7.7 Line-of-Sight Jitter

Line-of-sight (LOS) jitter is an important consideration for both imaging and non-imaging systems caused by internal or external dynamic loads acting on an optical system. For an imaging system, vibration disturbances cause the optical elements to vibrate, which causes the image of a stationary object to “jitter” on the image plane as depicted in Fig. 7.25. Due to the effects of jitter, the image is

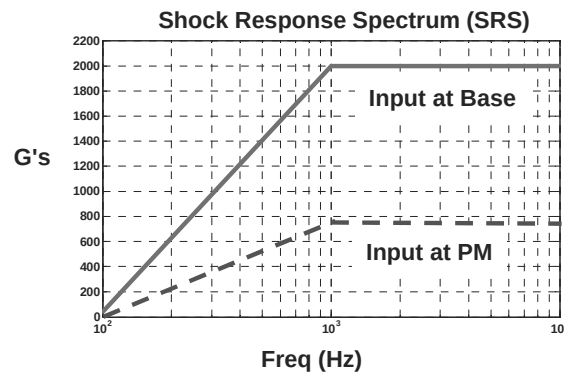


Figure 7.24 Resulting SRS curve at the primary mirror (PM) after attenuation due to distance and bolted joints.

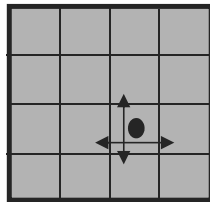


Figure 7.25 Transverse image motion on a focal-plane pixel array.

blurred or smeared on the detector that results in the loss of image quality and optical performance. Typical design goals for imaging systems are to limit the transverse image motion to a fraction of a pixel subject to dynamic loads. Reducing the LOS jitter to less than a quarter pixel is a good rule of thumb. For non-imaging systems, such as laser communication systems, laser beams need to accurately point over long distances to maintain the communication link in the presence of vibration disturbances. In this case, a good starting design point is to limit the angular jitter to a tenth of a beamwidth.

This section discusses the computation and the effects of transverse image motion in the plane of the detector. However, vibratory loads also induce longitudinal image motion or defocus that displaces the image along the optical axis. The impact of longitudinal image motion on optical performance is typically not as severe as transverse image motion of the same amplitude. In addition, computing the effect of longitudinal image motion on optical system performance is more complicated than for transverse image motion since the transfer functions are coupled.¹¹

7.7.1 LOS jitter analysis using FEA

LOS jitter may be computed using finite element analysis in the time or frequency domain. The LOS jitter simulation process is depicted in Fig. 7.26. A finite element model is developed that captures the structural dynamics behavior of the optical system subject to vibration disturbances and predicts the rigid-body

motions of the optical elements. LOS jitter optical sensitivity coefficients are used to multiply the rigid-body motions of the optical elements to compute the resulting image motion. Optical-system pointing errors may also be computed using these same techniques due to static loads, such as gravity and temperature changes.

Computation of the optical sensitivity coefficients can be performed using optical design software by manually perturbing each element/surface and computing the change in the position of the image, as illustrated in Fig. 7.27. Here the optical sensitivity coefficient is computed as the transverse image motion divided by the tilt of the primary mirror. Alternatively, optical sensitivity coefficients may be based on built-in tolerance algorithms in the optical design software or computed via analytical expressions.¹² The image point is typically defined as the on-axis chief ray or image centroid.

This method assumes that the image motion is a linear function of the optical element rigid-body displacements. Representation of the LOS jitter equations in the finite element model is performed by defining a LOS jitter node whose response is a linear summation of the rigid-body motions of the optical elements weighted by the optical sensitivity coefficients. The linear summation and weighting may be represented in FEA codes by use of a multi-point constraint (MPC) equation. This allows the response of a node either in the time or frequency domain to represent the image motion subject to user-defined dynamic disturbances. The use of FEA in predicting LOS jitter using these techniques is presented for an airborne imaging sensor and a pair of a laser communication systems.¹³⁻¹⁵ Commercial tools are available that automate the creation of the LOS jitter equations for use in finite element software.¹⁶

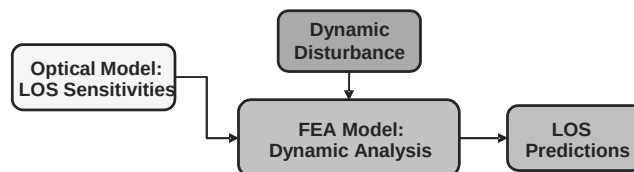


Figure 7.26 LOS-jitter-simulation process.

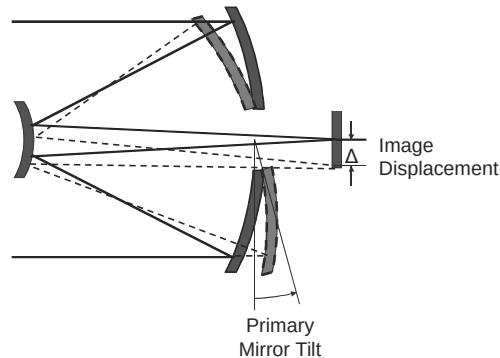


Figure 7.27 Primary mirror rigid-body tilt resulting in transverse image motion.

7.7.2 LOS jitter in object and image space

LOS jitter may be expressed in either object or image space. In image space, jitter is computed as the transverse motion of the image, Δ_x and Δ_y , on the detector plane. In object space, jitter is measured as the angular change, θ_{El} and θ_{Az} , in the position of an object located at infinity. The relationship between jitter in image and object space is shown in Fig. 7.28 and expressed as

$$\theta_{obj} = \frac{\Delta_{image}}{f_{eff}}. \quad (7.40)$$

The LOS jitter equations in matrix form for both image and object space are expressed below where the LOS jitter is computed by multiplying the rigid-body motions of the optical elements $\{X\}_{Optics}$ by the optical sensitivity coefficients $[L]$ for each optical element in 6 DOF:

$$\begin{Bmatrix} \Delta x \\ \Delta y \end{Bmatrix} = [L]_{Img} \{X\}_{Optics}, \quad (7.41)$$

$$\begin{Bmatrix} \theta_{El} \\ \theta_{Az} \end{Bmatrix} = [L]_{Obj} \{X\}_{Optics}. \quad (7.42)$$

Optical sensitivities are computed accordingly to relate optical-element rigid-body motions to the proper space—object or image—and are denoted by the matrices $[L]_{Img}$ and $[L]_{Obj}$.

7.7.3 Optical-element rigid-body motions

LOS jitter analysis requires the rigid-body motions of an optical element or surface to be computed in the finite element model. The manner in which the rigid-body motion is determined depends on the FEA modeling approach. If a single node is used to represent the optical surface/element, then rigid-body displacements are directly obtained. If the analyst uses a shell- or solid-element representation of the optical surface/element, then the rigid-body motion may be

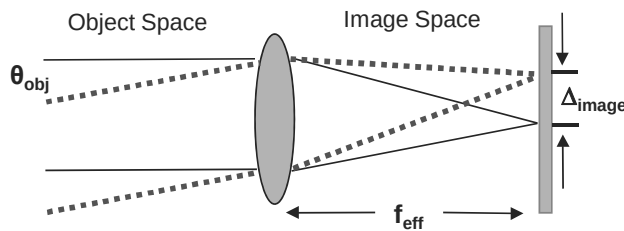


Figure 7.28 Image and object space LOS errors.

computed as the average rigid-body motion of the nodes on the optical surface. In this case, the use of an interpolation element in the finite element model is an effective approach to directly output the average rigid-body motions. Off-axis optical elements typically require special attention to relate the mechanical and optical coordinate systems using the techniques described in Section 4.1.2.

7.7.4 Cassegrain telescope LOS jitter example

Line-of-sight jitter equations are developed for a Cassegrain telescope shown in Fig. 7.29. The optical elements include the primary mirror M_1 , secondary mirror M_2 , and the image plane IP. The distance between the primary and the secondary mirror is 45.1 in., and the distance between the secondary mirror and the image plane is 58.9 in. The effective focal length of the system is 529.7 in.

Image space optical sensitivities are computed for each of the optical elements in six degrees-of-freedom and are listed in Table 7.2. The image motion equations for the image motion, Δ_x and Δ_y , are given as

$$\begin{aligned} \Delta_x = & 10.43(M_1\Delta_x) - 1059.4(M_1\Delta_\beta) - 9.43(M_2\Delta_x) \\ & + 117.8(M_2\Delta_\beta) - (\Delta_x \text{ of image plane}), \end{aligned} \quad (7.43)$$

$$\begin{aligned} \Delta_y = & 10.43(M_1\Delta_y) + 1059.4(M_1\Delta_\alpha) - 9.43(M_2\Delta_y) \\ & - 117.8(M_2\Delta_\alpha) - (\Delta_y \text{ of image plane}). \end{aligned} \quad (7.44)$$

7.7.5 LOS rigid-body checks

Successful LOS jitter analysis requires that the LOS equations have been properly constructed including ensuring that the optical element geometric location, coordinate systems, sign conventions, and units are consistent between the optical and finite element models. It is recommended that prior to performing a LOS jitter simulation that the LOS jitter equations are checked and verified. A simple method to do that involves performing rigid-body checks. These checks can be performed by translating and rotating the telescope as a rigid body in six degrees of freedom and computing the resulting LOS errors. The checks may be performed by hand, spreadsheet, simple stick finite element model, or full

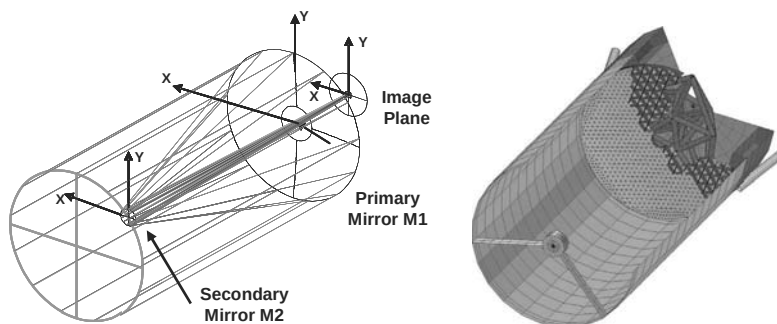


Figure 7.29 Cassegrain telescope optical and finite element models.

Table 7.2 Cassegrain telescope image space optical sensitivity coefficients.

Rigid-Body Motion	Sensitivity Coefficient	
	Δ_x	Δ_y
M1 Δ_x	10.43	0
M1 Δ_y	0	10.43
M1 Δ_z	0	0
M1 $\Delta\alpha$	0	1059.4
M1 $\Delta\beta$	-1059.4	0
M1 $\Delta\gamma$	0	0
M2 Δ_x	-9.43	0
M2 Δ_y	0	-9.43
M2 Δ_z	0	0
M2 $\Delta\alpha$	0	-117.8
M2 $\Delta\beta$	117.8	0
M2 $\Delta\gamma$	0	0
IP Δ_x	-1	0
IP Δ_y	0	-1
IP Δ_z	0	0
IP $\Delta\alpha$	0	0
IP $\Delta\beta$	0	0
IP $\Delta\gamma$	0	0

Sensitivity units: translation ($\mu\text{in}/\mu\text{in}$), rotation ($\mu\text{in}/\mu\text{rad}$)

telescope FEM. In the simple stick FEM, only the nodes of the optical surface/elements are necessary. They may be connected using 1D or rigid elements.

7.7.5.1 LOS rigid-body checks example

Two rigid-body checks are performed for the Cassegrain telescope with an object at infinity and collimated light entering the optical system. The first rigid-body check translates the telescope which for an object at infinity should result in no change in the position of the image. Second, tilting the telescope about the x or the y lateral axes should result in an apparent angular shift of the object that is equal to the angle of rotation of the telescope. These two rigid-body checks are performed below to verify that the LOS equations have been properly constructed.

First, the telescope is translated in the y direction 100 μinches . Substituting the appropriate values into Eqs. (7.43) and (7.44) yields the image motion:

$$\Delta_x = 0.0 \quad (7.45)$$

and

$$\Delta_y = 10.43(100) + 1059.4(0.0) - 9.43(100) - 117.8(0.0) - 1.0(100) = 0.0. \quad (7.46)$$

As expected, the position of the image does not change.

Second, a 1- μrad rigid-body rotation about the vertex of the secondary mirror is applied. Substituting in the resulting mechanical perturbations of the individual optical elements results in the following image motion:

$$\Delta_x = 0.0 \quad (7.47)$$

and

$$\begin{aligned} \Delta_y &= 10.43(0.0) + 1059.4(1.0) - 9.43(45.12) - 117.8(1.0) - 1.0(-12.95) \\ &= 529.7 \mu\text{in.} \end{aligned} \quad (7.48)$$

The LOS angular error may be computed from the image motion Δ_y by dividing by the effective focal length of the optical system:

$$\theta_{obj} = \frac{\Delta_y}{f_{eff}} = \frac{529.7 \mu\text{in}}{529.7 \text{ in}} = 1.0 \mu\text{rad}. \quad (7.49)$$

Thus the angular LOS error in object space is equal to the applied rigid-body rotation of the optical instrument as expected.

7.7.6 Radial LOS error

A single radial LOS error term may be calculated by vector summing the two image-motion terms Δ_x and Δ_y or the angular error terms θ_{El} and θ_{Az} , as illustrated in Fig. 7.30. This method may be used to combine static error terms (i.e., pointing errors) or used to approximate the combination of harmonic and/or random error terms. This approach is approximate for harmonic and random response since it does not account for the phase relationship between the two LOS error components.

An approach to compute a radial LOS error for harmonic and random motion that accounts for the phase relationship between the two components is to determine the phase angle at which the maximum radial LOS error occurs for a given frequency.

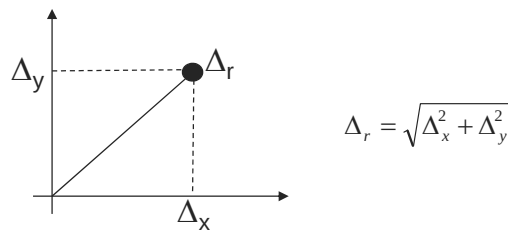


Figure 7.30 Radial LOS error computed as the vector sum.

Consider harmonic motion at a single driving frequency where the two components of image motion are expressed as

$$\begin{aligned}\Delta_x &= |\Delta_x| \cos(\Theta + \Phi_x), \\ \Delta_y &= |\Delta_y| \cos(\Theta + \Phi_y),\end{aligned}\quad (7.50)$$

where $\Theta = \omega t$ ranges from 0 to 2π radians over a full cycle, and Φ is the phase for each component.

Determining the phase angle where the radial LOS error is maximum may be performed mathematically by defining a least-squares error function,

$$\Delta_{rmax}^2 = \Delta_x^2 \cos^2(\Theta + \Phi_x) + \Delta_y^2 \cos^2(\Theta + \Phi_y), \quad (7.51)$$

taking the derivative, setting the derivative equal to zero, and solving Eq. (7.52) for the angle Θ :

$$\frac{d\Delta_{rmax}}{d\Theta} = 0 \rightarrow -\tan(2\Theta) = \frac{\Delta_x^2 \sin(2\Phi_x) + \Delta_y^2 \sin(2\Phi_y)}{\Delta_x^2 \cos(2\Phi_x) + \Delta_y^2 \cos(2\Phi_y)}. \quad (7.52)$$

The maximum radial LOS error may be computed for each driving frequency in a frequency response analysis. In general, the phase angle that produces the maximum radial LOS error at a given frequency will vary and thus the LOS error frequency response function will represent an upper bound. The radial LOS error frequency response function may be used to compute the radial LOS error due to random vibration loads. For complex structures with multiple modes participating in the response, the radial LOS error computed using this approach and the radial LOS error computed by vector summing produce similar results.

7.7.7 Identifying the critical structural modes

Identifying the structural modes that are the largest contributors to LOS jitter provide insight into the dynamic behavior of the optical structure and potential mechanical design changes. Both the PSD response curves and the backward and forward cumulative RMS plots are useful means to identify the frequencies of the sensitive structural modes. The cumulative RMS curves plot the total cumulative RMS response for a given frequency backwards to the end frequency or forward to the beginning frequency. Both the LOS PSD response and the cumulative RMS response plots were computed for the Cassegrain telescope example subject to random vibration excitation. The LOS PSD response is shown in Fig. 7.31, and the corresponding cumulative RMS plots are shown in Fig. 7.32

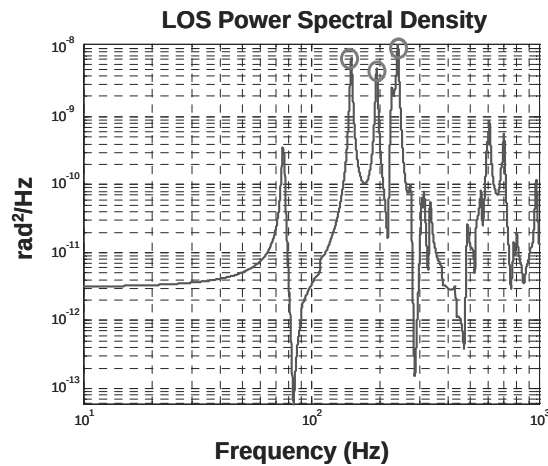


Figure 7.31 LOS PSD response for the Cassegrain telescope with the three peak modes highlighted.

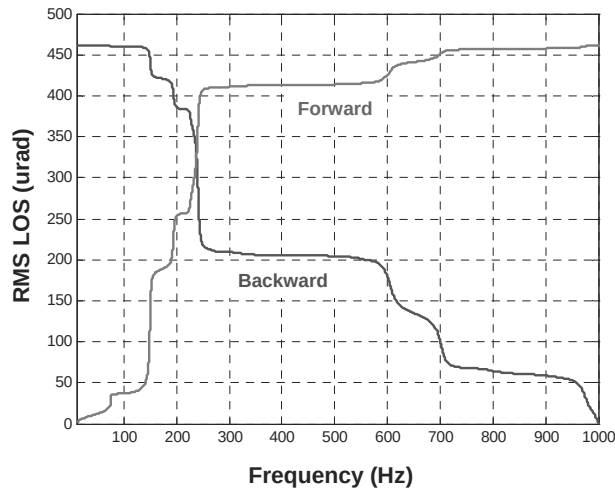


Figure 7.32 LOS cumulative sum curves (forward and backward) for the Cassegrain telescope.

Computing each mode's percent contribution to the total LOS error is a more exacting means to identify the specific modes that contribute to the LOS error.¹⁷ This analysis provides greater insight beyond that of the LOS PSD response and cumulative sum plots by quantifying the contributions of the primary modes and differentiating between the responses of closely spaced modes. In this example analysis, the RMS LOS error is computed one mode at a time, and divided by the total summation of all the modes to compute a percent contribution. The results of this analysis for the Cassegrain telescope example are listed in Table 7.3. The critical LOS modes for the telescope are modes 28, 45, and 60 that contribute 16.3%, 16.3%, and 46.5%, respectively, to the total LOS error.

Table 7.3 Percent LOS jitter contribution per mode for the Cassegrain telescope.

Mode	Freq	% LOS
25	149.6	0.0
26	149.61	0.0
27	149.76	0.0
28	149.9	16.3
44	187.87	0.0
45	193.96	16.3
59	239.47	0.0
60	240.43	46.5
61	244.95	0.0

Table 7.4 Optical element contributions for the significant modes of the Cassegrain telescope example for LOS error in the y-direction.

Mode#	Total-LOS-X	Total-LOS-Y	PM LOS-X	PM LOS-Y	SM LOS-X	SM LOS-Y	FPA LOS-X	FPA LOS-Y
28	0.0	-201.7	0.0	-194.2	0.0	-7.8	0.0	0.3
45	0.0	-73.6	0.0	-110.4	0.0	37.8	0.0	-1.0
60	0.0	-309.6	0.0	-308.4	0.0	-2.1	0.0	0.9

Once the critical modes have been identified, the motion of each of the optical elements in the LOS optical train can be determined by looking at the eigenvector for each mode. Multiplying the motions of the optical element by the optical sensitivity coefficients identifies the optical element(s) that contribute the most to the LOS error for a given mode. A breakdown of the individual optical element contributions for image motion in the y direction for the critical LOS modes of the Cassegrain telescope is shown in Table 7.4.

7.7.8 Effects of LOS jitter on image quality

Transverse image motion due to dynamic excitation smears the image intensity across the detector plane increasing the effective size of the PSF resulting in loss of optical resolution as illustrated in Fig. 7.33. A corresponding modulation transfer function (MTF) may be computed based on the blurred PSF that accounts for the effects of the jitter. Closed-form expressions exist to compute the MTF for various forms of image motion including constant velocity, sinusoidal, and random image motion^{18,19} that are presented below. An overall optical system MTF may be computed accounting for the effects of jitter by multiplying the nominal optical system MTF by the MTF computed due to the effects of jitter as given as

$$MTF_{\text{system}} = MTF_{\text{nominal}} * MTF_{\text{jitter}} \quad (7.53)$$

The effects of jitter on the performance of an airborne optical system is shown in Fig. 7.34. The optical system MTF is shown for the system on the ground and airborne where the effects of jitter are included in the overall MTF.

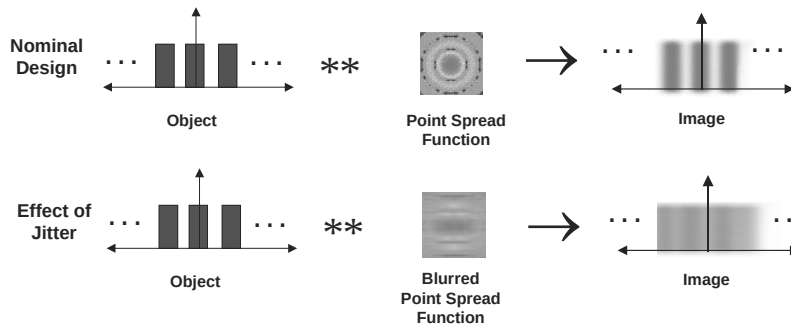


Figure 7.33 Image formation including the effects of jitter.

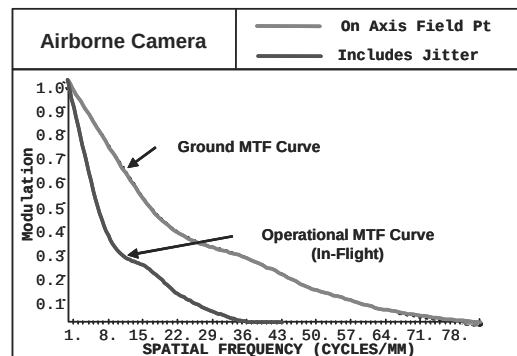


Figure 7.34 The effect of jitter on the MTF of an example airborne optical system.



Figure 7.35 Point spread function produced by constant velocity image smear.

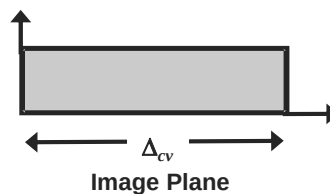


Figure 7.36 The line spread function describing the shape of the image blur for constant velocity motion.

7.7.8.1 Constant-velocity image motion

A camera taking a snapshot of a baseball moving at constant velocity is illustrated in Fig. 7.35. The resulting line spread function (cross-section view of the PSF) of the image results in a rectangle as shown in Fig. 7.36. The image

velocity v multiplied by the exposure or integration time τ_e yields the constant velocity image displacement Δ_{cv} . The MTF is computed as the Fourier transform of the line spread function of the blurred image as

$$MTF(\xi) = \frac{\sin \pi \xi \Delta_{cv}}{\pi \xi \Delta_{cv}}, \quad (7.54)$$

where ξ is the spatial frequency in cycles per millimeter.

7.7.8.2 High-frequency sinusoidal image motion

For sinusoidal image motion characteristic of a dominant harmonic forcing function, the loss in resolution is dependent upon the period of the integration time t_e to the period of the jitter t_j . The integration time refers to the photon-collecting time of the pixel array. High-frequency jitter occurs when the jitter period is less than the exposure/integration period or when $t_e > t_j$. The detector integrates over several cycles of image displacement, as illustrated in Fig. 7.37, where Δ_r is the amplitude of sinusoidal motion.

The line spread function or histogram of sinusoidal image motion is a horseshoe shape, as shown in Fig. 7.38. This is due to the image slowing down and reversing direction at the sinusoidal peaks and then reaching maximum velocity as it crosses the origin during oscillatory motion.

The corresponding MTF for high-frequency harmonic motion is computed as

$$MTF_{HFSinusoidal}(\xi) = J_0(2\pi\xi\Delta_r), \quad (7.55)$$

where J_0 is the zero-order Bessel function.

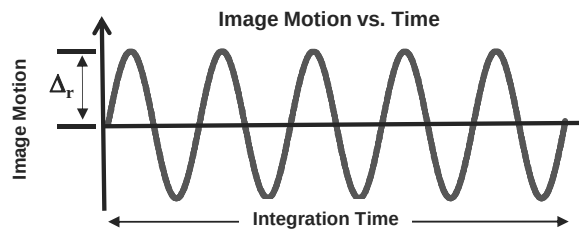


Figure 7.37 High-frequency sinusoidal image motion.

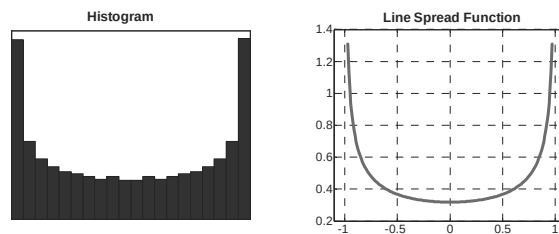


Figure 7.38 Image motion histogram and line spread function for high-frequency sinusoidal motion.

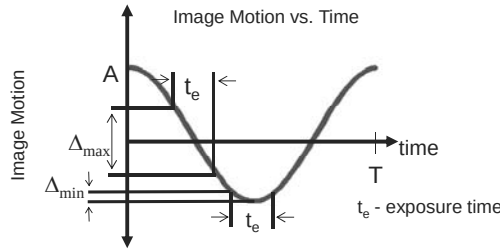


Figure 7.39 Low-frequency sinusoidal image motion.

7.7.8.3 Low-frequency sinusoidal image motion

For low-frequency jitter, i.e., when the jitter frequency is less than the exposure frequency or $t_e < t_j$, the magnitude of the image motion depends upon the phasing between the exposure frequency and the jitter frequency. Image displacement for sinusoidal motion as a function of time is shown in Fig. 7.39. The image displacement depends upon the position of the image at the start of the exposure period and the duration of the exposure. The maximum and minimum image motion for an exposure time of t_e is given by¹⁹

$$\Delta_{\max} = 2A \sin \left[\left(\frac{2\pi}{T} \right) \left(\frac{t_e}{2} \right) \right] \quad (7.56)$$

and

$$\Delta_{\min} = A \left\{ 1 - \cos \left[\left(\frac{2\pi}{T} \right) \left(\frac{t_e}{2} \right) \right] \right\}. \quad (7.57)$$

The resulting MTF may be bounded using the image motion computed in Eqs. (7.56) and (7.57), and substituted in Eq. (7.54). This assumes the resulting motion is linear, which is an approximation.

Alternatively, the MTF may be computed due to low-frequency sinusoidal motion assuming that the motion of the sine wave is at a stationary point at the mid-point of the integration.²⁰ The ratio between the integration period and the vibration period is represented by p .

$$MTF_{LFSinusoidal}(\xi) = J_0(2\pi\xi\Delta_r) + \sum_k \frac{1}{k\pi p} J_{2k}(2\pi\xi\Delta_r) \sin(2k\pi p). \quad (7.58)$$

7.7.8.4 Random image motion

For random image motion, the MTF is related to the RMS image motion as

$$MTF_{JitterRandom}(\xi) = e^{-2\pi^2 \Delta_{rms}^2 \xi^2}, \quad (7.59)$$

where Δ_{rms} is the RMS LOS jitter error in image space due to random excitation.

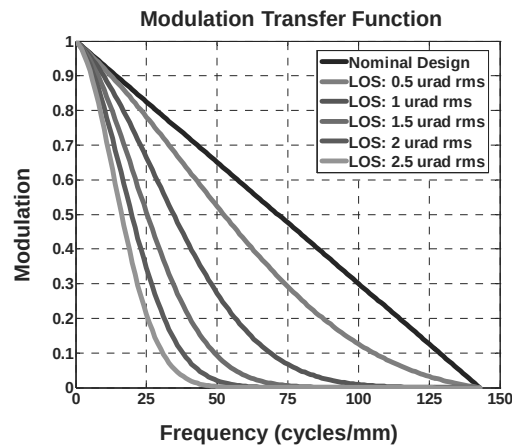


Figure 7.40 MTF curves for the nominal design and accounting for the effects of jitter.

This relationship assumes that the random motion is a statistically independent Gaussian process of zero mean. MTF curves are computed for varying levels of random LOS jitter errors, as shown in Fig. 7.40. The LOS error is reported as an angular error in object space using Eq. (7.40).

7.7.9 Impact of sensor integration time

The integration time of the detector can reduce the effects of harmonic image motion on optical performance by reducing the amplitude of low frequency image motion. This is analogous to increasing the shutter speed of a camera to reduce image smear over the exposure time when taking a picture of a moving object. The effects of integration time on LOS jitter depend on the frequency of the image motion relative to the time of integration. For high-frequency image motion, many cycles of image motion occur over the integration time and the resultant image motion on the detector is oscillatory and reaches full amplitude with zero average pointing error. For low-frequency image motion, the integration time limits the image motion to a fraction of a full cycle that reduces the amplitude of image motion and results in an average pointing error.

The effect of sensor integration time on LOS error is illustrated for a detector with a 50-ms integration time with high and low-frequency image motions of equal amplitude, as illustrated in Fig. 7.41. The 20- and 100-Hz frequencies represent the high-frequency image motions that reach full amplitude over the integration time with no average pointing error. For frequencies less than the integration frequency (2- and 5-Hz signals), the integration time reduces the amplitude of the sinusoidal motion that also results in an average image motion or pointing error over the integration time.

For random image motion that is comprised of both high and low-frequency harmonic motions, the total LOS error may be decomposed into a jitter component and an average pointing error component or drift term²¹ by

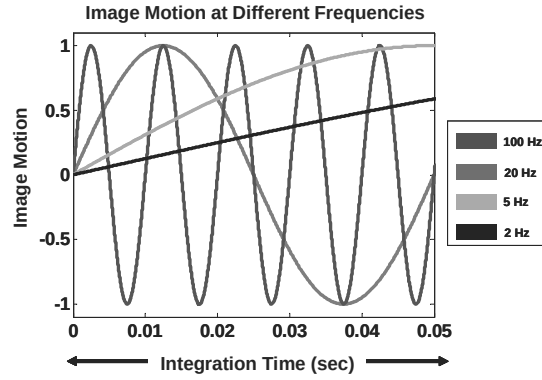


Figure 7.41 The impact of sensor integration time on various frequencies.

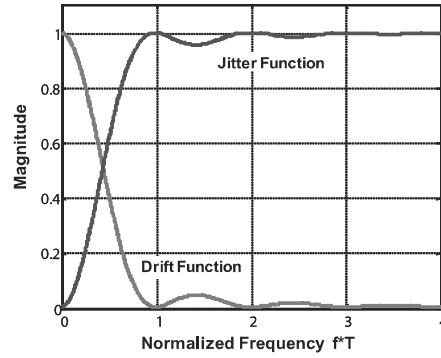


Figure 7.42 Jitter and drift integration time weighting functions.

multiplying the LOS PSD by weighting functions that accounts for the effects of integration time on the low-frequency image motion. The jitter weighting function W_d is expressed as

$$W_d = 1 - 2[1 - \cos(C)] / C^2, \quad (7.60)$$

where $C = 2\pi fT$, f is the frequency in Hz, and T is the integration time. The drift function is $1 - W_d$. The drift and jitter weighting functions are plotted in Fig. 7.42.

The weighting functions are used to multiply the LOS PSD response to compute both the jitter and drift PSD functions due to random disturbances. The jitter and drift RMS values may then be determined by taking the square root of the area under the jitter and drift PSD functions:

$$Jitter_{RMS} = \sqrt{\int_0^{\infty} W_d(f) PSD_{Resp}(f) df}, \quad (7.61)$$

$$Drift_{RMS} = \sqrt{\int_0^{\infty} (1 - W_d(f)) PSD_{Resp}(f) df}. \quad (7.62)$$

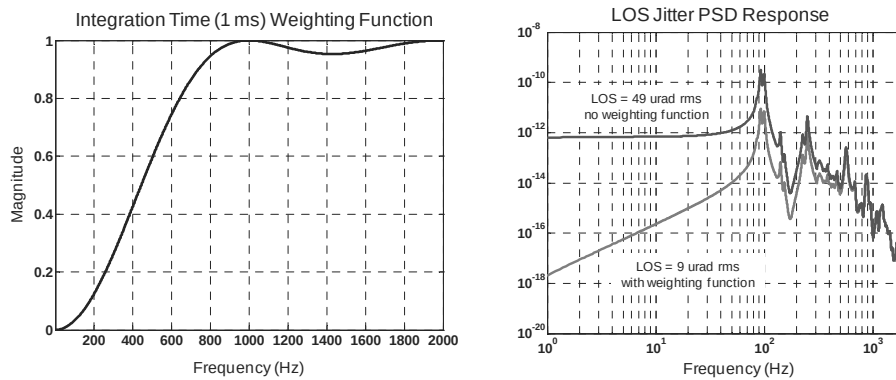


Figure 7.43 Effect of 1-ms integration time on LOS jitter computations.

The effects of a 1-ms integration time on LOS jitter random response is illustrated in Fig. 7.43. The nominal RMS LOS jitter is 49 μrad . Accounting for the effects of the integration time reduces the LOS jitter to 9 μrad . Use of the weighting functions in a random vibration LOS jitter analysis allows sensor integration time to be treated as a variable in the design process.

7.8 Active LOS Stabilization

Active LOS stabilization techniques are commonly employed to stabilize the pointing of an optical beam in the presence of vibration disturbances. For example, fast steering mirrors embedded in the optical train coupled with an optical detector can compensate for rigid-body and elastic flexing of the optical system, as illustrated in Fig. 7.44. Steering mirrors and pointing gimbals coupled with inertial measurement units have the ability to compensate for rigid-body motion of an optical system as shown in Fig. 7.45. More advanced active LOS stabilization simulations require details of the control system and are beyond the scope of this text. However, approximate techniques may be used to represent the active LOS stabilization system coupled with finite element analyses to evaluate the overall stabilization trade space.

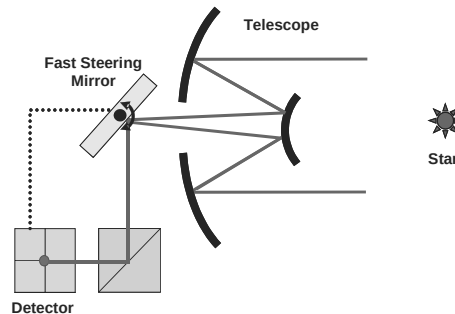


Figure 7.44 Image motion stabilization using a high-speed detector and a fast-steering mirror.

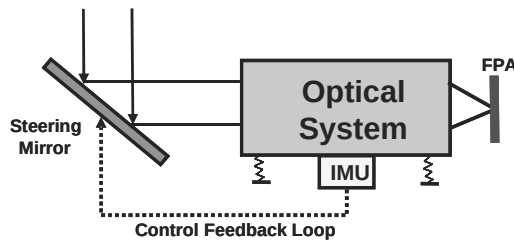


Figure 7.45 Rigid-body stabilization using a steering mirror and an inertial measurement unit.

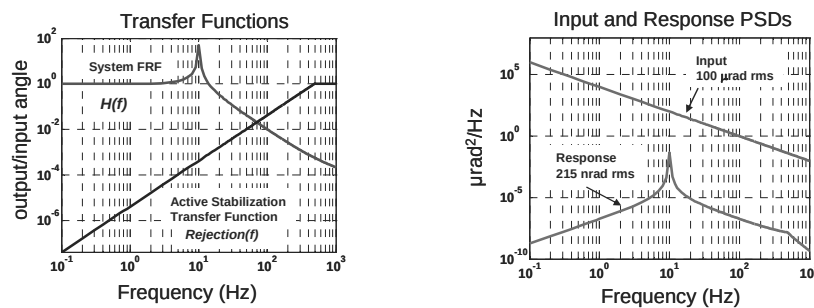


Figure 7.46 Use of a rejection transfer function to predict LOS error.

7.8.1 Image motion stabilization

In the case of image motion stabilization illustrated in Fig. 7.44, the optical detector senses the LOS image motion errors directly. These errors may then be compensated by the fast-steering mirror. A control loop transfer function known as a rejection function may be computed based upon the signal-to-noise ratio, camera frame rate, and the fast-steering mirror bandwidth. The rejection function may then be multiplied with the LOS jitter system transfer function to determine the resulting LOS jitter of the system:

$$\text{LOS PSD}_{\text{Resp}} = [H(f) * \text{Rejection}(f)]^2 \text{PSD}_{\text{in}}(f). \quad (7.63)$$

In this analysis, the bandwidth and shape of the control-system rejection function become additional design variables in the LOS jitter simulations.

An example of the use of a rejection function in the calculation of LOS jitter is shown in Fig. 7.46. The frequency response and rejection functions are plotted on the left that are then squared and multiplied by the angular PSD input levels shown on the right. This results in an angular response of 215 nrad rms.

7.8.2 Rigid-body stabilization

Rigid-body stabilization analysis may be performed in a similar manner to image stabilization analysis. Here, the rigid-body motion of the sensor is measured instead of the image motion and the errors due to flexing or bending of the

optical mounts and support structures, known as elastic errors, are unsensed and cannot be corrected. This form of analysis requires separating the rigid-body and the elastic LOS errors, which can be performed by creating two sets of LOS error equations. The first set is developed using the techniques discussed previously to compute the total LOS error of the sensor that includes both the rigid-body and the elastic response. The second set of LOS jitter equations is developed to compute only the rigid-body LOS errors. This is done by connecting each of the optical elements of the sensor to the base of the structure, short-circuiting the structure's elastic compliance. The elastic LOS error is computed as the difference between the total LOS error and the rigid-body LOS error. The rejection function is then used to multiply the rigid-body LOS error to account for the effects of rigid-body stabilization. The net LOS jitter error may be determined by combining the residual rigid-body LOS error with the elastic LOS error. Depending on the rejection function, the residual and elastic errors may be combined by root-sum-squaring or by operating in the amplitude domain and using complex math to account for the phase relationship.

7.9 Structural-Controls Modeling

LOS stabilization simulations are commonly performed using control system techniques that account for the detailed performance characteristics of the stabilization system. Accounting for the FEA-derived structural dynamic characteristics of the system can be included in these simulations using state-space matrices.

In this approach, the FEA-derived eigenvalues, eigenvectors, and damping values are recast into a set of matrices and represented in state-space form for use in the control simulations. State-space variable models are comprised of coupled first-order differential equations. The general vector-matrix form is expressed below using matrices **A**, **B**, **C**, and **D**:

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{Ax} + \mathbf{Bu}, \\ y &= \mathbf{Cx} + \mathbf{Du}.\end{aligned}\tag{7.64}$$

The equations of motion for structural dynamics in state-space formulation are:

$$\begin{Bmatrix} \dot{\zeta} \\ \ddot{\zeta} \end{Bmatrix} = \begin{bmatrix} 0 & I \\ -\omega^2 & -2b\omega \end{bmatrix} \begin{Bmatrix} \zeta \\ \dot{\zeta} \end{Bmatrix} + \begin{bmatrix} 0 \\ \Phi^T \end{bmatrix} \{\mathbf{P}\},\tag{7.65}$$

$$\begin{Bmatrix} \mathbf{U} \\ \dot{\mathbf{U}} \\ \ddot{\mathbf{U}} \end{Bmatrix} = \begin{bmatrix} \Phi & 0 \\ 0 & \Phi \\ -\Phi\omega^2 & -2\Phi b\omega \end{bmatrix} \begin{Bmatrix} \zeta \\ \dot{\zeta} \end{Bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \Phi\Phi^T \end{bmatrix} \{\mathbf{P}\},\tag{7.66}$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & \mathbf{I} \\ -\omega^2 & -2b\omega \end{bmatrix}; \quad \mathbf{B} = \begin{bmatrix} 0 \\ \Phi^T \end{bmatrix}; \quad \mathbf{C} = \begin{bmatrix} \Phi & 0 \\ 0 & \Phi \\ -\Phi\omega^2 & -2\Phi b\omega \end{bmatrix}; \quad \mathbf{D} = \begin{bmatrix} 0 \\ 0 \\ \Phi\Phi^T \end{bmatrix}. \quad (7.67)$$

The $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, and $\mathbf{D}$ matrices include the modal frequencies ω , the modal damping ratio b , and the eigenvectors Φ . The physical coordinates are denoted by $\mathbf{U}$, and the physical force vector by $\mathbf{P}$. This assumes the modes are mass normalized, which yields the modal mass matrix equal to the identity matrix $\mathbf{I}$. The size of the matrices may be minimized by including only the DOF of the eigenvector with applied input forces in matrix $\mathbf{B}$, and including only the DOF where outputs are computed in matrix $\mathbf{C}$.

7.10 Vibration Isolation

The use of mechanical vibration isolation systems are used to protect sensitive optics from high-frequency vibration. The isolators are located between the source of vibration and the sensor and act as a mechanical low-pass filter. There are many forms of mechanical isolators including pneumatic, elastomeric, coil, wire cable, and others. Each of these isolator options has advantages and disadvantages, and selection depends on the specific application and requirements.

The basic elements of vibration isolation are illustrated with a SDOF system and associated frequency response function shown in Fig. 7.47. In this discussion, the mass of the SDOF is assumed to be the optical system. The objective of the isolation system is to isolate the individual optics from dynamic disturbances that cause performance degradation and/or compromise structural integrity. The isolation system is designed such that the natural frequency is much lower than the natural frequencies of the optical mounts and supporting structures within the optical system. At low-frequency ratios r , where r is the

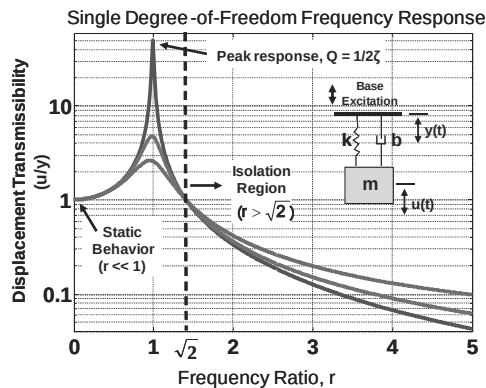


Figure 7.47 The fundamentals of vibration isolation for a SDOF system.

ratio of the forcing frequency to that of the natural frequency of the isolation system, the optical system moves with the base motion. As the forcing frequency nears the natural frequency of the isolation system, the optical system moves essentially as a rigid-body on the isolation system with an amplified response. In the region where r is greater than the square root of two (known as the region of isolation), the higher-frequency excitation is attenuated by the isolation system and the dynamic response of the optical mounts and supporting structure is reduced. The degree of isolation can be varied by changing both the natural frequency of the isolation system and the isolator damping. The lower the isolation frequency is, the greater the attenuation of the high-frequency inputs. Greater isolation is also achieved by using less damping, which increases the roll-off characteristics of the frequency response curve. However, reducing the isolator frequency and damping increases the rigid-body or low-frequency response of the system and this must be balanced with the need for increased high frequency isolation.

7.10.1 Multi-axis vibration isolation

The SDOF isolation principles discussed in the previous section may be applied in the design of a multi-axis or six-DOF vibration isolation system. For a multi-axis isolation system, as shown in Fig. 7.48, the optical sensor will experience six rigid-body modes of vibration.

First-order design objectives for a multi-axis isolation system are to decouple and minimize the frequency spread of the six rigid-body isolator modes. Decoupling means that the translational and the rotational rigid-body modes are independent, and linear excitation does not cause angular response. This may be accomplished by locating the isolators in a plane that passes through the sensor's center of gravity. Minimizing the frequency spread between the six rigid-body modes provides for equal isolation characteristics for each of the six degrees of freedom. These design goals are good starting points for a multi-axis isolation system. The actual design configuration should be tailored for the specific application and performance requirements.

Design and performance of a multi-axis vibration isolation system are also dependent on the geometric configuration of the isolators, the payload mass properties, and the locations available to mount the isolators to the payload and platform. Thus, the design of the isolator configuration should be performed in conjunction with the payload design and in the selection of the platform mounting locations to optimize performance.

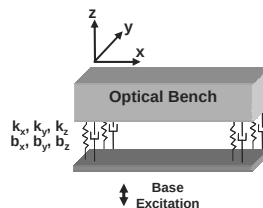


Figure 7.48 Multi-axis vibration isolation of an optical sensor.

7.10.2 Vibration isolation system example

An optical bench is mounted on four vibration isolators, as shown in Fig. 7.49. The design goal is to create a 5-Hz 6-DOF decoupled isolation system. Vibration isolation performance may be assessed by plotting the six-by-six frequency response matrix along with the diagonal frequency response functions. The frequency response matrix reveals if coupling exists between the translational and rotational modes by the presence of off-diagonal terms. Plotting the diagonal frequency response functions provides the frequency spread of the rigid-body isolator modes. Both the six-by-six frequency response matrix and the diagonal frequency response functions are plotted in Fig. 7.50. In this initial case, linear-to-angular coupling exists as seen in the off-diagonal terms in the frequency response matrix. The mode spread of the rigid-body isolator frequencies range from 3–13 Hz.

Design improvements in the performance of the isolation system are achieved by mounting the vibration isolators in the plane of the optical bench's center of gravity as shown in Fig. 7.51. The resulting frequency response matrix and diagonal frequency response functions are shown in Fig. 7.52. The coupling between the rigid-body vibration isolator modes has been eliminated. The frequency of the translational modes is 5 Hz, and the frequency of the rotational modes is 9 Hz.

The design goal of a 5-Hz 6-DOF vibration isolation system is achieved by moving the isolator mounting locations to a position that is twice the radius of gyration of the optical bench (Fig. 7.53) that provides for equal natural frequencies in the translational and rotational degrees of freedom. The performance of this isolation system configuration is shown in Fig. 7.54.

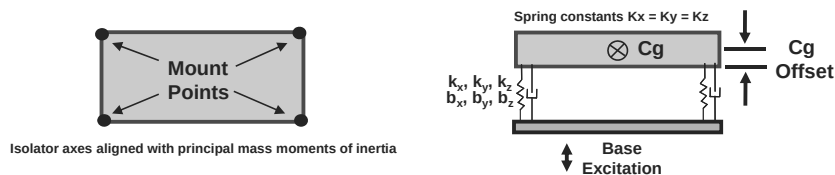


Figure 7.49 Optical bench mounted on four vibration isolators.

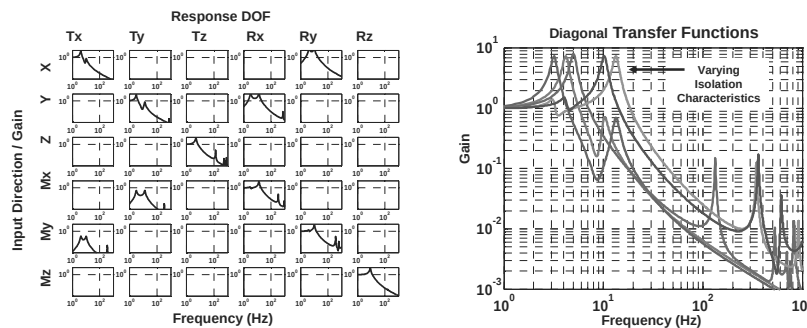


Figure 7.50 Multi-axis vibration isolation initial performance characteristics.



Figure 7.51 Optical bench mounted on four vibration isolators in the plane of the center of gravity of the optical bench.

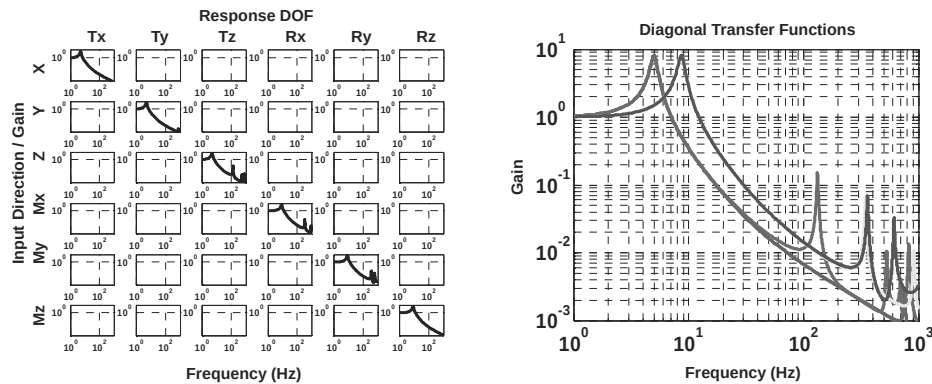


Figure 7.52 Multi-axis vibration isolation performance characteristics with the isolators in the plane of the center of gravity.

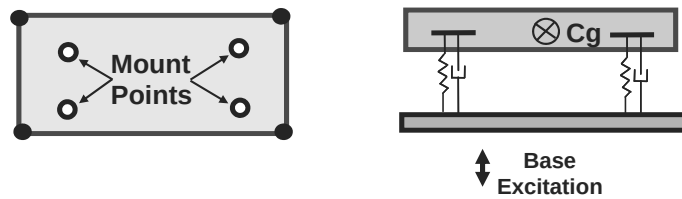


Figure 7.53 Relocating the mounts of the vibration isolators on the optical bench.

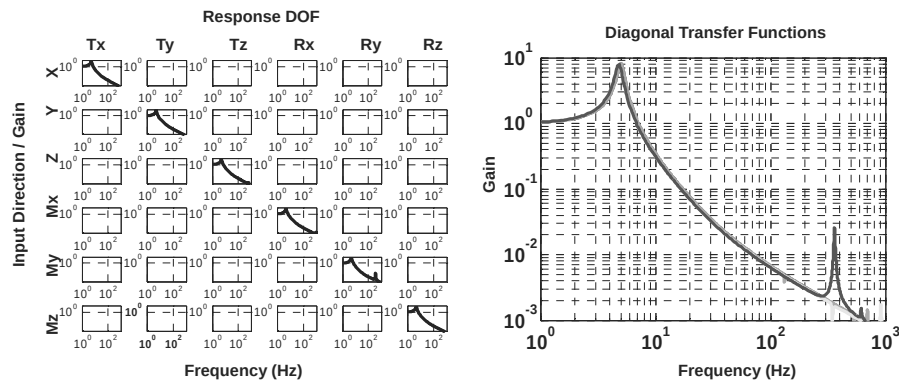


Figure 7.54 Multi-axis vibration isolation performance characteristics for a decoupled and 5-Hz six-DOF design configuration.

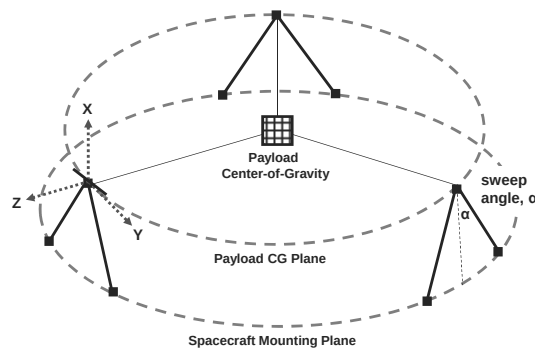


Figure 7.55 Hexapod isolation configuration.

7.10.3 Hexapod vibration isolation systems

A hexapod arrangement of isolators is a common vibration isolation configuration as shown in Fig. 7.55. A starting design configuration to decouple the rigid-body isolator modes is to arrange the three sets of bipods 120 deg apart about the center of gravity with the bipod intersection point lying in the plane of the center of gravity (the isolators need not be physically mounted at the center of gravity but mounted such that the virtual intersection point lies in the plane of the center of gravity). The size of the mounting radius and the angles of the bipods may then be used to tailor the vibration isolation characteristics. For example, increasing the sweep angle of the bipod stiffens the in-plane and twist modes and softens the vertical and out-of-plane rotational modes. Increasing the mounting radius stiffens the rotational modes without impacting the translational modes.

The bipod lean angle may also be varied to decouple the isolator modes when the bipod intersection point does not lie in the plane of the center of gravity. These first-order design guidelines assume the isolator acts as an idealized axial load carrying member, i.e., a single-degree-of-freedom isolator. Higher-order effects include non-zero rotational stiffness at the isolator ends (pivot stiffness), and local lateral and surge modes of the isolator that degrade idealized performance characteristics.

Use of numerical optimization techniques have successfully been employed²² to tailor the isolator design variables and characteristics that account for the geometric constraints on mounting locations to meet isolator design requirements.

7.10.4 Vibration isolation roll-off characteristics

The principles of vibration isolation were introduced using a SDOF mass-spring-damper system, also known as a two-parameter isolator. The magnitude of attenuation or transmissibility of the isolator is based on the roll-off characteristics or slope of the transmissibility curve at high frequency. The high-frequency roll-off of a two-parameter isolator is -20 dB/dec.

Improved isolator performance may be achieved using a three-parameter isolator. Such an isolator includes an additional spring element in series with the

damper element. This system is also known as an elastically coupled viscous damper. The additional spring element allows the damper element to minimize the response near resonance but eliminates the effect of the damper at high frequencies, resulting in a high-frequency roll-off of -40 dB/dec.²³⁻²⁵ A comparison of the transmissibility functions for the two- and three-parameter isolator is shown in Fig. 7.56.

The details of vibration isolation and implementation are beyond the scope of this book. However, more sophisticated and complex passive-isolator techniques exist that may be employed to tailor performance requirements against increased design complexities, aggressive disturbance environments, and the use of active stabilization for high-performance imaging and communication sensors.

7.11 Optical Surface Errors Due to Dynamic Loads

For relatively lightweight and flexible optics such as those considered for large-aperture space systems, the elastic response and resulting optical surface errors due to dynamic loads may not be ignored as in previous sections. This section discusses the complexities associated with predicting the dynamic behavior of an optical surface and techniques to predict the pointing, focus, and overall surface RMS errors due to harmonic and random vibration loads.

7.11.1 Dynamic response and phase considerations

Computing the surface errors of an optic subject to dynamic loading is sufficiently more complex than static loads. Consider, for example, the behavior of three nodes on an optical surface (shown in Fig. 7.57) experiencing harmonic

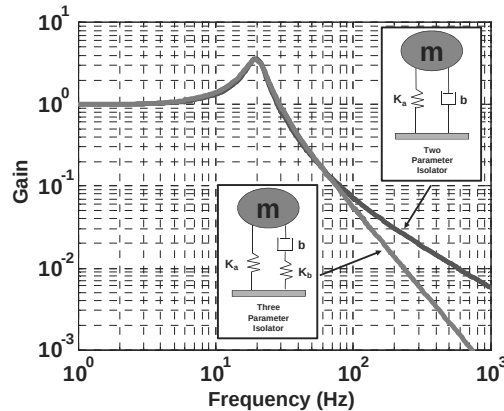


Figure 7.56 Two-parameter and three-parameter isolator performance characteristics.

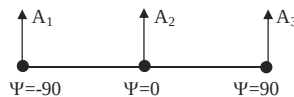


Figure 7.57 Dynamic displacement of three nodes of equal magnitude with varying phase angles.

motion of the form represented by

$$U_k = A_k \cos(\omega t - \psi_k). \quad (7.68)$$

The amplitude of displacement of the three nodes are equal ($A_k = A$) but due to damping the responses of each node are out-of-phase with phase angles ranging from $\psi = -90$ deg to 90 deg. The resulting surface shape is a function of the phase angles and varies at any given point in time over the full cycle of harmonic motion. Four surface shapes as a function of time are shown in Fig. 7.58.

For random response of an optical surface, the RMS displacement for each node on the optical surface may be determined. However, the surface shape remains unknown since all the phase information is lost. For example, for three nodes on a surface where the random RMS displacement has been computed, the analyst has an effective envelope or range that each node may be displaced as illustrated in Fig. 7.59. In this example, the three nodes have the same RMS displacement response. The RMS data provides no information regarding how the nodes move relative to each other and hence the resulting surface shape at any point in time is unknown.

7.11.2 Method to compute optical surface dynamic response

A technique to compute the surface RMS for an optical surface subject to harmonic and random motion is presented. The maximum surface RMS error for a given harmonic forcing frequency may be determined by computing the phase at which the maximum surface RMS occurs. This is performed by creating a least-squares error function

$$Err = \sum_{k=1}^n w_k A_k^2 \cos(\Theta - \psi_k)^2, \quad (7.69)$$

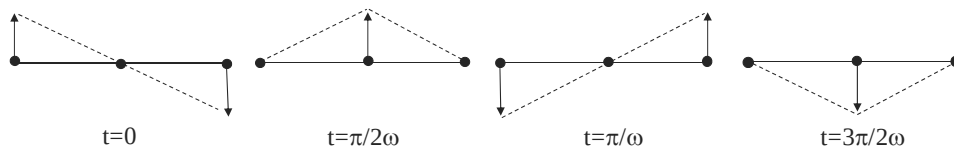


Figure 7.58 The dynamic shape of the surface depends on the phase angle and point in time during the harmonic oscillation.

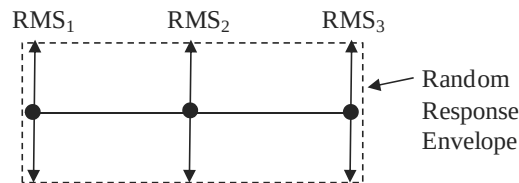


Figure 7.59 Envelope of random displacement response for three nodes on a surface.

where Θ replaces ωt and varies from 0 to 2π , w_k is the area weighting of each node, and A_k is the magnitude of the user-selected displacement component with phase angle ψ_k .

The maximum surface RMS is computed by taking the derivative of the error function with respect to Θ and setting it equal to zero, then solving for Θ , substituting to determine each node's response at that phase angle, and then computing the surface RMS. At a given frequency, the surface shape is now defined by the phase angle that determines the maximum surface RMS. For a frequency response analysis, this may be repeated for each frequency step to compute a maximum surface RMS frequency response function. In general, the maximum phase angle for each driving frequency will vary from frequency to frequency, hence this approach is conservative. The surface RMS frequency response function may then be used in subsequent random vibration analyses to compute random surface RMS error.

7.11.3 Dynamic surface response and modal techniques

A technique to compute optical surface errors subject to dynamic loads operates on the mode shapes²⁶ of the optical surface. In this analysis, the modes are decomposed into various surface errors including rigid-body, focus, and higher-order elastic deformations.

The first step in the analysis is to compute the natural frequencies and mode shapes of the optical element using FEA. The mode shapes are then decomposed into surface quantities of interest similar to a static decomposition. This step is typically performed using post-processing tools external to the FEA code. Modal superposition is then used to compute the combined modal response for each of the decomposed surface errors of the optical element in the presence of dynamic loads.

Consider, for example, a single optical surface with multiple modes j and mode shapes Φ_j . Each mode shape may be decomposed into

RB_j = rigid-body vector for the j^{th} mode,

$Focus_j$ = focus for the j^{th} mode,

$Residual_j$ = the residual vector for the j^{th} mode = $\Phi_j - RB_j - Focus_j$.

The net surface rigid-body, focus, and residual surface deformations are obtained via modal superposition using

$$\begin{aligned} \text{RigidBody} &= [\Phi_{RB}] \{Z\}, \\ \text{Focus} &= [\Phi_{Focus}] \{Z\}, \\ \text{ResidualSurface}_{\text{RMS}} &= \text{RMS}([\Phi_{Residual}] \{Z\}). \end{aligned} \quad (7.70)$$

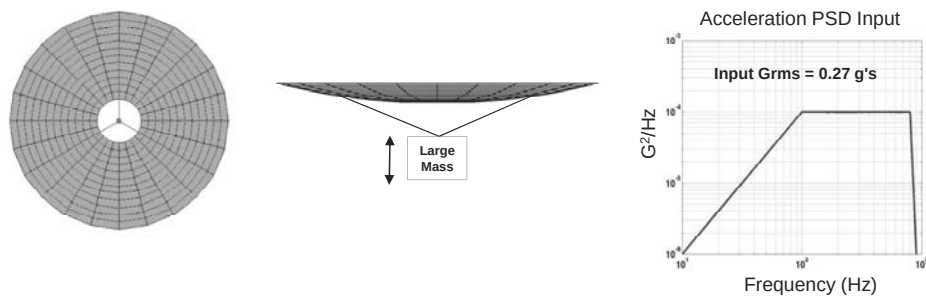


Figure 7.60 Primary mirror subject to a base acceleration PSD.

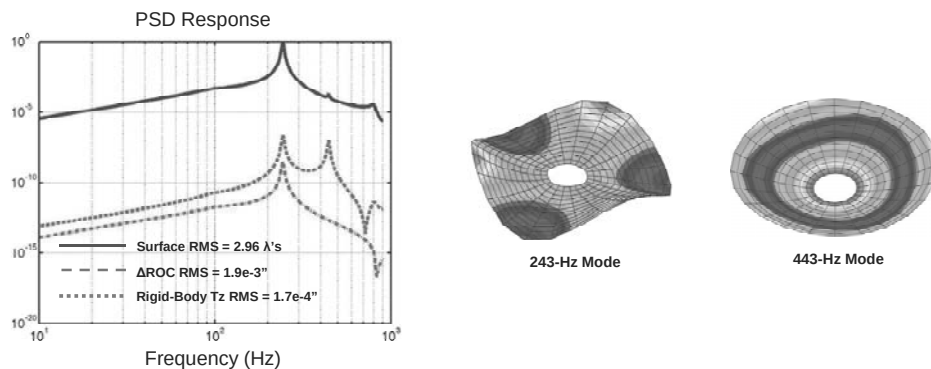


Figure 7.61 Primary mirror axial rigid-body motion (Tz), power change (Δ ROC), and surface RMS errors computed due to random excitation. First two mode shapes are on the right side.

where $\{Z\}$ is the vector of modal participation factors and is complex.

The results of an optical surface deformation analysis of a primary mirror subject to random base acceleration is shown in Fig. 7.60. The axial rigid-body motion, the radius of curvature, and the residual surface RMS errors are shown in Fig. 7.61.

This technique may be used to compute the RMS surface errors for each optical surface in the optical system. Total system performance can be approximated by root-sum-squaring all the contributions. The method in the next section allows a more accurate prediction of system response by accounting for the phase relationships between the optical surfaces.

7.11.4 System wavefront error due to dynamic loads

Optical system wavefront error may be estimated for an optical system by using a variation on the modal methods discussed in the previous section coupled with optical sensitivity matrices.²⁷ The process begins with decomposing each optical surface mode shape into Zernike polynomials as depicted in Fig. 7.62. The Zernike surface errors are then related to wavefront error through a wavefront

error sensitivity matrix. The Zernike sensitivity matrix may be computed using an optical model and adding a single Zernike term of surface error and computing the resulting wavefront error in the form of a vector of Zernike polynomials. The resulting system wavefront error as a function of dynamic loads then relies upon multiplication and modal superposition as illustrated in the flow chart in Fig. 7.63. This technique requires a linear system and that the Zernike polynomials are a good fit to the mode shapes of the optical elements.

$$\text{Total Response} = \Phi_1 + \Phi_2 + \Phi_3 + \dots + \Phi_n$$

$\Phi_1 = \{Z_1, Z_2, Z_3 \dots Z_n\}$
 $\Phi_2 = \{Z_1, Z_2, Z_3 \dots Z_n\}$
 $\Phi_3 = \{Z_1, Z_2, Z_3 \dots Z_n\}$
 $\vdots$
 $\Phi_n = \{Z_1, Z_2, Z_3 \dots Z_n\}$

Figure 7.62 The decomposition of mode shapes into Zernike polynomials.

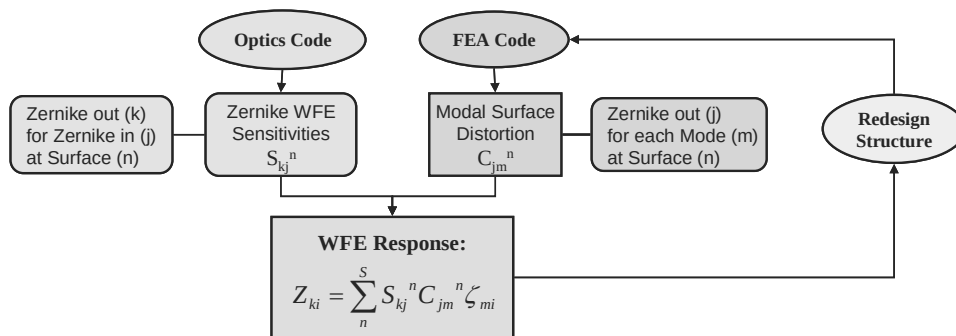


Figure 7.63 Flow chart illustrating wavefront error calculations subject to dynamic loading.

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◀Chapter 8▶

Mechanical Stress and Optics

Mechanical stress is developed in optical systems from various internal and external influences, including inertial, pressure, dynamic, temperature, and mechanical loads. Excessive stress can lead to permanent deformations of optical mounts and support structures by exceeding the yield strength of the material resulting in misalignments of optical elements and loss in optical performance.

Stress must also be minimized to avoid loss in structural integrity and ultimate failure of parts and components including flexures, adhesive/epoxy bonds, and optical elements. Detailed stress analyses and the selection of an application dependent glass design strength are often necessary to avoid brittle fracture of glass optics. Determination of the design strength involves accounting for the fracture mechanics failure mechanism, the detailed stress distribution, the specific glass type, the presence of surface flaws, and subcritical crack growth due to environmentally enhanced fracture. Time-to-failure curves may be constructed to determine a design strength to meet lifetime service requirements.

In addition, the presence of mechanical stress in optical glass affects optical system performance by creating anisotropic variations in the index of refraction due to the photoelastic effect. The presence of stress birefringence creates both wavefront and polarization errors in the optical system.

8.1 Stress Analysis Using FEA

Finite element methods are routinely employed to predict mechanical stress in optical substrates and support structures. FEA stress analysis is typically more labor-intensive and time-consuming as compared to displacement and dynamic models. Extra attention to detail and higher-fidelity models are required to capture peak stresses in locations of fillets and holes. In addition, more elements are required because stress is numerically computed as a derivative of the displacement (i.e., strain) and is one order lower in accuracy than the prediction of displacements. Mesh convergence studies are typically performed to ensure that the element fidelity meets the desired level of accuracy, which requires varying the mesh density over several model iterations.

Performing a FEA stress analysis for a point or contact load will result in a stress singularity at the point of the applied load. In this instance, the use of Hertzian contact stress equations¹ is recommended to predict the local stresses based on FEA predicted loads. Analytical solutions are always recommended to validate FEA stress analyses and to provide first-order estimates during preliminary design stages.^{2,3}

8.1.1 Coarse FEA models and stress concentration factors

A coarse finite element model coupled with the use of stress concentration factors is an alternate approach to a high-fidelity model. In this approach, the coarse model is used to predict the average stress across the critical stress region. The peak stress is then estimated by multiplying the average stress by a stress concentration factor (K_t) that may be found in reference books⁴:

$$\sigma_{peak} = K_t * \sigma_{nominal}. \quad (8.1)$$

A comparison of this approach versus a high-fidelity mesh is presented for a bipod flexure mount shown in Fig. 8.1. The stress difference between the highly detailed solid element mesh (115,200 elements for a single bipod leg) and the coarse beam element mesh (24 elements for a single bipod leg) whose stress results are scaled by a stress concentration factor is 7%, well within the expected accuracy for that level of detail. The beam model has benefits for rapid-turnaround design trades beneficial early in the design cycle where geometry can be easily changed and solution times are fast. The solid element model provides greater accuracy and is most suitable for a detailed stress analysis of a mature design.

8.1.2 FEA post-processing

There are several issues the engineer should understand prior to plotting the stress results in FEA post-processing software to ensure proper interpretation. In most software, the stress is calculated at the centroid, corner, or the integration points of the element. The post-processor can then plot these stresses in several different coordinate systems including the element, local, or global coordinate systems. For elements that share a common node, there are multiple stress values at that node.

FEA post-processors offer the option of averaging the nodal stresses. Depending on the mesh fidelity, the stress results can vary significantly between the averaged and unaveraged stress values. For example, the stress in a plate with a central hole is shown in Fig. 8.2. The theoretical peak stress in the plate is 240 psi. The coarse mesh on the left averages the nodal stresses, which predicts a maximum stress of 144 psi that significantly underpredicts the theoretical maximum. The center contour plot uses the same coarse mesh but turns off nodal averaging. This produces a discontinuous contour plot but increases the peak



Figure 8.1 FEA beam model (left) and solid element model (right) of bipod flexure.

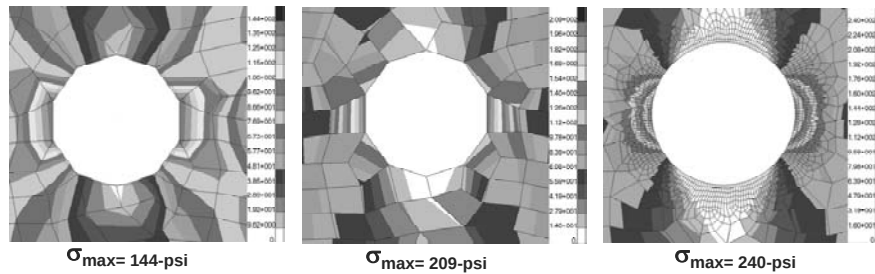


Figure 8.2 Finite element stress contours and peak stresses for a plate with a central hole.

stress to 209 psi. The stress results using a high-fidelity mesh density with stress-averaging on is shown in the plot on the right side of Fig. 8.2. Here the stress has converged to the theoretical solution of 240 psi. In general, for a detailed mesh that has met stress convergence criteria, the stresses predicted using no averaging and averaging would be very similar.

In addition, the analyst should be sure to plot the appropriate stress in the FEA post-processing software for the given application. For ductile materials, von Mises stress should be plotted for comparison to yield or microyield. For brittle materials, maximum principal stress should be plotted for failure analyses. If the load is reversible, the maximum and minimum principle stress should be plotted. Directional stress plots are useful to understand the behavior of the structure so that design improvements can be made.

Also, unlike directional stresses, Von Mises and principal stresses cannot be averaged between elements. Von Mises and principal stresses must be recomputed from averaged directional stresses. The analyst should run several simple test cases to fully understand all FEA post-processing options in the software code. When reporting stresses, the type of stress, the options used, and the load case description should be noted on the plot.

8.2 Ductile Materials

Optical metering structures, flexures, and housings are commonly made from metal materials that are inherently ductile, including aluminum, steel, Invar, and titanium. Failure analyses for ductile materials involves comparing the predicted von Mises stress to the yield or ultimate design allowable. In typical mechanical structures, a 0.2% permanent strain is used to define the yield stress σ_y .

8.2.1 Microyield

A more conservative design criterion used for optical metering structures and mounts is microyield (also known as the precision elastic limit). Microyield is defined as the stress required to produce a plastic strain of one part per million (10^{-6} offset strain). There is no direct relationship between yield and microyield, and microyield values can vary with processing, heat treating, and alloy composition.^{5,6}

The use of microyield as a design criteria is somewhat arbitrary intended to ensure that the applied stress does not permanently misalign the optic beyond the allowable. A direct approach to determine a design allowable is to relate the permanent strain induced by the stress field to the allowable displacement tolerance on the individual optical element. A nonlinear analysis can be performed to compute the permanent strain using the materials stress–strain curve. A design stress may then be selected that is based on optical element tolerances.

8.2.2 Ultimate strength

Designing to ultimate strength generally requires a nonlinear analysis to account for the nonlinear stress–strain curve. However, it is conservative practice to use a linear elastic analysis to compute the stress in a part past the yield point. This practice is based upon a decrease in the slope of the stress–strain curve that is common for metal materials following the yield point. In this case, where a localized region has yielded, the load-carrying capacity of the material is reduced, and the load is carried by the surrounding material. A nonlinear analysis would account for the local reduction in stiffness and would predict lower stresses than the linear analysis.

8.3 Analysis of Brittle Materials

For brittle materials such as glass and ceramics, a fracture mechanics approach is used to describe failure. The failure mechanism involves the growth of flaws or cracks in materials under tensile loads. Failure occurs when the stress intensity K_I , which describes the level or intensity of the stress distribution just ahead of the crack tip, as illustrated in Fig. 8.3, exceeds the fracture toughness or critical stress intensity of the material K_{IC} ,

$$K_I > K_{IC}. \quad (8.2)$$

Under an applied load, the stress at the crack tip is concentrated due to the crack geometry and the lack of plastic deformation. When the stress value exceeds the strength of the bonds in the glass molecular network, crack propagation occurs. The stress intensity may be computed for a given crack size

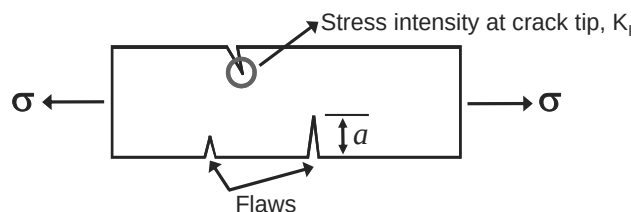


Figure 8.3 Failure occurs when the stress intensity at the crack tip exceeds the fracture toughness of the material.

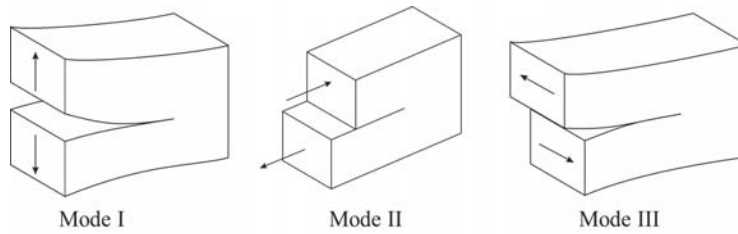


Figure 8.4 Three modes of crack separation.

and geometry using Griffith's law, expressed as

$$K_I = Y\sigma\sqrt{a}, \quad (8.3)$$

where Y is a nondimensional crack geometry factor, σ is the nominal stress, and a is the flaw size.

There are three basic modes of crack separation, as shown in Fig. 8.4: mode I is known as an opening mode where stress acts normal to the crack plane, mode II is an in-plane shearing mode or sliding, and mode III is out-of-plane shearing or tearing. Critical stress intensity values can be computed and compared to the fracture toughness for each mode. Mode I is the most commonly used for analysis purposes.

8.3.1 Fracture toughness

Fracture toughness varies among common glass types, as shown in Table 8.1 and is an indication of the strength variation among glass materials. For comparison purposes, the fracture toughness of ductile materials such as aluminum, titanium, and steel range from 20 to 100 times larger.⁷

Fracture toughness data is not always available for the glass types in a given optical system. However, the fracture toughness for commercial optical glasses (e.g., Schott, Ohara, Hoya) may be determined using the manufacturers published data on lapping hardness.⁸ Lapping hardness is defined as the volume of material removed under a predefined set of processing conditions including pressure, velocity, coolant, and abrasive. Rates of removal, surface roughness, and subsurface damage are a function of a glass's mechanical properties, including

Table 8.1 Fracture toughness values of various optical glass materials.

Fracture Toughness, K_{IC} psi $\sqrt{\text{in}}$	
Fused Silica	674
BK7	774
SF5	519
SK16	710
LaK10	865
F2	500
SF58	346

the elastic modulus, hardness, and fracture toughness. Lapping hardness may be derived from the mechanical properties controlling elastic deformation (elastic modulus E), resistance to flow (hardness H_k), and resistance to cracking (fracture toughness K_{IC}) through a combined figure of merit given by the following:

$$E^{7/6} / (K_{IC} H_K^{23/12}) \quad (8.4)$$

The glass manufacturers each publish data on the lapping hardness of their glasses. Using this data and the curves that have been published (showing the relationship between the above figure of merit and lapping hardness), fracture toughness may be estimated.

8.3.2 FEA methods to compute the stress intensity

Two techniques to compute the stress intensity at a crack tip using finite element analysis are summarized below. Both involve having knowledge of the physical size of the crack within the optic. The first approach involves embedding a specialized crack-tip element in a model of standard finite elements that represents the actual geometry of the crack. The model then reports the stress intensity at the crack tip for a given applied load that may be compared to the fracture toughness of the material.

The second approach is based on the strain-energy release rate. The crack area is modeled using standard elements. The individual crack is modeled as a “slit” of unequivalenced nodes in a fine mesh. The initial FEA solution calculates the strain energy SE_1 . The crack is then extended a small amount in the model, giving a change in the crack surface area of ΔA . The analysis is repeated to find the strain energy SE_2 . The strain energy release rate G and stress-intensity factor K_I can be found from the following expressions:

$$G = \frac{(SE_2 - SE_1)}{\Delta A}, \quad (8.5)$$

$$K_I = \sqrt{EG}. \quad (8.6)$$

8.4 Design Strength of Optical Glass

Brittle materials such as glass do not possess a single characteristic strength. The strength of the material is dependent on the distribution of cracks or surface flaws. This, coupled with the inherent brittleness (source of catastrophic or rapid failure), translates into conservative design strengths. Rule-of-thumb tensile design strengths for optical glass are typically 1000 psi for nominal glass materials. This rule of thumb serves as a simplistic design strength for common optical glass that enables a quick assessment of a given design. However, determining a material and application dependent design allowable is often necessary for several reasons including when stress levels exceed 1000 psi, when using relatively brittle glass types (as denoted by the fracture toughness), or when

optical elements are subject to relatively high stress levels over a significant period of time that are susceptible to the effects of environmentally enhanced fracture.

8.4.1 Surface flaws

The strength of glass is governed by the random distribution of surface flaws that vary in size, orientation, and location in relation to regions under stress. The grinding and finishing operations of the optical substrate leave a cracked layer near the glass surface known as subsurface damage, resulting in the flaw distribution. The surface flaws increase in size from cleaning, handling, and environmental effects. Due to the scatter in the flaw size, there is no deterministic strength for brittle materials (unless the flaws are extremely uniform) and in general the strength of glass is a function of the size of the glass component and the area under stress. Flaws or cracks propagate under tensile loads to a critical value and then experience uncontrolled crack growth until the part physically fractures.

8.4.2 Controlled grinding and polishing

The strength of a glass component may be maximized by minimizing the surface flaws during the manufacturing process. This is achieved through a controlled grinding and polishing process that decreases the depth of the surface flaws after each step of the operation, as shown in Fig. 8.5.⁹ Removal rates are designed to remove a depth of material that is equal to or greater than the maximum flaw depth.¹⁰ The maximum flaw depth may be approximated as three times the average diameter of the grinding particle.

Maximizing the strength of edges in an optical element is typically done by beveling followed by polishing with a fine grit or wiping the edge with a concentration of hydrofluoric acid (common time is 15 minutes using 40% concentration).¹¹ The engineer should ensure that the fabrication processes and finishing operations of the glass part use these techniques or equivalent to maximize the strength of the part both on the surface and the edge.

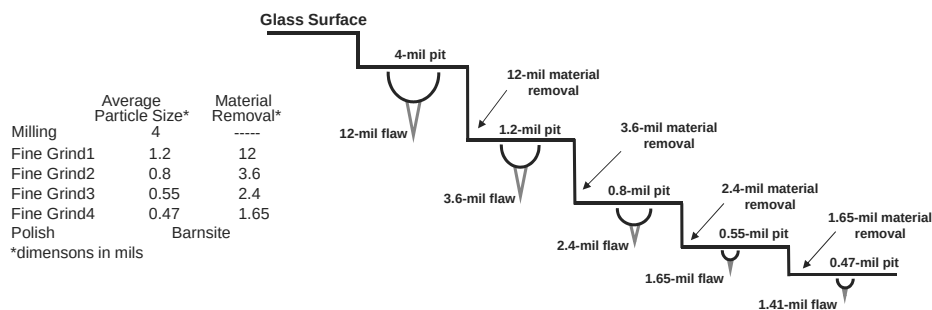


Figure 8.5 Example of a controlled grind and polishing process.

8.4.3 Inert strength

A design allowable can be based upon the inert strength (also known as the modulus of rupture) of the glass. The inert strength of glass is the stress required to cause instantaneous failure of the material and assumes that cracks do not grow over the service life of the glass (i.e., no subcritical crack growth). The inert strength of the material σ_i may be expressed by rearranging Eq. (8.3) and substituting in the critical stress intensity K_{IC} for the stress intensity K_I :

$$\sigma_i = \frac{K_{IC}}{Y\sqrt{a}}. \quad (8.7)$$

8.4.3.1 Residual stress and inert strength

The inert strength of the material is reduced by the presence of residual stress at the crack tip generated by the creation of surface flaws¹²:

$$\sigma_i = \left[\frac{r}{(r+1)^{(r+1)/r}} \right] \frac{K_{IC}}{Y\sqrt{a}}. \quad (8.8)$$

The variable r characterizes the nature of the residual stress field: for line flaws, $r = 1$, and for point flaws, $r = 3$. For analysis purposes, it is conservatively assumed that the flaws are point flaws consistent with polished glass. This results in the following relationship for the inert strength:

$$\sigma_i = \frac{0.47K_{IC}}{Y\sqrt{a}}. \quad (8.9)$$

The above equation may be used to determine an appropriate design strength that accounts for the fracture toughness of the material, an assumed flaw size and flaw geometry factor. Typically, a conservative flaw size is assumed that accounts for the grinding and polishing operation, cleaning and handling effects, and environmental influences at end-of-life. A factor of safety can be included to cover uncertainty and provide margin. Alternatively, for a known applied stress, Eq. 8.9 may be used to compute the size of the allowable flaw to cause fracture.

8.4.3.2 Inert strength based on material testing and Weibull statistics

Strength testing in conjunction with Weibull probabilistic methods may be used to characterize the inert strength of an optical element. Testing removes the need to have knowledge of the flaw size, flaw geometry factor, critical stress intensity factor, and the residual stress field. This approach assumes that the surface flaws that limit the strength is the same for the specimens as it is for the components and that the surface flaw population is unvarying with time in service (i.e., no

subcritical crack growth). A statistically relevant sample size of specimens are tested to failure, and the data is fit to a two-parameter Weibull distribution. The probability of failure for a given test specimen may be determined for an applied stress σ , expressed as

$$P_f = 1.0 - e^{\left[-\left(\frac{\sigma}{\sigma_o}\right)^m\right]}, \quad (8.10)$$

where P_f is the probability of failure, σ_o is the characteristic strength (63.2% of the specimens fail at this stress level), and m is the Weibull modulus, an indicator of the scatter of the data.

The strength distribution of the test specimen may be used to derive the strength distribution of the actual glass component using a ratio of the surface areas under tensile stress. An effective surface area A_{eff} may be computed for a component with a varying stress field such as that computed via finite element analysis using the following relationship:

$$A_{eff} = \int \left(\frac{\sigma}{\sigma_{max}} \right)^m dA, \quad (8.11)$$

where σ represents the surface tensile stress over area dA , and σ_{max} is the maximum surface tensile stress. The characteristic strength of the component is then computed by¹³

$$\frac{\sigma_o^{specimen}}{\sigma_o^{component}} = \left(\frac{A_{component}}{A_{specimen}} \right)^{\frac{1}{m}}. \quad (8.12)$$

The above relationship allows the characteristic strength of the actual glass component to be statistically characterized assuming the Weibull modulus is constant. For test specimen data to be extrapolated to component geometries, the surface of the test specimens must be prepared exactly as the component. This ensures that the Weibull modulus used in the scaling law is constant between the test specimens and component. This general method, while not exact, offers many benefits as it is often impractical to test actual components in their true loading condition in sufficient quantity to yield reliable results. The method is adequate for multi-axial, tensile loaded specimens, provided that the second or third principal stresses are significantly less than the principal tensile stress. More sophisticated analyses that take into account the effect of multi-axial tensile stresses on flaws are required if this is not the case.

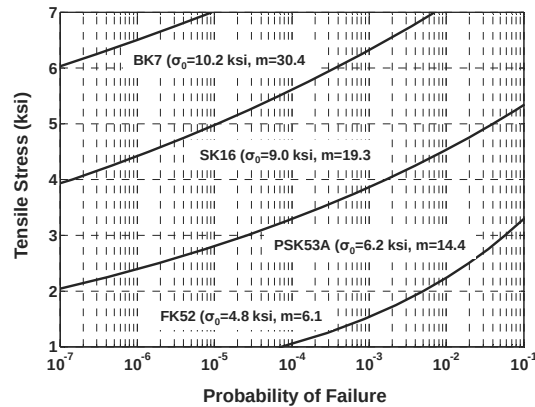


Figure 8.6 Probability-of-failure curve for several Schott glasses.

Probability of failure versus tensile stress curves may be used to select a design strength based on a chosen probability of failure. The probability of failure versus tensile stress is shown for several Schott glasses in Fig. 8.6. The data used in this figure is based on a double ring test using a surface area of 113 mm² at room temperature.¹⁴ The surface of the glass specimens were polished using a loose silicon carbide grain with an average particle size of 9 μm.

8.4.4 Environmentally enhanced fracture

Environmentally enhanced fracture, also known as stress corrosion, is a form of static fatigue that causes cracks to grow over time and occurs when the glass is under a state of stress and in the presence of moisture. In this condition, subcritical crack growth occurs and increases the size of the crack reducing the strength of the part. Environmentally enhanced fracture is due to the reaction of the glass material with water at the crack tip. Small amounts of water vapor can significantly reduce the lifetime of the component under static load conditions. Thus, the strength of glass is a function of the humidity level and the time in which the component has been loaded. Knowledge of how cracks grow in the presence of moisture and under stress is required to determine time-to-failure predictions from which a design strength can be based.

8.4.4.1 Crack growth studies

Environmentally enhanced crack growth may be studied using controlled growth of well-defined surface flaws. The crack velocity V is measured and plotted versus the stress intensity at the crack K_I on a semi-log scale known as the V - K_I curve. The regions of crack propagation for a typical glass V - K_I curve are shown in Fig. 8.7.¹⁵ Crack propagation is initiated once the stress intensity at the crack tip exceeds a given threshold. Region I is the region of stable crack growth where the surface flaw is reacting with the moisture. The slope of this line is a measure of the fatigue resistance parameter N of the material. The fatigue resistance

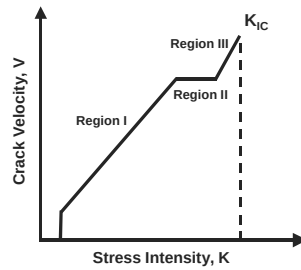


Figure 8.7 Regions of a typical V–K curve.

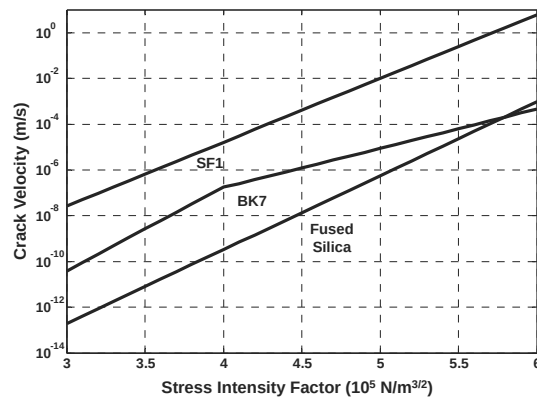


Figure 8.8 V–K curves for SF1, BK7, and fused silica (Region I).

parameter, also known as the flawed growth exponent, is a measure of a material's ability to resist crack growth in the presence of an applied load. Region II is a transition region and uncontrolled crack growth occurs in region III. Once the stress intensity reaches the fracture toughness of the material K_{IC} , the part fractures. Crack propagation in region I for several glasses, based on data from Weiderhorn,¹⁶ is shown in Fig. 8.8.

8.4.4.2 Static and dynamic fatigue testing

Static and dynamic fatigue testing are alternative ways to study environmentally enhanced crack growth. Static fatigue testing is performed by loading test specimens at various moisture levels and recording the time to failure. (The inert strength may be determined by cooling specimens to liquid nitrogen temperatures where the effects of water are negated.) During static fatigue testing, the time to failure is recorded for a constant load. For higher applied loads, crack growth is more rapid as compared to lower applied loads that result in faster times to failure.

Dynamic fatigue testing is an accelerated method where stress is applied to the specimen at a constant rate. When the stress rate is fast, there is less time for crack growth, and the specimen fails at a higher stress. Conversely, when the stress is applied slowly, the crack has an opportunity to propagate, and the specimen fails at a lower stress. The fatigue resistance parameter may be

determined by plotting the average stress at failure versus the rate of stress using a log–log scale and computing the slope of the line. Dynamic fatigue testing is often performed in the harshest environmental conditions, such as liquid water, to provide conservative estimates of the fatigue resistance parameter for analytical lifetime estimates. Values for the fatigue resistance parameter for a number of select glass types¹⁷ are shown in Table 8.2. Materials with a larger fatigue resistance parameter have greater resistance to environmentally enhanced crack growth.

Residual stress at the crack tip reduces the fatigue resistance parameter that results in a significant reduction in the ability of the material to resist environmentally enhanced crack growth. An apparent fatigue resistance parameter N' (also known as the residual stress flaw growth exponent) can be computed that accounts for the effects of residual stress.¹² The apparent fatigue resistance parameter and the nominal fatigue resistance parameter are related by

$$N' = (rN + 2) / (r + 1), \quad (8.13)$$

where r is a measure of the residual stress field. The value of r is often conservatively assumed to represent a residual stress field produced for a point flaw ($r = 3$) that results in the following:

$$N' = (3N + 2) / 4. \quad (8.14)$$

8.4.4.3 Lifetime and time-to-failure analyses

Glass design strengths may be based upon time-to-failure analyses that account for the effects of environmentally enhanced fracture and subcritical crack growth during the service life. This is a more involved and comprehensive method to predict the lifetime of a glass component, taking into account the reduced strength of the material over time for a given state of stress. Using crack growth rates, analytical expressions exist to compute the total time-to-failure t for a component under a constant static stress with a known surface flaw, expressed as¹⁸

$$t = \frac{2}{\sigma^2 Y^2} \int_{K_{II}}^{K_{IC}} \left(\frac{K_I}{V} \right) dK_I, \quad (8.15)$$

Table 8.2 Fatigue resistance parameters for various materials.

Materials	N
BK7	20
Zerodur	31
Fused Silica	35
Zinc Selenide	40
Calcium Fluoride	50
Zinc Sulfide	76
ULE	27
Magnesium Fluoride	10

where K_{II} is the initial stress intensity factor, and V is the crack velocity. For crack geometries consistent with those found on glass surfaces, and assuming a power law form of the V–K curve,

$$V = A \left(\frac{K_I}{K_{IC}} \right)^N, \quad (8.16)$$

where A and N are constants. The following expression produces the time-to-failure:

$$t = 2 \left(K_{II}^{2-N} - K_{IC}^{2-N} \right) / \left[(N-2) A \sigma^2 Y^2 \right]. \quad (8.17)$$

Time-to-failure curves may be developed based on an initial flaw size using Eq. (8.17). The initial flaw size may be estimated using the inert strength of the material from which an initial K_{II} is determined. Curves may be drawn for different flaw sizes, as illustrated in Fig. 8.9, and to compare different materials, as shown in Fig. 8.10.

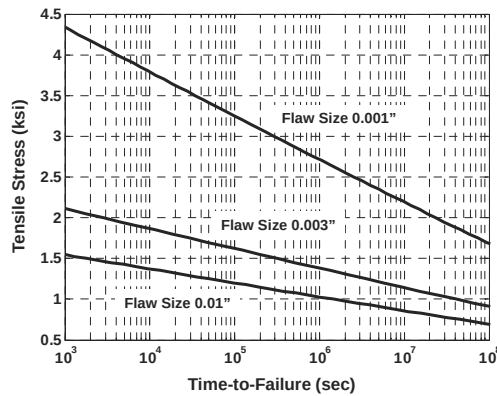


Figure 8.9 Time-to-failure vs. initial flaw size for SF1.

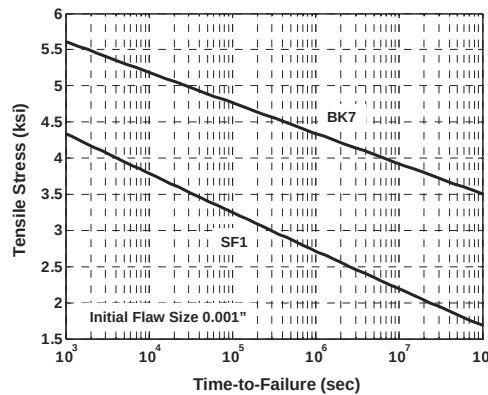


Figure 8.10 Time-to-failure curves for BK7 and SF1.

8.4.4.4 Lifetime prediction and probability of failure

Design strength diagrams may be created that plot time-to-failure versus tensile stress as a function of probability of failure. The initial stress-intensity at the crack tip can be estimated by scaling the critical stress intensity by the ratio of the applied stress and the fracture stress σ_{IC} ,

$$K_{II} = K_{IC} \left(\frac{\sigma}{\sigma_{IC}} \right). \quad (8.18)$$

Using the stress at fracture σ_{IC} for σ in Eq. (8.10) and solving for σ_{IC} , and then substituting into Eq. (8.18) yields the following relationship¹⁹:

$$K_{II} = K_{IC} \left(\frac{\sigma}{\sigma_o} \right) \left[\log \left(\frac{1}{1 - P_f} \right) \right]^{-1/m}. \quad (8.19)$$

The initial stress intensity value is then used in the time-to-failure equation [Eq. (8.17)] to develop a family of design curves. These design curves account for subcritical crack growth and allow a design strength to be selected based on the desired probability of failure. Example time-to-failure design curves as a function of probability of failure based on polished BK7 inert strength data¹² are shown in Fig. 8.11.

Lifetime predictions and design strengths are commonly based on the Weibull A-basis inert strength (99% reliability with 95% confidence). The time-to-failure as a function of the inert strength is expressed in Eq. (8.20)¹⁸.

$$t = 2\sigma_i^{N-2} / \left[(N-2)A\sigma^N Y^2 K_{IC}^{N-2} \right]. \quad (8.20)$$

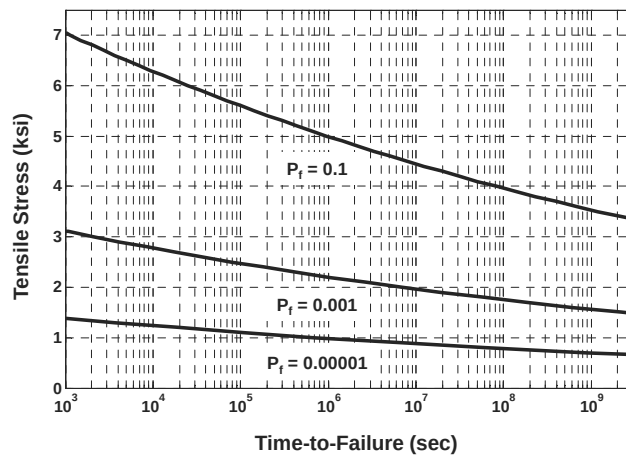


Figure 8.11 BK7 design strength curve.

8.4.4.5 Effects of residual stress on time-to-failure

The effects of residual stress have a significant impact on time-to-failure calculations. The inclusion of residual stress, however, complicates the computations and requires many parameters to be determined through material testing.²⁰

An analytical solution developed by Pepi²¹ enables the practicing engineer to compute a design strength using time-to-failure calculations that requires a much limited set of test data. In this derivation of the time-of-failure calculations, only the inert strength and the apparent fatigue resistance parameter must be known:

$$t = 0.0001RH * \left(\frac{\sigma * FS_a}{\sigma_i} \right)^{-(N'-2)}, \quad (8.21)$$

where RH is the relative humidity (for 100% humidity, $RH = 1$), and FS_a is an approximation factor given as

$$FS_a = 2 \times 10^{-5} \sigma + 0.98. \quad (8.22)$$

This provides a practical solution during the design stages of a program that precludes the need for extensive material testing. A comparison of time-to-failure curves using the exact solution and the approximate expression from Pepi is computed for fused silica in Fig. 8.12.

All of the time-to-failure calculations previously discussed assume that the glass material is isotropic and homogeneous, the flaw distribution used in the inert strength calculations are the same distributions in the actual component, any of the tests do not change the existing flaw distribution, and that flaws change only through environmentally enhanced fracture.

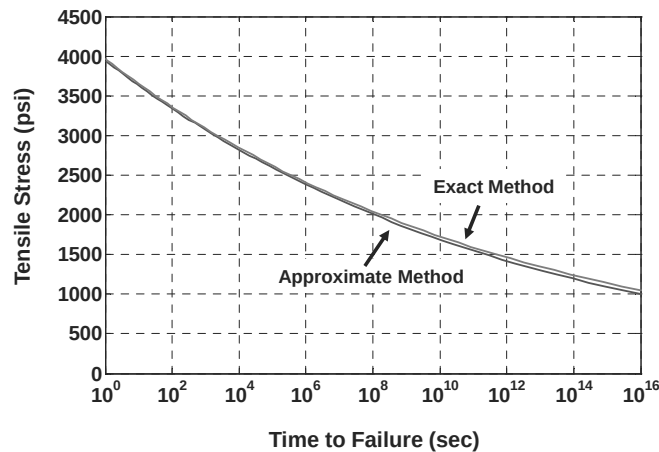


Figure 8.12 Comparison of exact and approximate time-to-failure curves for fused silica.

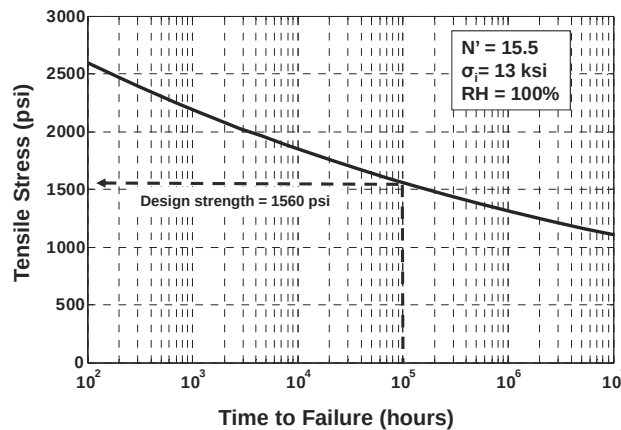


Figure 8.13 BK7 window design strength time-to-failure curve.

8.4.4.6 BK7 design strength example

The design strength of a BK7 window is to be determined using Eq. (8.21) for an application that has a lifetime requirement of 100,000 hours. The fatigue resistance parameter for BK7 from Table 8.2 is 20. An apparent fatigue resistance parameter N' of 15.5 is computed using Eq. (8.14) to account for the effects of residual stress. The window is assumed to be exposed to 100% relative humidity. The Weibull A-basis inert strength of the window is 13 ksi. No area scaling is performed for this example. Using Eq. (8.21), the time-to-failure curve for the BK7 window is shown in Fig. 8.13. A window design strength of 1560 psi is determined from the curve for 100,000 hours of operation.

8.4.5 Proof testing

Proof testing is used to verify a level of strength in a part by applying a known stress field to the actual glass component. The applied stress is often two or three times larger than the expected stress during the service environment. Thus, the test is often referred to as an overload proof test. Once a component has passed a proof test, the initial strength distribution, an upper limit on the most critical flaw in the component, and the maximum stress intensity at the beginning of the service life have been established. The benefits of proof testing are that components that pass a properly executed test provide the best time-to-failure estimates because no aspect of the analysis is unresolved. Proof-test diagrams are often constructed that establish proof testing levels that guarantee a minimum time-to-failure for a given applied load.

For a proof test to be applicable and relevant, the applied stress distribution on the glass must be equivalent to the expected service stress distribution in operation. Furthermore, the test must not modify the initial flaw distribution in the part that would weaken the component. The tests are thus typically performed in an inert environment to avoid environmentally enhanced fatigue, and the applied load is unloaded as quickly as possible.

8.4.6 Cyclic fatigue

Determining a design strength for glass that is subject to cyclical mechanical loads is much more complicated than crack growth under static load conditions; the details of which are beyond the scope of this book. Early approaches computed the time-to-failure due to cyclic loading as a function of the static-crack-growth curves. This approach assumes that the cyclic stress field may be represented as a summation of incremental static loads, that the compressive nature of the load has no effect on crack growth, and the static and cyclic failure mechanisms are equivalent.²² A true fatigue effect has been subsequently identified in brittle solids leading to a decelerated or accelerated crack growth under cyclic loads as compared to static loads. It has been shown that during repeated tension–compression cyclic loading, whereby the crack surfaces are repeatedly placed in physical contact during the compression stroke of the cycle, that crack growth may be accelerated.²³ Conversely, crack growth may be decelerated by factors such as debris particles wedging between the contact surfaces. For practical design considerations, it is recommended that the glass be exposed to very conservative cyclical stress levels.

8.5 Stress Birefringence

Stress birefringence is a consequence of mechanical stress acting on a transmissive optical material due to the photoelastic effect. Light experiences two refractive indices or double refraction (birefringence) when travelling through a birefringent material. This is illustrated for a glass plate under uniaxial stress in Fig. 8.14. The applied stress modifies the indices of refraction in the directions parallel and perpendicular to the direction of the applied stress in the plane of the plate. For a general triaxial state of stress varying within a lens element due to applied mechanical loads, the optical properties become anisotropic and inhomogenous. This results in wavefront and polarizations errors within the optical system.

Stress birefringence is an issue for many types of optical systems, including systems for optical lithography, data storage, high-energy lasers, LCD projectors, and telecommunications. For these types of optical systems, integrated modeling techniques help enable design trades of optical performance as a function of glass type and mounting methods. An example of the effects of stress birefringence is

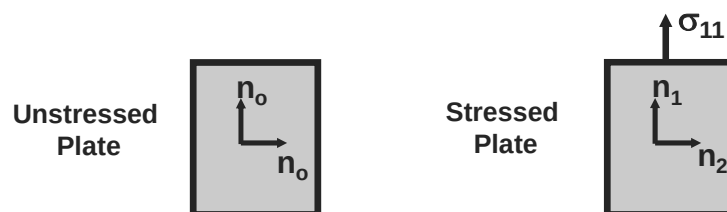


Figure 8.14 Mechanical stress modifies the indices of refraction of transmissive materials.

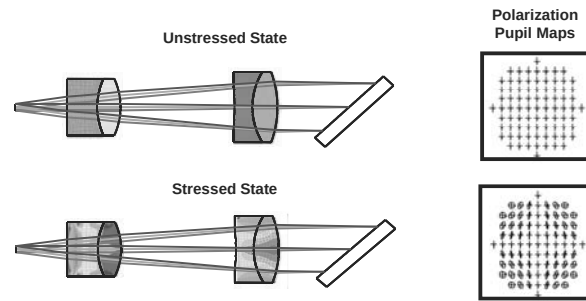


Figure 8.15 Polarization pupil maps representing the effect stress has on the state of polarization for a WDM demultiplexer.

demonstrated for a telecommunication demultiplexer in Fig. 8.15. On the left side of the figure, rays are shown passing through the finite element model of the lens element for the unstressed and stressed states. On the right side, the polarization pupil maps reveal that the stress field converts the incident linearly polarized light to circularly polarized light at points near the edges of the pupil.

8.5.1 Mechanical stress and the index ellipsoid

The index ellipsoid (also known as the ellipsoid of wave normals or the optical indicatrix) provides a geometrical interpretation of the optical properties of a material by defining the indices of refraction as the semi-axes of the ellipsoid, as illustrated in Fig. 8.16. In general, the application of mechanical stress modifies the shape of the index ellipsoid and hence the indices of refraction. The index ellipsoid is defined by a second-degree surface, or quadric, expressed by the following equation:

$$\sum_{ij} B_{ij} x_i x_j = 1. \quad (8.23)$$

The coefficients of the surface are defined by the dielectric impermeability tensor, which is expressed in matrix form below:

$$B_{ij} = \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{12} & B_{22} & B_{23} \\ B_{13} & B_{23} & B_{33} \end{bmatrix}. \quad (8.24)$$

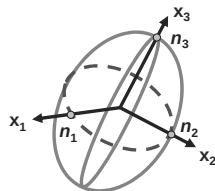


Figure 8.16 Index ellipsoid.

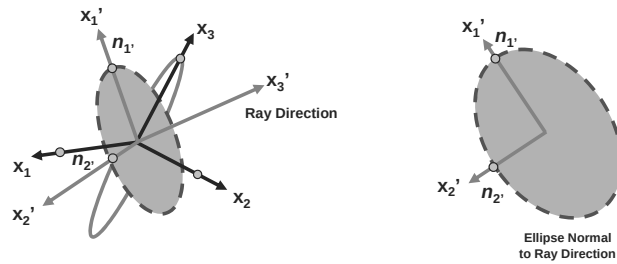


Figure 8.17 Computing the indices of refraction for an arbitrary ray direction.

For a ray of arbitrary direction traversing a birefringent material, light propagation is affected by the indices of refraction normal to the ray direction. These indices of refraction may be geometrically constructed as the semi-axes of the ellipse centered at the index ellipsoid origin and normal to the ray direction, as shown in Fig. 8.17.

Mechanical stress changes the indices of refraction by altering the size, shape, and orientation of the index ellipsoid, as given by the following fourth-rank tensor transformation:

$$\Delta B_{ij} = q_{ijkl} \sigma_{kl}, \quad (8.25)$$

where q is the piezo-optical coefficient matrix and a material property, and σ is the stress tensor. For a general crystalline material, this tensor has 36 independent components, whereas for an isotropic material, there are only two independent components.

Changes in the dielectric impermeability tensor due to the photoelastic effect may also be defined using mechanical strain, expressed as

$$\Delta B_{ij} = p_{ijrs} \varepsilon_{rs}, \quad (8.26)$$

where p is the elasto-optical coefficient, and ε is the strain tensor.

In general, changes in the dielectric impermeability tensor due to the effects of stress are small and are considered perturbations to the index ellipsoid. Thus, they may be superimposed on the natural birefringence for all crystal systems by adding the terms to the nominal coefficients of the index ellipsoid equation.

8.5.2 Stress birefringence for isotropic materials

The change in the dielectric impermeability tensor may be computed for isotropic and crystalline materials using the appropriate piezo-optical coefficient matrix. Each material type behaves differently under stress. For example, under a uniform state of stress, isotropic materials become uniaxial. Conversely, various classes of cubic crystals that nominally exhibit isotropic properties may become biaxial. Uniaxial and biaxial refer to the number of axes in which a given ray traveling parallel to the axis will experience no birefringence. This section develops the equations for isotropic materials.

The piezo-optical coefficient matrix for a homogeneous and isotropic material is as follows:

$$q = \begin{bmatrix} q_{11} & q_{12} & q_{12} & 0 & 0 & 0 \\ q_{12} & q_{11} & q_{12} & 0 & 0 & 0 \\ q_{12} & q_{12} & q_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & q_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & q_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & q_{44} \end{bmatrix}, \text{ where } q = q_{11} - q_{12}. \quad (8.27)$$

In the plane normal to the ray direction, the two indices of refraction are defined by the semi-axes of the ellipse centered at the origin of the ellipsoid. When no stress is acting on the material, the index ellipsoid is a sphere, and the index plane normal to the ray direction is a circle, and there is no birefringence. However, under a mechanical stress defined in an arbitrary xy coordinate system,

$$\sigma^o = [\sigma_{xx}^o \quad \sigma_{yy}^o \quad \sigma_{zz}^o \quad \sigma_{xy}^o \quad \sigma_{yz}^o \quad \sigma_{xz}^o]^T, \quad (8.28)$$

where the z axis is defined along the ray direction, the material becomes birefringent, and the circle centered at the origin of the sphere becomes an ellipse.

The angle γ , which defines the orientation of the semi-axes of the ellipse, coincides with the direction of the principal stresses σ_{11} and σ_{22} in the plane normal to the ray. This angle is computed as

$$\gamma = \frac{1}{2} \tan^{-1} \frac{2\sigma_{xy}^o}{\sigma_{xx}^o - \sigma_{yy}^o}, \quad (8.29)$$

yielding the following stress tensor defined along the in-plane principal stress directions:

$$\sigma = [\sigma_{11} \quad \sigma_{22} \quad \sigma_{zz} \quad 0 \quad \sigma_{yz} \quad \sigma_{xz}]^T. \quad (8.30)$$

The change in the dielectric impermeability tensor is given by

$$\Delta B_{ij} = \begin{bmatrix} q_{11}\sigma_{11} + q_{12}(\sigma_{22} + \sigma_{zz}) \\ q_{11}\sigma_{22} + q_{12}(\sigma_{11} + \sigma_{zz}) \\ q_{11}\sigma_{zz} + q_{12}(\sigma_{11} + \sigma_{22}) \\ 0 \\ q_{44}\sigma_{yz} \\ q_{44}\sigma_{xz} \end{bmatrix}. \quad (8.31)$$

The index changes in the plane normal to the ray direction may be computed by differentiating ΔB and assuming the changes in index are comparatively small, which yields the following relationships:

$$\Delta n_1 = -\frac{1}{2} n_o^3 \Delta B_{11} = -\frac{1}{2} n_o^3 [q_{11} \sigma_{11} + q_{12} (\sigma_{22} + \sigma_{zz})] \quad (8.32)$$

and

$$\Delta n_2 = -\frac{1}{2} n_o^3 \Delta B_{22} = -\frac{1}{2} n_o^3 [q_{11} \sigma_{22} + q_{12} (\sigma_{11} + \sigma_{zz})], \quad (8.33)$$

where n_o is the nominal index of refraction.

For a wavefront incident on a birefringent material, the incident electric field vector is decomposed into electric field components along the x and y directions, each traveling paths with different indices of refraction, as illustrated in Fig. 8.18. The difference in the optical path ΔOPD between the two ray components is given by the difference in the index of refraction multiplied by the distance the ray traveled L :

$$\Delta OPD = (\Delta n_2 - \Delta n_1) L = -\frac{1}{2} n_o^3 q (\sigma_{11} - \sigma_{22}) L, \quad (8.34)$$

where a single piezo-optical coefficient q is defined as $q = q_{11} - q_{12}$.

The stress-induced wavefront error may be approximated by averaging the optical path of the two electric field components:

$$OPD = \frac{\Delta n_1 + \Delta n_2}{2} L. \quad (8.35)$$

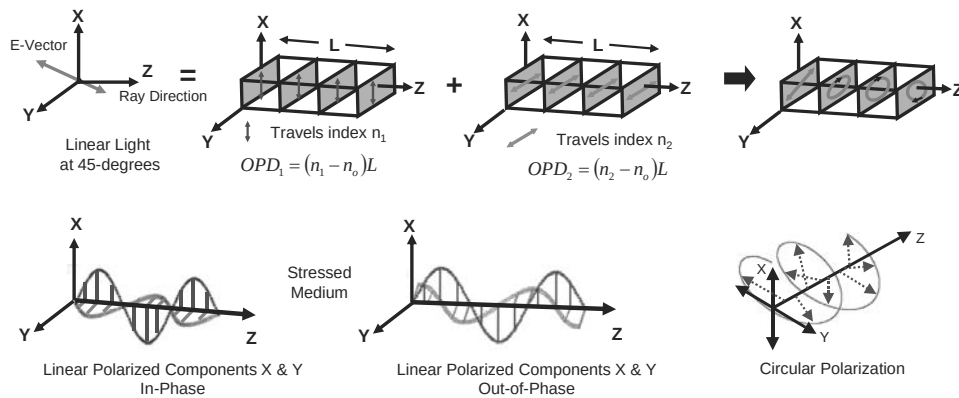


Figure 8.18 For a birefringent material, electric field components travel along paths of different indices of refraction resulting in changes in the state of polarization.

8.5.3 Stress-optical coefficients

The stress-optical coefficients k_{ij} are an alternative form to relate mechanical stress to changes in the dielectric impermeability tensor. The stress-optical coefficients are related to the piezo-optical coefficients q_{ij} by the following relationship:

$$k_{ij} = -\frac{1}{2}n_o^3 q_{ij}. \quad (8.36)$$

The changes in the refractive index as a function of the stress-optical coefficients are expressed as

$$\Delta n_1 = k_{11}\sigma_{11} + k_{12}(\sigma_{22} + \sigma_{zz}) \quad (8.37)$$

and

$$\Delta n_2 = k_{11}\sigma_{22} + k_{12}(\sigma_{11} + \sigma_{zz}). \quad (8.38)$$

This yields the following OPD difference between the orthogonal components of the electric field:

$$\Delta OPD = k(\sigma_{11} - \sigma_{22})L, \quad (8.39)$$

where $k = k_{11} - k_{12}$.

The stress-optical coefficients for several Schott glasses at a wavelength of 546 nm are given in Table 8.3.²⁴ Depending on the values of the coefficients, a material may be more sensitive to polarization changes under a state of stress than wavefront error and vice versa. The Schott glasses SF57 and BaK6 are suitable examples. For SF57, the stress-optical coefficient k is approximately zero at 546 nm. Thus, minimal changes in polarization occur due to the effects of stress. However, a comparatively large wavefront error may be induced in SF57 due to the large values of k_{11} and k_{12} .

Table 8.3 Stress-optical coefficients for selected Schott glasses.

SCHOTT GLASS STRESS-OPTICAL COEFFICIENTS ($\lambda = 546 \text{ nm}$; UNITS $10^{-6} \text{ mm}^2/\text{N}$)			
GLASS	$-k_{11}$	$-k_{12}$	k
FK3	1.0	4.9	3.9
PK2	0.4	3.1	2.7
BK7	0.5	3.3	2.8
BaK6	0.8	3.2	2.4
LaK21	1.0	2.8	1.8
PSK53A	1.5	2.6	1.1
SF1	4.5	6.2	1.7
SF57	6.7	6.7	0.0
SF59	9.0	7.6	-1.4

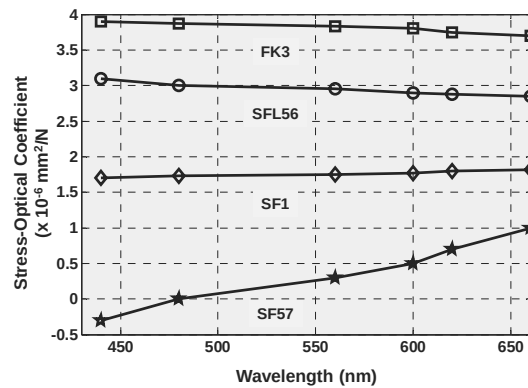


Figure 8.19 Stress-optical coefficient versus wavelength for select materials.

Conversely, the glass BaK6 has a comparatively large value of k but relatively small values of k_{11} and k_{12} , which will minimize wavefront error but increase the polarization errors. The stress-optical coefficients are a function of wavelength and are shown for selected materials in Fig. 8.19.

8.5.4 Computing stress birefringence for nonuniform stress distributions

For nonuniform mechanical stress distributions as shown in a 3D model of a lens assembly in Fig. 8.20, numerical techniques may be used to calculate the effects on optical performance. These techniques rely on computing the changes in the indices of refraction Δn_1 , Δn_2 , and orientation γ at incremental points along a given ray path through the stress field, as illustrated in Fig. 8.21. This analysis is equivalent to computing the effects of a ray traversing a series of uniaxial crystals or a series of crossed waveplates of varying birefringence. The integrated effects for a given ray through the nonuniform stress distribution may be performed by the use of Jones calculus.²⁵ At incremental points along the ray path, Jones rotation and retarder matrices are defined. The retarder matrix is used to modify the optical phase of the two orthogonal electric field components, defined as

$$R(\delta) = \begin{bmatrix} e^{i\delta_1} & 0 \\ 0 & e^{i\delta_2} \end{bmatrix}, \quad (8.40)$$

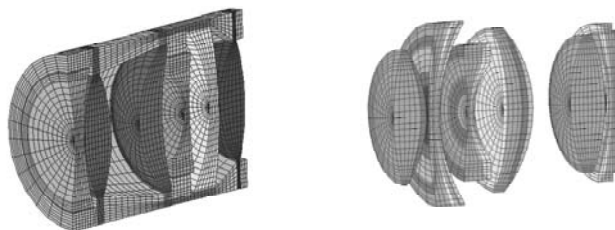


Figure 8.20 Lens assembly finite element model (left) and stress distribution (right).

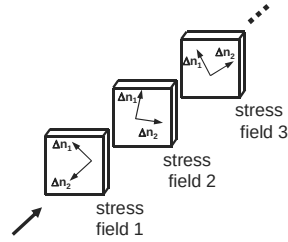


Figure 8.21 Individual ray traversing a nonuniform stress field.

where the phase change, expressed in radians, is given by

$$\delta_1 = \frac{2\pi\Delta n_1 L}{\lambda} \text{ and } \delta_2 = \frac{2\pi\Delta n_2 L}{\lambda}. \quad (8.41)$$

The rotation matrix is used to rotate between a user-defined coordinate system and the principal coordinate system, and is defined as

$$R(\gamma) = \begin{bmatrix} \cos \gamma & \sin \gamma \\ -\sin \gamma & \cos \gamma \end{bmatrix}. \quad (8.42)$$

A Jones matrix is computed for each incremental stress field i , using the relationship

$$M_i = R(\gamma)_i^T R(\delta)_i R(\gamma)_i, \quad (8.43)$$

and a system-level matrix M_s is developed by multiplying each incremental matrix M_i , given as

$$M_s = M_i \dots M_2 M_1. \quad (8.44)$$

The system Jones matrix M_s for a given ray represents the effective optical retarder properties. A grid of ray paths may be evaluated through a finite element derived stress state to compute the system Jones matrix for each path as illustrated in Fig. 8.22.

From the Jones matrix, the optical properties including birefringence, crystal axis orientation, and ellipticity may be derived²⁶ that results in 2D birefringence, crystal axis orientation, and ellipticity maps that may be compared against optical performance metrics. Example birefringence and crystal axis orientation (CAO) 2D maps are shown on a lens in Fig. 8.23.

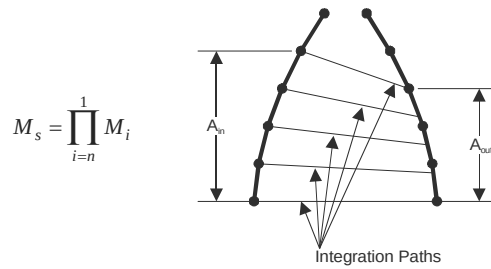


Figure 8.22 Integration paths used to compute a system Jones matrix for a grid of rays through a finite element stress field.

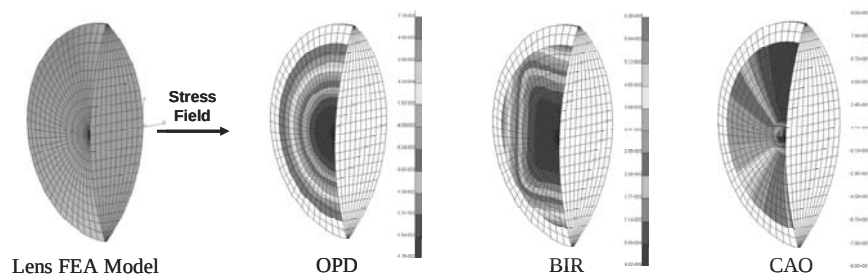


Figure 8.23 Lens FEA stress results converted into 2D maps of optical path difference (OPD), birefringence (BIR), and crystal-axis-orientation (CAO).

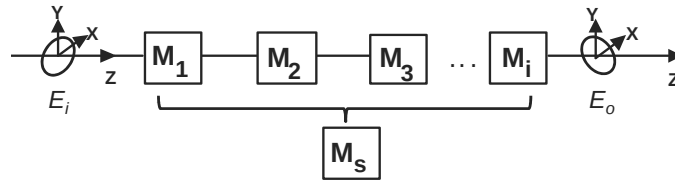


Figure 8.24 Integrated effects of the stress field may be represented by a Jones matrix.

The system-level Jones matrix may also be used to determine the output polarization state of an incident polarized beam using the following expression:

$$E_o = M_s E_i, \quad (8.45)$$

where E_i and E_o represent the input and output Jones vectors. This is depicted schematically in Fig. 8.24. Jones vectors are used to describe the magnitude and phase of the two orthogonal components of the electric field.

The wavefront error may be approximated by averaging the optical path of the two electric field components for each incremental length L_i and summed for a given ray path:

$$OPD_i = \left(\frac{\Delta n_1 + \Delta n_2}{2} \right)_i L_i \rightarrow WFE = \sum_{i=1}^n OPD_i. \quad (8.46)$$

It is assumed in stress birefringence analyses that both electric field components of an incident ray travel in straight lines and exit at the same point. In reality, the two perpendicular electric field components travel in different paths within the birefringent medium. The ray deviations due to the effects of stress birefringence are generally considered insignificant.

8.5.5 Stress birefringence example

Linearly polarized light, defined by the Jones vector in Eq. (8.47), is incident upon a BK7 window with a uniaxial stress field ($\sigma_{11} = 500$ psi and $\sigma_{22}, \sigma_z = 0$ psi), as illustrated in Fig. 8.25. The output polarization state is computed using Jones calculus. The stress-optical coefficients are defined as $k_{11} = -0.34 \times 10^{-8}$ in²/lb_f and $k_{12} = -2.27 \times 10^{-8}$ in²/lb_f:

$$E_i = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \quad (8.47)$$

The angle θ between the coordinate system defining the incident Jones vector and the principal stress directions is 45 deg and is used to compute the Jones rotation matrix:

$$\mathbf{R}(\gamma) = \begin{bmatrix} 0.707 & 0.707 \\ -0.707 & 0.707 \end{bmatrix}. \quad (8.48)$$

The state of birefringence and the change in the indices of refraction are illustrated in Fig. 8.26 and are given as

$$\begin{aligned} \Delta n_1 &= k_{11}\sigma_{11} + k_{12}(\sigma_{22} + \sigma_{zz}) \\ &= (-0.34 \times 10^{-8})(500) = -1.70 \times 10^{-6} \end{aligned} \quad (8.49)$$

and

$$\begin{aligned} \Delta n_2 &= k_{11}\sigma_{22} + k_{12}(\sigma_{11} + \sigma_{zz}) \\ &= (-2.27 \times 10^{-8})(500) = -1.14 \times 10^{-5}. \end{aligned} \quad (8.50)$$

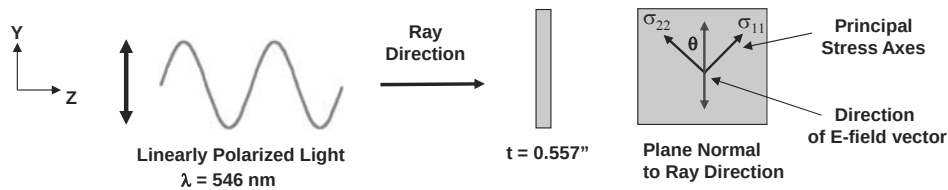


Figure 8.25 Linearly polarized ray incident on a window.

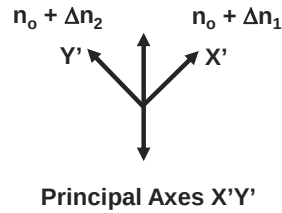


Figure 8.26 Birefringence along the principal stress axes in the window.

The resulting phase change in radians for the principal directions is given by

$$\delta_1 = \frac{2\pi\Delta n_1 t}{\lambda} \approx -0.28 \text{ rad} \quad (8.51)$$

and

$$\delta_2 = \frac{2\pi\Delta n_2 t}{\lambda} \approx -1.84 \text{ rad}, \quad (8.52)$$

which yields the retarder matrix

$$\mathbf{R}(\delta) = \begin{bmatrix} e^{i(-0.28)} & 0 \\ 0 & e^{i(-1.85)} \end{bmatrix}. \quad (8.53)$$

The output polarization state in Jones vector format is computed using Eqs. (8.43) and (8.45):

$$E_{out} = \begin{bmatrix} 0.616 + 0.345i \\ 0.346 - 0.618i \end{bmatrix}. \quad (8.54)$$

The magnitudes of E_x and E_y are both 0.707, and the difference in phase is $\pi/4$ rad or 90 deg. Thus, the mechanical stress acting on the window in this example converts linearly polarized light into circularly polarized light.

8.5.6 Stress birefringence and optical modeling

Three-dimensional ray tracing of an inhomogeneous and anisotropic index field due to the presence of mechanical stress requires knowledge of the dielectric impermeability tensor at each point in the optical element. This may be accomplished by converting the finite element stress distribution into an index ellipsoid map using the piezo- or stress-optical coefficient matrix and the nominal optical properties of the material, as illustrated in Fig. 8.27. A user-defined surface may be developed that couples the optical design software to a finite element stress field where interpolation routines are used to determine the optical properties at any point within the optical element during ray tracing.

An approximate approach for representing the effects of a 3D mechanical stress state in an optical model is using Code V stress birefringence interferogram files. The interferogram files are 2D maps that are an approximate technique to model the effects of a spatially varying stress field based on a linear retarder model.²⁷ The birefringence and crystal axis orientation are derived from a Jones matrix representation that may be assigned to optical surfaces in the optical model. Use of the stress birefringence interferogram files are best suited for collimated light incident on nonpowered optical elements but may be used at the engineer's judgment to approximate the impact of stress on optical performance for powered optics and noncollimated light.

Polarization errors due to the effects of temperature on a doublet collimating lens for a telecommunication component was explored as a function of glass type and mounting method using stress birefringence interferogram files.²⁷ The resulting birefringence and wavefront maps are shown for both front and rear elements in Fig. 8.28.

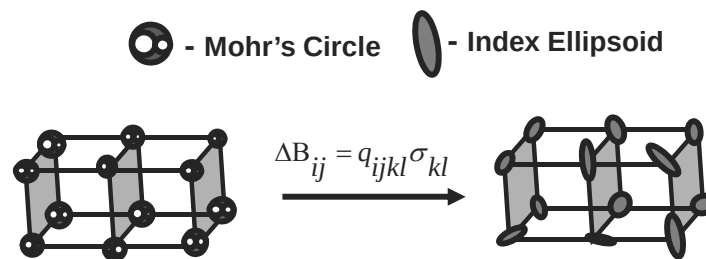


Figure 8.27 Converting a stress distribution into a 3D birefringence model.

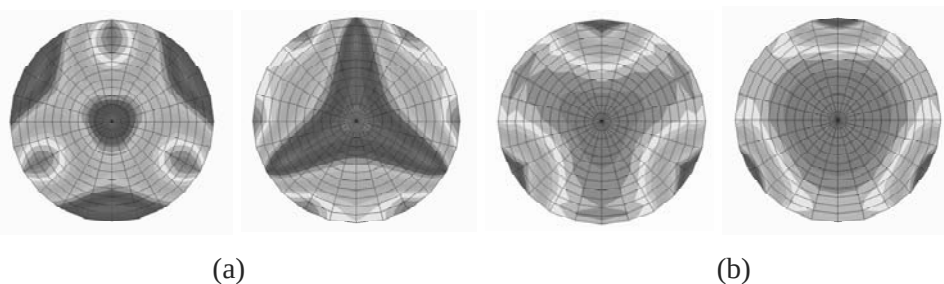


Figure 8.28 Optical errors due to mechanical stress in doublet: (a) birefringence maps, and (b) wavefront error maps for front and rear element, respectively.

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◀Chapter 9▶

Optothermal Analysis Methods

Optical systems often need to operate and survive over a range of temperatures and thermal conditions. Temperature variations adversely affect the performance of an optical system in two ways. First, the thermal expansion and contraction due to thermal variations results in position and shape changes of optical elements, and second, temperature variations change the indices of refraction of optical materials. In addition to performance considerations, thermal effects can create structural failures in optical systems, including yielding or ultimate failure of flexures, debonding of adhesives, and the fracture of glass elements.

Thermal management is a critical aspect of optical system design to ensure performance requirements and integrity are met over the operational and nonoperational service environments. Thermal management includes the use of passive and active control strategies, including appropriate materials selection to maintain temperatures at acceptable levels. For complex systems with demanding performance and environmental requirements, integrated thermal-structural-optical models are beneficial to quantitatively assess thermal-management strategies.

9.1 Thermal Design and Analysis

Thermal design and analysis of high-performance optical systems should begin during the conceptual design stages. First-order analyses and sensitivity studies may be performed using closed-form and simple parametric thermal models accounting for conduction, convection, and radiation modes of heat transfer to identify appropriate thermal-management strategies. Passive thermal control techniques are preferred for their low cost, reliability, and simplicity.

As the thermal design matures, more complex thermal models are developed to predict detailed temperature profiles for a range of operating extremes. These higher-fidelity models may account for detailed geometry, internal heat sources (such as dissipation from electrical components), external heat sources (such as solar and IR fluxes), heat capacitances, joint conductivity, absorptivity/emissivity, the specular/diffusive of coatings and surrounding surfaces, beginning and end-of-life thermal properties, and thermal control solutions such as heaters, insulation, heat pipes, cold plates, and radiators. These analyses are used to ensure that component, assembly, and system thermal requirements are met.

Verification and validation of thermal management strategies is established through coupled thermal-structural-optical performance analyses, along with component-, assembly-, and system-level thermal testing.

9.2 Thermo-Elastic Analysis

Temperature changes in an optical system create thermal strains that cause dimensional and positional changes in the optical components due to the expansion and contraction of materials. This includes changes in optical element thickness, diameter, radii of curvature, and higher-order surface deformations. These departures from the nominal optical system prescription affect optical performance.

9.2.1 Thermal strain and the coefficient of thermal expansion

Thermal strains are created when a material is heated or cooled. Assuming linear behavior, the thermal strain ε_T is expressed as

$$\varepsilon_T = \frac{\Delta L}{L} = \alpha \Delta T, \quad (9.1)$$

where ΔL is the change in length, L is the nominal length, ΔT is the temperature change, and α is the linear coefficient of thermal expansion. The CTE relates the rate at which strain changes with respect to a unit change in temperature. The CTE varies significantly among materials and is commonly expressed in parts per million (ppm). For example, the CTE of structural materials varies from Invar36 at ~ 1.0 ppm/C to titanium at 8.8 ppm/C and aluminum at 23.6 ppm/C. The CTE of optical glasses include fused silica at 0.5 ppm/C, BK7 at 7.1 ppm/C, and FK54 at 14.6 ppm/C. The CTE of plastics and epoxies may be an order of magnitude greater than metals and glasses with CTEs in the hundreds of ppm/C.

There are instances where the thermal strain is nonlinear over the temperature range as shown in Fig. 9.1 and must be accounted for in the analysis. This is most typical when the optical system experiences large temperature swings but may be due to materials with relatively high nonlinear behavior.

The thermal strain for a nonlinear material may be computed using a secant or a tangent CTE. The secant CTE α_s is defined as the linear slope of the thermal strain versus temperature curve between a reference temperature T_0 and a final

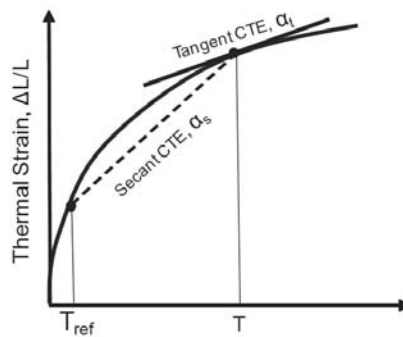


Figure 9.1 Nonlinear thermal strain curve showing tangent and secant CTE definitions.

temperature T and may be used to compute the thermal strain:

$$\varepsilon_T = \frac{L - L_0}{L_0} = \alpha_S (T - T_0). \quad (9.2)$$

In this calculation, the change in length is based from a fixed reference length L_0 .

The tangent CTE, also known as the instantaneous CTE, is the slope of the thermal strain versus temperature curve at a given temperature. The total thermal strain may be computed by integration that accounts for the incremental growth and the changing tangent CTE from the initial temperature to the final temperature:

$$\varepsilon_T = \int_{L_0}^L \frac{dL}{L} = \ln \left(\frac{L}{L_0} \right) = \int_{T_1}^{T_2} \alpha_t dT \quad (9.3)$$

The thermal strain as a function of temperature for a given material is typically found in the literature from which the tangent and secant CTE can be readily derived. Depending on the FEA code, the tangent or secant CTE is entered as a function of temperature to compute the nonlinear structural response quantities.

In lieu of a nonlinear analysis, the secant CTE may be used in a linear analysis to compute the thermo-elastic response quantities over a given temperature range. A variation of the above approach is to use an effective thermal load vector ΔT_{eff} to generate the equivalent thermal strain at the final temperature state for an assumed CTE. The effective thermal load vector is computed as

$$\Delta T_{eff} = \frac{\varepsilon_t}{\alpha_0}, \quad (9.4)$$

where α_0 is an arbitrary coefficient of thermal expansion. This approach can be repeated for several temperatures of interest to compute the nonlinear response using a series of linear subcases instead of performing a nonlinear analysis. Using a secant CTE or thermal load vector in a linear analysis assumes that all other mechanical properties are linear over the temperature range.

9.2.2 CTE inhomogeneity

The spatial variation of the coefficient of thermal expansion can create unacceptable errors for large optical substrates and support structures. The CTE inhomogeneity may be due to the fabrication of the raw materials and/or the micro-mechanical variations in grain size and orientation. For a uniform temperature change, the effects of CTE inhomogeneity are equivalent to thermal

gradients acting on the optical system. A known spatial variation of CTE may be represented in a finite element model by assigning elements different materials that have varying CTE values. In this approach, only one CTE variation can be analyzed in a single FEA solution, as a new stiffness matrix must be computed due to the change in material properties.

An alternative approach that provides efficiencies for studying multiple CTE spatial variations in a single FEA solution is to represent the CTE variations as thermal load vectors, as illustrated in Fig. 9.2. Multiple thermal load vectors may then be executed in a single FEA solution to evaluate variations in spatial CTE and perform sensitivity studies to assess the impact. This technique can also be used to study random variations in the CTE. A program or spreadsheet may be used to generate the random thermal load vectors.

The equivalent thermal load vector ΔT_{equ} may be computed by the following relationship:

$$\Delta T_{equ}(x, y, z) = \frac{\alpha(x, y, z)}{\alpha_0} \Delta T, \quad (9.5)$$

where $\alpha(x, y, z)$ is the CTE at coordinate location x , y , and z , and α_0 is the nominal CTE value specified in the finite element model as a constant material property.

A combined thermal load vector may be computed to account for the CTE variation with temperature and CTE inhomogeneity using

$$\Delta T(x, y, z, T_2) = \frac{1}{\alpha_0} \int_{T_1}^{T_2} \alpha(x, y, z, t) dt, \quad (9.6)$$

where $\alpha(x, y, z, T_2) = \alpha(x, y, z, T_1) + \Delta\alpha(T_2)$, which assumes that the thermal variation is not spatially dependent and that $\Delta\alpha(T_2)$ is equal to the temperature-dependent deviation in CTE from $\alpha(x, y, z, T_1)$.

For the special case of a uniform plate, an effective thermal soak ΔT_{eff} and a through-the-thickness thermal gradient T'_{eff} , may be defined as follows:

$$\Delta T_{eff}(x, y, T_2) = \frac{1}{\alpha_0 t} \int_{t/2}^{t/2} \int_{T_1}^{T_2} \alpha(x, y, z, t) dt dz \quad (9.7)$$

and

$$T'_{eff}(x, y, T_2) = \frac{12}{\alpha_0 t^3} \int_{-t/2}^{t/2} \int_{T_1}^{T_2} [\alpha(x, y, z, t)] dt dz. \quad (9.8)$$

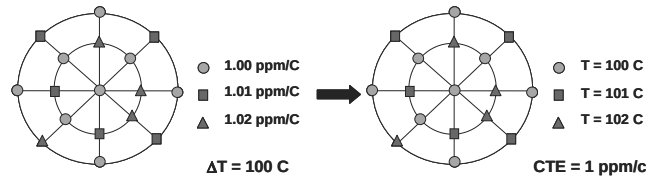


Figure 9.2 Equivalent models accounting for spatial variations in CTE.

9.3 Index of Refraction Changes with Temperature

In addition to thermo-elastic effects, temperature affects optical system performance by changing the index of refraction of an optical material; this results in wavefront errors in the optical system. The thermo-optic coefficient dn/dT defines the change in the index of refraction of an optical material as a function of temperature. There are two commonly used thermo-optic coefficients, relative and absolute. The relative dn/dT value, denoted by dn_{rel}/dT , is the change in the index of refraction of the optical material relative to air. The absolute dn/dT , denoted by dn_{abs}/dT , is the change in the index of refraction relative to vacuum. The relative and absolute thermo-optic coefficients are related as follows:

$$\frac{dn_{abs}}{dT} = n \frac{dn_{air}}{dT} + n_{air} \frac{dn_{rel}}{dT}. \quad (9.9)$$

Thermo-optic coefficients vary widely among glass types with positive and negative values. The absolute thermo-optic coefficients for several Schott glasses¹ at room temperature at a wavelength of 546 nm are listed in Table 9.1.

The thermo-optic coefficient is a function of both wavelength and temperature as given by the Sellmeier Dispersion equation¹:

$$\frac{dn_{abs}(T)}{dT} = \frac{n_{rel}^2(T_{ref}) - 1}{2n_{rel}(T_{ref})} \left[D_0 + 2D_1(T - T_{ref}) + 3D_2(T - T_{ref})^2 + \frac{E_0 + 2E_1(T - T_{ref})}{\lambda_{AWVL}^2 - \lambda_{TK}^2} \right], \quad (9.10)$$

Table 9.1 Absolute thermo-optic coefficients for various glass types in ppm/C.

Glass	dn/dT
Fused Silica	10.0
BK7	1.6
LaF2	-0.7
SF1	6.4
FK51	-7.0
LaK23	-2.0
BaF3	2.1
LaSF3	5.2
PSK2	1.3
SK51	-2.0

where T_{ref} is the reference temperature at 20 °C from which refractive properties are evaluated, λ is the wavelength in a vacuum in microns, λ_{TK} is the average effective resonance wavelength in microns, and D_0 , D_1 , E_0 , and E_1 are material-dependent constants. The absolute thermo-optic coefficient is plotted as a function of wavelength and temperature for Schott glass FK5 in Fig. 9.3.

The impact of thermo-optic effects is to create wavefront errors in the optical system. The change in optical path for a wavefront traveling through a window with a local temperature change is shown in Fig. 9.4. The optical path difference (OPD) or wavefront error is computed using the following relationship,

$$OPD = \frac{dn}{dT}(\Delta T)t, \quad (9.11)$$

where ΔT is the local temperature change, and t is the thickness of the window.

Use of the relative thermo-optic coefficient is applicable when performing a thermal analysis of an optical system in air experiencing a uniform temperature change where both the air and the optics are at the same temperature. In this case, changes in the refractive index of the air do not have to be included. Use of the absolute thermo-optic coefficient is more general and can account for changes in the indices of refraction of both the optical elements and the optical medium that may be at different temperatures, typical of optical systems under thermal loads.

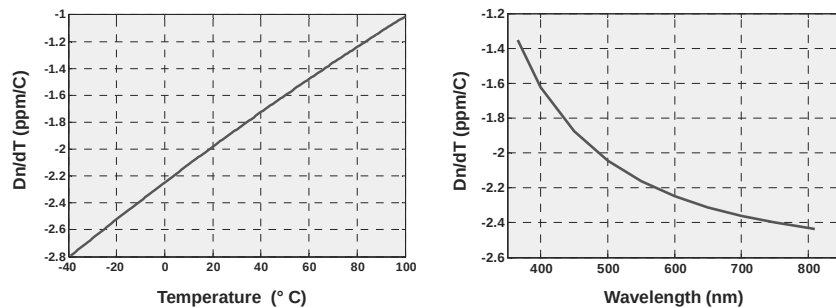


Figure 9.3 Absolute thermo-optic coefficient for Schott Glass FK5 as a function of temperature (left) and wavelength (right).

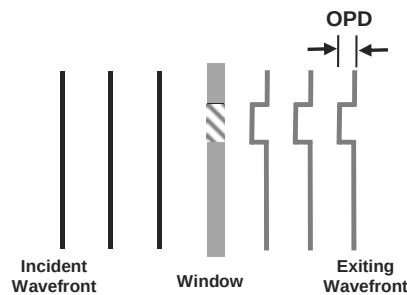


Figure 9.4 OPD error produced by a local temperature change in a window.

A third form of the thermo-optic coefficient that is relative to a fixed air temperature² that may be different than the temperature of the optics is known as the constant-reference thermo-optic coefficient, expressed as

$$\frac{dn_{crrrel}}{dT} = \left(\frac{1}{n_{med} T_{ref}} \right) \frac{dn_{abs}(T)}{dT}. \quad (9.12)$$

This relationship may be derived using Eqs. (9.9) and (9.10). The constant-reference thermo-optic coefficient allows a thermo-optic analysis to be performed for a system in air when the optical elements are at a different temperature than the air without specifying the index change of the air.

9.4 Effects of Temperature on Simple Lens Elements

For a single thin lens element in air, the change in focal length Δf for a uniform change in temperature, as illustrated in Fig. 9.5, is given by³

$$\Delta f = \left[\alpha - \left(\frac{1}{n-1} \frac{dn_{rel}}{dT} \right) \right] f \Delta T, \quad (9.13)$$

where f is the focal length of the lens. The term in the brackets is known as the optothermal expansion coefficient or the thermal-glass constant η . This coefficient accounts for changes in the curvature of the optical surfaces and changes in the refractive index of the material.

The change in focal length may be compensated or balanced by the change in position of the image plane due to the thermal expansion and contraction of the metering structure, illustrated in Fig. 9.6. The difference between the change in focal length from the optical element and the change in the image plane due to the thermal expansion of the metering structure results in an overall focus shift Δ_{focus} of the optical system expressed by

$$\Delta_{focus} = (\eta_s - \alpha_m) f \Delta T \quad (9.14)$$

where α_m is the CTE of the metering structure material.

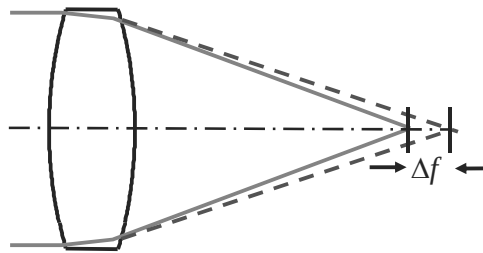


Figure 9.5 Change in focal length due to uniform temperature change.

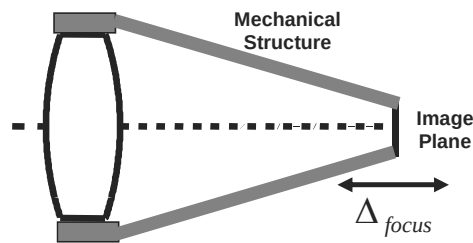


Figure 9.6 An athermal mount balances the focus shift from the optical elements with the thermo-elastic motion from the mechanical mount.

Table 9.2 Thermal-glass constants and coefficient of thermal expansion values for select materials in ppm/C.

Visible Glasses, η		IR Materials, η		Mount Materials, α	
BK7	-1	CaF2	-51	Aluminum	23
BaSF51	8	ZnS	26	Stainless Steel	10
SF5	0	ZnSe	36	Beryllium	11
FK52	-27	Germanium	130	Titanium	9
SK4	-2	Silicon	63	Invar36	1

The thermal-glass constant for infrared materials is typically much larger than the CTE of conventional housing materials. Thus, it is generally much more difficult to passively compensate infrared systems as compared to visible systems. A few select thermal-glass constants for visible and IR glasses are listed in Table 9.2.

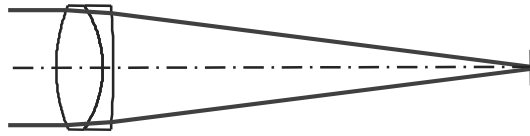
A thermal-glass constant η_s can be specified for lens elements in contact using the following expression:

$$\eta_s = \sum_{i=1}^n \frac{f_s}{f_i} \eta_i, \quad (9.19)$$

where f_s is the system focal length, and f_i are the focal lengths of the individual elements.

9.4.1 Focus shift of a doublet lens example

The focus error for a doublet lens in air imaging an object at infinity is computed for several potential metering structure materials. The doublet has an $f/\#$ of 3.0 and is shown in Fig. 9.7. The properties of the two glass materials are shown in Table 9.3. The analysis assumes a wavelength of 546 nm. Using Eq. (9.15), a doublet thermal-glass constant is computed as 3.2 ppm/C.

**Figure 9.7** Doublet lens element.**Table 9.3** Doublet properties.

Doublet	α	dn_{rel}/dT	η	f (mm)
Lens 1	6.5	4.3	-0.4	42
Lens 2	8.1	7.9	-2.8	-68.1

Table 9.4 The change in focus for three housing materials over temperature.

MATERIAL	Δ_{focus}	ΔT
Aluminum	-2.04	4.8
SS416	-0.67	14.6
Invar36	0.22	44.5

The design requirement is to select a mounting material that minimizes the focus error such that diffraction-limited performance is maintained over the broadest temperature range. Diffraction-limited performance is given by the Rayleigh quarter-wave criterion and is known as the diffraction-limited depth-of-focus, which is expressed in Eq. (9.16). Using this equation, the allowable focus error δ is 9.8 μm , which represents the longitudinal distance in which the image may be offset from the ideal image location:

$$\delta = \pm 2\lambda (f / \#)^2 = \pm 9.8 \mu\text{m}, \quad (9.16)$$

Hence, to maintain diffraction-limited performance, the following condition must be met:

$$\Delta_{focus} = (\eta_s - \alpha_s) f \Delta T \leq 9.8 \mu\text{m}. \quad (9.17)$$

Three materials were considered for the housing: aluminum, stainless steel 416, and Invar36. The change in focus per degree Celsius and the total temperature range ΔT in which the system maintains diffraction-limited performance is given in Table 9.4. The results show that Invar meets the performance requirements over the largest temperature range.

9.4.2 Radial gradients

For radial gradients where the temperature is assumed constant through the thickness of the optical element, the wavefront error or OPD at a given point may be approximated for a system in air by the following expression⁴:

$$OPD = \left[\alpha(n-1) + \frac{dn_{rel}}{dT} \right] t \Delta T, \quad (9.18)$$

where ΔT is the temperature gradient from the center to a point on the edge of the lens. The coefficient of thermal expansion accounts for the change in OPD due to thickness changes, and the thermo-optic coefficient accounts for the change in the index of refraction. This relationship assumes that the thickness of the lens from the center to the edge is constant but is useful for first-order estimates of powered optics. The bracketed expression in Eq. (9.18) is referred to as the thermo-optic constant G and is an approximate measure of the sensitivity of the optical element to radial gradients:

$$G = \alpha(n-1) + \frac{dn_{rel}}{dT}. \quad (9.19)$$

Note that in Eq. (9.18), the contributions from the CTE and thermo-optic coefficients are added, whereas in Eq. (9.13) the contributions are subtracted. Thus, lens assemblies whose glass types tend to minimize focus shifts due to thermal soak conditions are typically more sensitive to radial gradients and vice versa. The thermal-glass and thermo-optical constants for a few select glasses are shown in Table 9.5.

9.5 Thermal Response Using Optical Design Software

Thermal capabilities within optical design software provide for first-order thermo-elastic and thermo-optic analyses allowing the lens design and optical materials to be optimized for system performance. This includes being able to simulate thermal soak conditions that accounts for the expansions and contractions of the spacers and mount. Use of dummy surfaces allows various mechanical mounting arrangements to be represented.

Radial gradients may be simulated across optical elements using Code V. In this case, the temperature is specified as a function of radial points along the lens as shown in Fig. 9.8.

Axial gradients may be approximated using multiple surfaces to define a lens element, as shown in Fig. 9.9. Surfaces are defined at axial locations through the lens and assigned different temperatures.

Table 9.5 Comparison of thermal-glass and thermo-optic constants.

	η	G
N-FK5	11.2	3.5
N-BK7	1.3	6.7
N-LAK12	8.2	4.8
SF11	-10.2	17.7
N-FK51	25	0.8

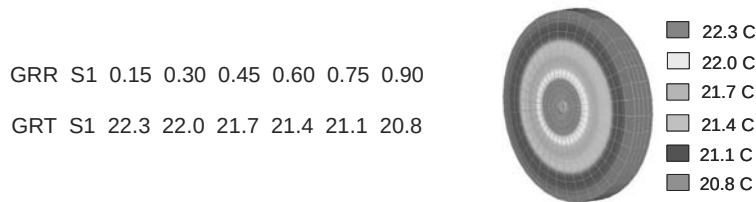


Figure 9.8 Representing radial gradients in Code V optical design software.



Figure 9.9 Representing axial gradients in a lens using multiple surfaces.

9.5.1 Representing OPD maps in the optical model

Optical design codes offer various formats to represent externally derived OPD maps in the optical model. Code V provides the use of wavefront interferogram files (analogous to the surface interferogram files discussed in Chapter 4). The OPD data may be represented by Zernike polynomials or in a uniform rectangular array. The interferogram files are commonly assigned to dummy or optical surfaces that add OPD to the rays intersecting the surface but do not affect the surface shape. Zemax provides the Zernike Fringe Phase and the Zernike Standard Phase surface definitions to represent OPD maps. These surfaces support standard surface-shapes along with the ability to define additional phase terms to deviate and add OPD to the assigned surface. The Zemax Grid Phase surface provides for the representation of OPD in the form of a uniform array of data using a plane surface. Stress-induced OPD errors may also be represented in this manner.

When assigning OPD data to an optical model it is critical that the proper sign conventions are understood. In Code V, for example, a positive wavefront error represents a leading wavefront, as shown in Fig. 9.10. In addition, it is important to align and place the data at the correct location and with the proper orientation. An example using wavefront interferogram files is discussed in the athermal design and analysis of a telecommunications wavelength division multiplexer.⁵

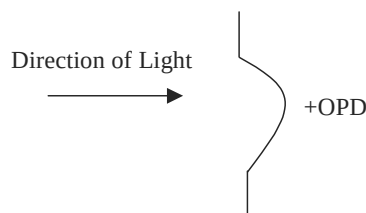


Figure 9.10 Sign convention for Code V wavefront interferogram files.

9.6 Thermo-Optic Analysis of Complex Temperature Fields

Techniques to account for the effects of complex thermal gradients and the resulting thermo-optic errors are discussed. These include the creation of OPD maps that may be represented in the optical model using techniques discussed in the previous section or a method to represent the complex gradient field directly in the optical model for evaluation via ray tracing.

9.6.1 Thermo-optic finite element models

Thermo-optic finite element models are a useful modeling technique to compute OPD maps due to complex temperature profiles. A thermo-optic model is created by modifying the material properties and boundary conditions of a 3D finite element model of the optical element.⁶ The material properties are modified by replacing the coefficient of thermal expansion with the thermo-optic coefficient (depending on the application, the engineer must determine use of the relative or absolute thermo-optic coefficient), setting the elastic modulus to a value of one, and setting the shear modulus and Poisson's ratio to zero to decouple the in-plane and out-of-plane effects. The nodes on the front surface are constrained in the three translational degrees of freedom, while the remaining nodes in the model are constrained in the axes normal to the optical axis (here, the optical axis is assumed to be along the z axis). The applied load vector is the temperature field.

A schematic of a thermo-optic finite element model is shown in Fig. 9.11. At the rear surface of the optical element, a finite element solution will yield a displacement map representing the optical path difference for an exiting wavefront. This displacement profile may then be post-processed similarly to optical surface deformations. For example, the OPD map may be fit to Zernike polynomials or interpolated to a uniform grid. This technique assumes that the rays traverse the optical element along the optical axis or z direction. Hence, for powered optics this technique is an approximation. This method is illustrated for a lens element in Fig. 9.12.

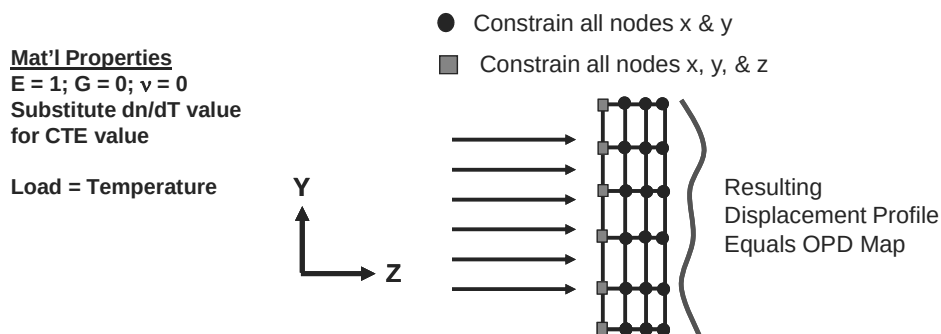


Figure 9.11 A thermo-optic finite element model.

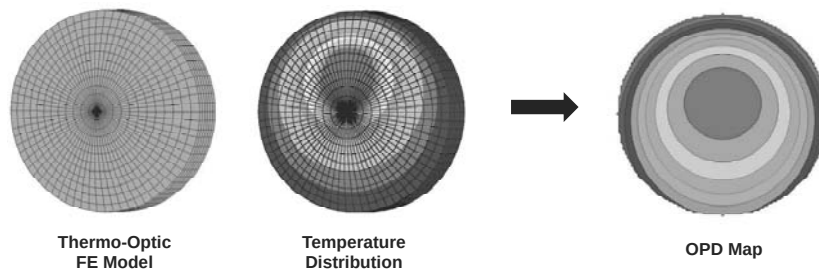


Figure 9.12 Thermo-optic finite element model resulting in an OPD map.

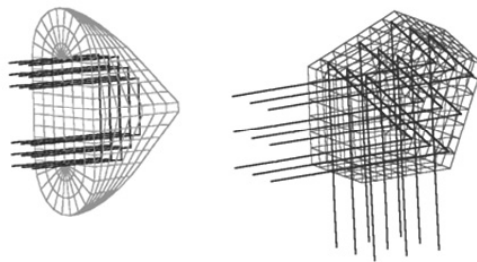


Figure 9.13 Porro and penta prism finite element models.

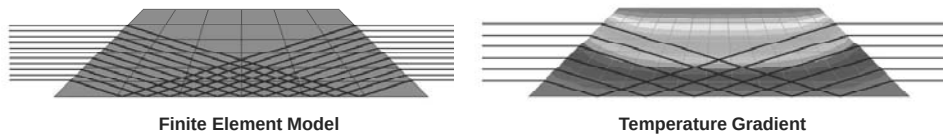


Figure 9.14 Dove prism finite element model and temperature distribution.

9.6.1.1 Multiple reflecting surfaces

Thermo-optic finite element models may also be used to compute wavefront maps for optical elements with multiple reflecting surfaces such as beam splitters and prisms. Finite element models of a Porro and Penta prism are shown in Fig. 9.13. Thermo-optic models for optical elements with multiple reflecting surfaces are typically more difficult to create than those for single-pass elements. The specific ray paths need to be represented within the 3D FEA model using 1D elements such as rod elements. The OPD map of the exiting wavefront is computed by summing the axial displacements of the rod element representing a given ray path. This requires mapping temperatures to the nodes of the rod elements from a 3D temperature distribution of the optical element. Temperature mapping techniques discussed in Section 9.8 may be applied here. This modeling technique is demonstrated for a Dove prism in Fig. 9.14.

9.6.2 Thermo-optic errors using integration techniques

A more rigorous and accurate means to compute OPD maps that accounts for complex temperature profiles is to incrementally sum the OPD along pre-defined integration paths,⁷ as shown in Fig. 9.15. This approach requires defining the ray path through the lens element and knowing the temperature at specific points along the ray path, requiring post-processing of optical element temperature fields.

Typically the integration points do not correspond to node points in the thermal analysis model. Determining the temperature for each point may be performed by using 3D shape-function interpolation⁸ from the finite element model, as illustrated in Fig. 9.16.

The integration may be performed through the whole lens or at several slices through the lens element. For a single field point, one OPD map can effectively represent the OPD errors, as shown in Fig. 9.17(a)–(b). However, for multiple field points, the use of 2D maps to represent a 3D effect is approximate. In this case, using multiple OPD maps [Fig. 9.17(c)–(d)] assigned to axial slices through a lens element improves the accuracy.

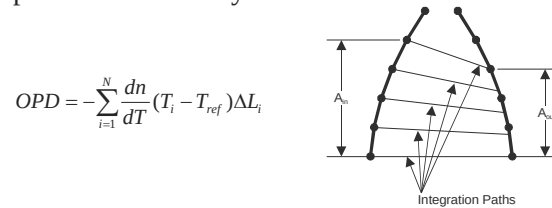


Figure 9.15 Computing OPD maps by integrating through a lens element.

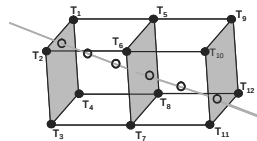


Figure 9.16 Shape function interpolation may be used to determine the temperatures at points inside a finite element.

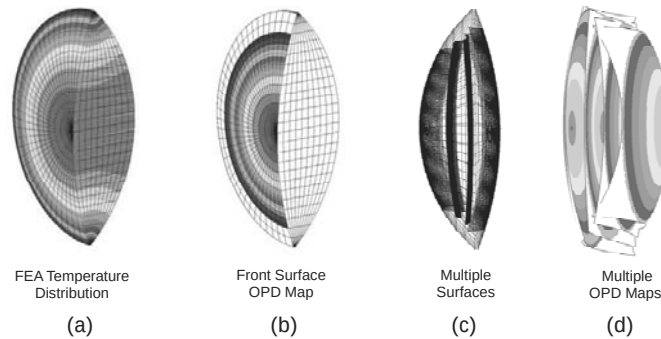


Figure 9.17 (a) Temperature profile; (b) front lens surface OPD map; (c) lens broken into multiple surfaces; (d) multiple OPD maps.

9.6.3 User-defined surfaces

The most accurate means discussed to account for refractive index variations is to create a user-defined surface (UDS) that links optical design software to an external program to define the index of refraction at each location during ray tracing.⁹ Complex thermo-optic effects may then be represented by converting a known temperature distribution into an index of refraction field. Shape function interpolation may be used to determine the index of refraction at any location in a lens element.

9.7 Bulk Volumetric Absorption

As light travels through a transmissive optical element in a typical imaging system, the majority of the light is transmitted. However, a small fraction of the light is absorbed in what is referred to as bulk volumetric absorption. This absorbed energy produces thermal heating. The mechanism responsible for the absorption is the oscillation of the electrons within the atomic structure, which consists of negatively charged electrons floating in bands around the positively charged nucleus. An electrostatic force or spring binds the electrons to the nucleus. The forcing function acting on the atomic oscillator is the time-varying electric field of an incident wavefront, which causes the electrons to vibrate analogous to a mechanical oscillator. The increase in motion of the electrons is dampened by the neighboring atoms and molecules of the material, resulting in the dissipation and absorption of energy. The greatest interaction occurs at the condition of resonance when the electrical field of the incident wavefront has the same frequency as a characteristic frequency of the optical material. The interaction described above is responsible for heating via absorption, and is also responsible for the variation in the index of refraction of a material as a function of wavelength, known as dispersion.

The amount of energy absorbed via bulk volumetric absorption may be approximated by the use of Beer's law. The absorbed energy A is a function of the absorption coefficient κ_λ and the distance the wavefront travels in the optical element d :

$$A = 1 - e^{-\kappa_\lambda d} . \quad (9.20)$$

An absorption coefficient for a given material and wavelength may be computed using the materials' transmission characteristics. For example, Schott glass SF1 transmits 99.3% of 420-nm light for a thickness of 5 mm, as shown in Fig. 9.18. Using Beer's law, an absorption coefficient of 0.0014 mm^{-1} is computed, from which the amount of energy absorbed by a 50.8 mm thick piece of SF1 may be determined. This approach may be extended to account for a spectral range by summing the watts absorbed by each individual wavelength, as shown in Fig. 9.19. Solar energy is absorbed as a function of wavelength by both the coatings and the window substrate. Overlaying the normalized solar spectrum curves with the coating and bulk volumetric absorption curves shows the wavelength bands where the energy is absorbed.

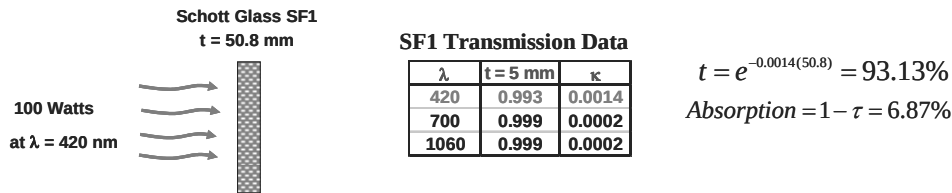


Figure 9.18 Bulk volumetric absorption.

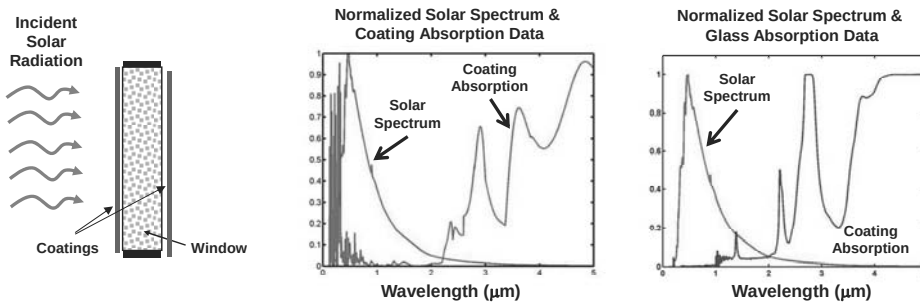


Figure 9.19 Solar absorption by the coatings and window substrate.

Accounting for bulk volumetric absorption in solid transmissive elements using thermal analysis software may be performed using a two-stage thermal modeling process.¹⁰ The first step is to develop a heat-rate model using plate elements of zero thickness to determine the energy absorbed. The second step involves transferring the nodal heat rates determined in the plate elements to a thermal-network model. Then, a steady-state analysis is performed to compute the temperature distribution in the optical element.

Several modeling techniques may be employed to shape, mask, and direct the radiation within the thermal model. This includes using multiple heat sources of varying intensity, optical elements to focus and direct the beam, masks to contour the beam, and absorption coefficients that vary with angle of incidence.

Optical design software may also be used to compute the irradiance profile at detector surfaces using nonsequential ray tracing that can be mapped as thermal loads to optical elements for subsequent thermal analysis.

9.8 Mapping of Temperature Fields from the Thermal Model to the Structural Model

Performing a thermo-elastic analysis requires the mapping of temperatures from the thermal to the structural model. When the models do not share a common mesh, various techniques may be employed to define temperatures at each of the structural nodes. The thermal mesh typically employs a coarser mesh than the structural mesh (see Fig. 9.20) that results primarily from the different physical phenomena each model seeks to represent.

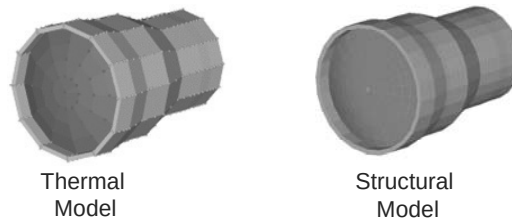


Figure 9.20 Coarse thermal mesh (left) and fine structural mesh (right).

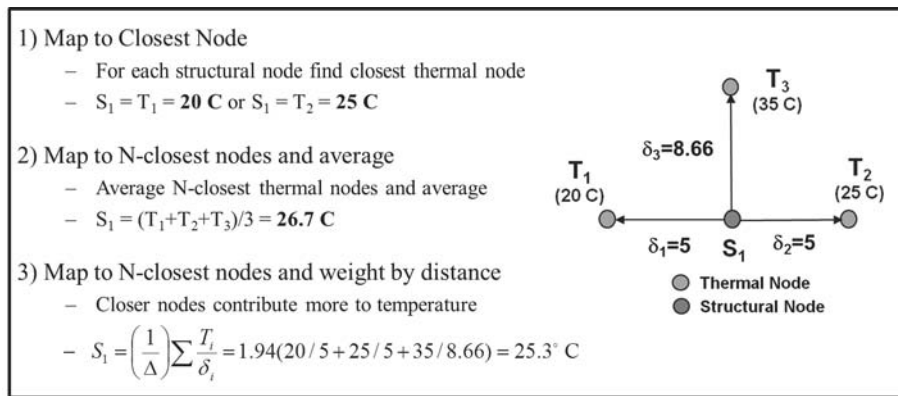


Figure 9.21 Mapping temperatures using nearest-node methods.

9.8.1 Nearest-node methods

One approach to map temperatures from the thermal to the structural model is to transfer the temperature of the nearest thermal node to the structural node. Use of nearest-node methods is appropriate only for the interior nodes of continuous media because the method cannot account for boundaries, gaps, and element properties in the neighboring nodes. A derivative of this approach, intended for greater accuracy, computes a nodal average of the nearest thermal nodes to a given structural node or weights the nearest nodes contribution by distance. Examples of these approaches are shown in Fig. 9.21.

9.8.2 Conduction analysis

A convenient method to map temperatures to the structural model is to perform a finite element conduction analysis. This approach assumes that there are more structural nodes than thermal nodes, and that a structural node exists near or, preferably, at each thermal node location. The temperatures computed by the thermal model serve as boundary conditions in the conduction analysis, as illustrated in Fig. 9.22. Advantages of this technique include accounting for gaps and element properties. However, this technique may generate temperature errors near boundaries, as illustrated in Fig. 9.23. Temperatures are mapped from a coarse thermal model [Fig. 9.23(a)] to a structural model [Fig. 9.23(b)]. The additional thermal paths within the structural model create an erroneous spiked temperature distribution that may yield unacceptable results.

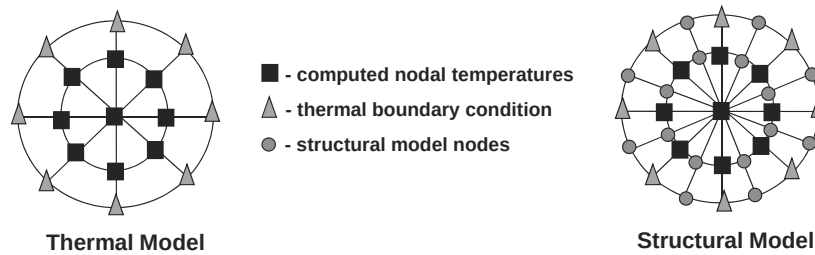


Figure 9.22 Temperature mapping using a conduction analysis.

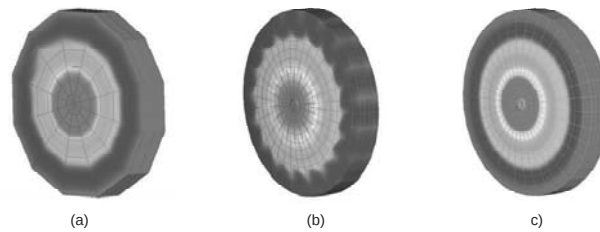


Figure 9.23 (a) Thermal model temperature distribution; (b) temperatures mapped to the structural model using a conduction analysis; (c) temperatures mapped to the structural model using finite element shape functions.

9.8.3 Shape function interpolation

A technique that overcomes the limitations of the previous methods is to use the finite element shape functions to interpolate temperatures from the nodes of the thermal model to the nodes of the structural model.⁸ An example of interpolating temperatures using shape function interpolation is shown in Fig. 9.23(c). In this approach, the temperature at a given structural node is determined based on the temperature of the nodes of the element in which the structural node is located. A numerical example of the use of shape functions to interpolate temperatures using triangle finite elements is illustrated in Fig. 9.24. Variations of shape function interpolation include fitting functions or polynomials to the temperature field that are then used to compute temperatures at each of the structural nodes.

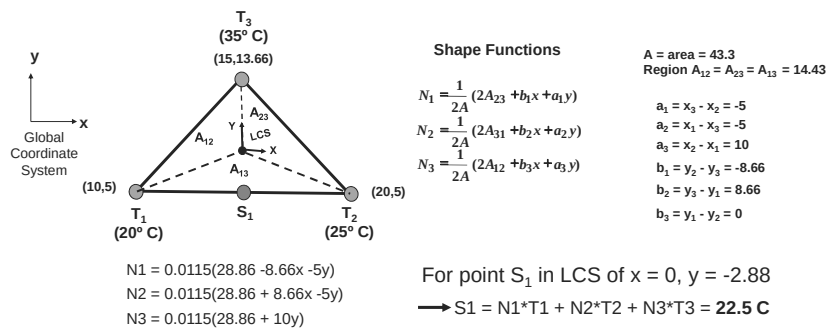


Figure 9.24 Triangle finite element shape functions used to interpolate temperatures.

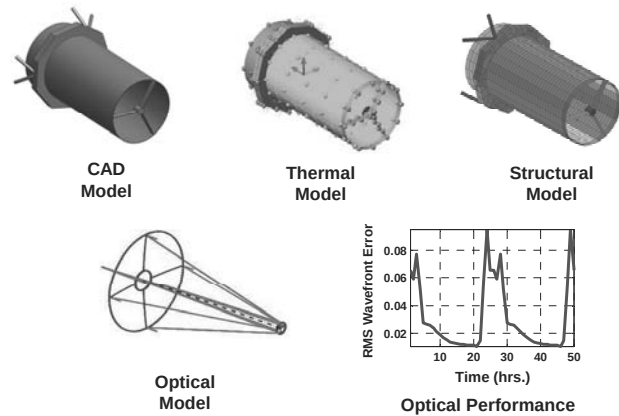


Figure 9.25 STOP analysis example predicting wavefront error for a spaceborne telescope orbiting the earth.

Temperature mapping via shape function interpolation is utilized in the structural-thermal-optical performance (STOP) analysis of a spaceborne telescope orbiting the earth shown in Fig. 9.25. Temperatures are mapped using FEA shape functions from the commercial software package Thermal Desktop[®] to the FEA model. A thermo-elastic analysis is performed to compute the primary and secondary mirror rigid-body errors and deformed surface shapes that are passed to the optical model for optical performance evaluations every hour.

9.9 Analogous Techniques

Numerical techniques applicable to solving thermal conduction problems may be used to solve mass-diffusion-type problems.¹¹ Fick's law is the governing differential equation for problems of mass diffusion, which is of the same form as Fourier's law, the governing differential equation for heat conduction. Fick's law is expressed as

$$\frac{\dot{m}}{A} = -D \frac{\partial C}{\partial x}, \quad (9.21)$$

where D is the diffusion coefficient, C is the moisture concentration, and $\dot{m}$ is the mass flux per unit time.

Fourier's law of heat conduction is given as

$$\frac{q}{A} = -k \frac{\partial T}{\partial x}, \quad (9.22)$$

where k is the thermal conductivity, T is the temperature, and q is the heat flux per unit time. Thus, using heat conduction modeling tools, mass-diffusion-type problems may be solved by using the appropriate substitutions.

9.9.1 Moisture absorption

A common application of the above techniques is to compute the dimensional changes in plastic optics due to moisture absorption, which is a form of mass diffusion and governed by Fick's law. Thermal-modeling tools may be used to compute the moisture concentration using the following thermal-to-moisture analogy: moisture concentration is substituted for temperature, diffusivity replaces conductivity, the moisture gradient replaces the temperature gradient, and the moisture flow equals the thermal flux. In the above analysis, the thermal capacitance should be set to unity. For a known moisture concentration, a thermo-elastic analysis may be performed to compute dimensional changes. Here, the coefficient of moisture expansion (CME) is used in place of the CTE, and the moisture concentration is used as the thermal load.

9.9.2 Adhesive curing

For optical elements mounted with adhesives, shrinkage during curing may cause misalignment and introduce stress in the optical element. Adhesive curing is a form of mass diffusion and the techniques applied to moisture absorption/desorption may be applied to compute the effects of shrinkage. Modeling curing requires the solvent concentration to be used in place of the temperature distribution and the coefficient of solvent shrinkage substituted for the CTE value. Generally, the coefficient of solvent shrinkage must be obtained from test data for each adhesive in consideration.

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◀Chapter 10▶

Analysis of Adaptive Optics

This chapter presents methods and concepts relevant to finite element simulation of adaptive optical systems.

10.1 Introduction

In an adaptive optical system, image motion and aberrations are reduced by moving and deforming one or more optical surfaces. Such adjustment may be made to compensate for temperature gradients, gravity effects, or other load conditions. Such adjustments are often made in response to a measurement of the optical performance of the system. Fig. 10.1 shows a schematic of an adaptive telescope in which aberrations caused by a turbulent atmosphere are corrected. Before reaching the image plane, some of the light is split into a wavefront sensor. Measurements from the wavefront sensor are sent to a controller that predicts how the deformable mirror should be actuated to best correct any aberrations. In such a system, the ability of the algorithm used to predict accurate control commands to best correct the induced aberrations is critical to the net optical performance. It is equally important, however, for the deformable mirror to be able to deform into shapes that will be required to correct the unwanted aberrations. Therefore, predictions of the deformable mirror's performance are of great interest to engineers designing such a system.

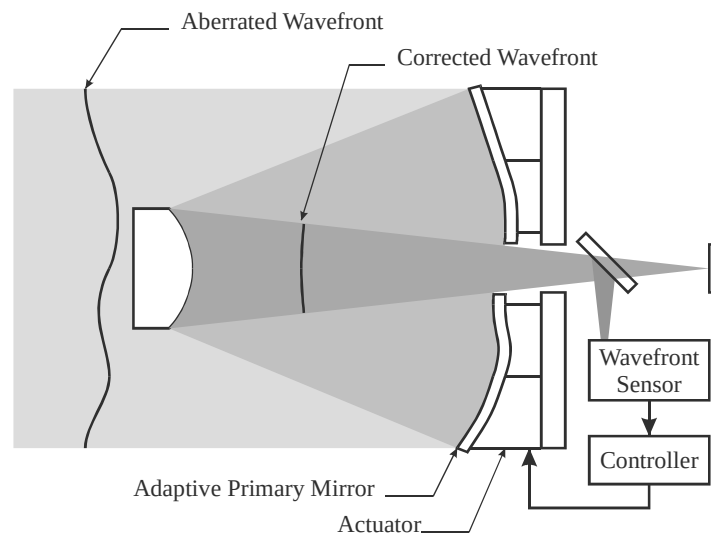


Figure 10.1 Schematic illustration of an adaptive optical system.

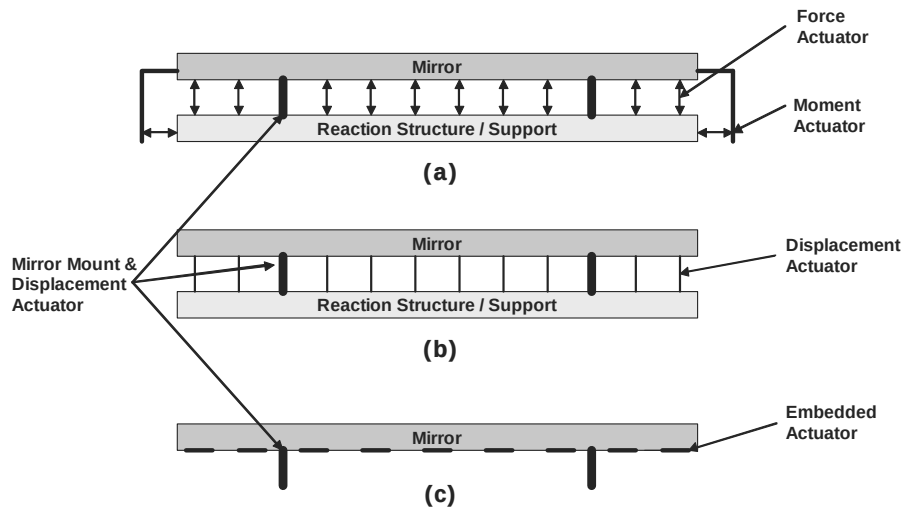


Figure 10.2 Common adaptive mirror architectures.

Three common forms of adaptive mirrors discussed in the literature are shown in Fig. 10.2. Fig. 10.2(a) illustrates architecture characterized by enforced displacement actuators for motion control and an accompanying array of force actuators. The displacement actuators exhibit relatively high stiffness, and, therefore, act as mounts and a means to control the rigid body motion of the optic with high authority. The force actuators are generally low-authority implementations that deliver a desired equal and opposite force pair between two points. A relatively stiff reaction structure is used to react to the actuation loads developed during adaptive figure control. Notice that moment actuators may be implemented through the use of a force actuator and an accompanying moment arm, as shown. Fig. 10.2(b) illustrates an architecture characterized by an array of high-authority displacement actuators. The displacement actuators react against a relatively stiff reaction structure during adaptive figure control and loading of the assembly. Fig. 10.3(c) illustrates an architecture employing embedded actuators within an optic that is mounted on displacement actuators. Use of this architecture eliminates the need for a reaction structure since the actuation loads are internally reacted within the optic. The embedded actuator implementation, however, generally requires many more actuators than the two architectures illustrated by Figs. 10.2(a) and 10.2(b).

10.2 Method of Simulation

The method of simulation of adaptive control that is presented here is limited to static single-pass adaptive control. Issues associated with closed loop control such as feedback and dynamic response are outside the scope of this text. It is also assumed that the corrected performance is linearly related to each actuator's influence on the system.

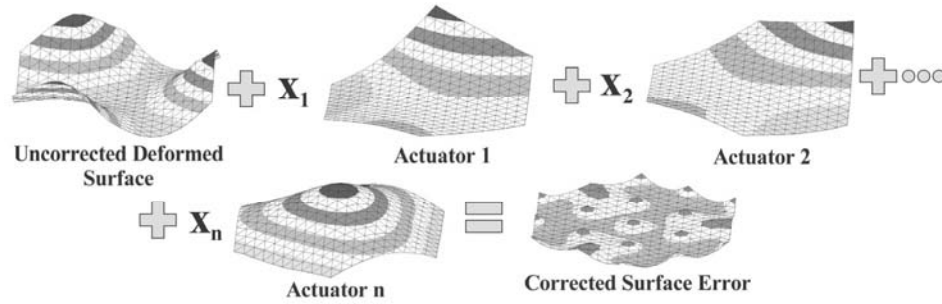


Figure 10.3 Illustration of adaptive control simulation. The unactuated deformed surface plus all of the influence functions scaled by the actuator inputs equals the corrected surface.

It is important to note that many scenarios that are not thought of as adaptive control problems can use the same simulation techniques as are used for adaptive control applications. The support of a large optic mounted on multiple airbags with adjustable pressures is one such example.

10.2.1 Determination of actuator inputs

The goal in the determination of the actuator inputs is to compute a set of actuator forces or displacements that minimizes the surface error of a deformed optical surface.¹ Once actuator inputs have been found, the residual surface error, correctability, and other performance metrics may be found. Fig. 10.3 shows that the corrected surface is equal to the sum of the uncorrected deformed surface plus each of the actuator influence functions scaled by each actuator's respective actuator input. We start by writing the expression for the i^{th} node's corrected displacement ds_i^{Corr} for the optical-surface FEM that has been corrected by the vector of actuator-control inputs x_j :

$$ds_i^{\text{Corr}} = ds_i + \sum_j x_j dx_{ji}, \quad (10.1)$$

where ds_i is the uncorrected displacement of the i^{th} node, x_j is the variable actuator input for the j^{th} actuator, and dx_{ji} is the displacement of the i^{th} node for the j^{th} actuator's influence function. The influence function for a particular actuator is the deformed surface due to a unit input of that actuator while all other inputs are zero. Now, we write an expression of the surface mean square of the corrected optical surface E :

$$E = \sum_i w_i \left(ds_i + \sum_j x_j f_{ji} \right)^2, \quad (10.2)$$

where w_i is the area weighting of the i^{th} node. The weighting factors must be scaled such that their sum is equal to unity:

$$w_i = \frac{A_i}{\sum_i A_i} . \quad (10.3)$$

To solve for the actuator inputs that minimize the mean square error E , we take derivatives of Eq. (10.2) with respect to each actuator input x_j and set each resulting equation equal to zero. This results in the following linear system:

$$[H]\{X\} = \{F\} , \quad (10.4)$$

where

$$H_{jk} = \sum_i w_i f_{ji} f_{ki} \quad (10.5)$$

and

$$F_k = \sum_i w_i d s_i f_{ki} . \quad (10.6)$$

Once the actuator inputs have been found from Eq. (10.4), Eq. (10.2) may be used to compute the mean-square surface error. The surface RMS error is the square root of the mean-square error. The corrected nodal displacements given in Eq. (10.1) may be used for surface RMS, surface peak-to-valley computation, fitting to Zernike polynomials, or interpolation to an array. The latter two processes may be used to generate input to an optical analysis program.

10.2.2 Characterization metrics of adaptive optics

A common metric with which adaptively corrected optical performances are characterized is correctability. Correctability is a relative measure of the decrease in surface deformation from an uncorrected state to a corrected state. However, correctability may be defined in a variety of different ways. Each definition quantifies a unique component of correctability for a given disturbance and actuator configuration. To illustrate the various definitions of correctability we first define the RMS surface characterizations in Table 10.1. With these definitions, we can define three measures of correctability, as shown in Table 10.2.

The total correctability C_{Tot} is usually the desired performance measure. However, it is useful to understand the other two values. The rigid-body correctability C_{RB} shows how much of the total error would be corrected by removal of only the best-fit rigid body motions of the optical surface. The elastic correctability C_{Elas} is a measure of the correction of the surface error attributed to elastic deformation.

Table 10.1 Surface RMS definitions.

<u>SURFACE RMS SYMBOL</u>	<u>DEFINITION</u>
R_U	Surface RMS of the uncorrected surface.
R_B	Surface RMS of the uncorrected surface minus the best fit plane motions.
R_C	Surface RMS of the corrected surface.

Table 10.2 Correctability definitions.

<u>CORRECTABILITY SYMBOL</u>	<u>VALUE</u>	<u>DEFINITION</u>
C_{Tot}	$\frac{R_U - R_C}{R_U}$	Total actuator correctability.
C_{RB}	$\frac{R_U - R_B}{R_U}$	Actuator correctability due to perfect rigid-body motion removal.
C_{Elas}	$\frac{R_B - R_C}{R_B}$	Actuator correctability of elastic component of disturbance.

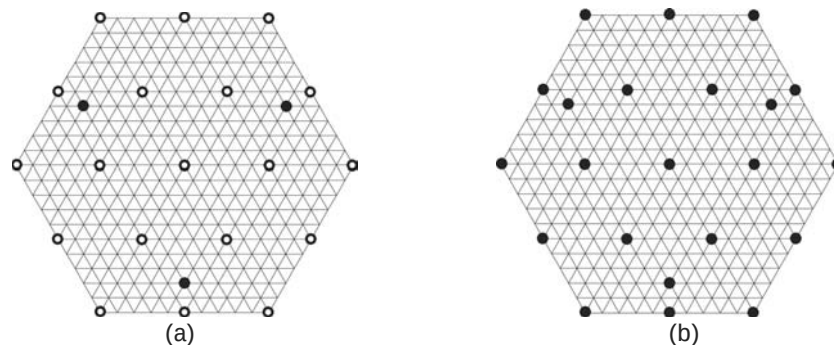


Figure 10.4 Finite element models of a hexagonal mirror segment to be used in an adaptive control simulation: (a) first actuator configuration and (b) second actuator configuration. Solid circles indicate displacement actuator locations, while open circles indicate force actuator locations.

10.2.2.1 Example: adaptive control simulation of a mirror segment

The importance of the differences between the correctability definitions listed in Table 10.2 is best illustrated by example. The finite element model of a hexagonal mirror segment (shown in Fig. 10.4) is to be used in an adaptive

a gravity load normal to its surface. The goal of the analysis is to predict the best-corrected surface and compute the various measures of correctability.

The uncorrected deformed surfaces due to the gravity load, influence functions of the center actuator, and corrected deformed surface are shown for each actuator configuration in Figs. 10.5–10.7, respectively. The surface RMS errors and correctabilities are shown in Table 10.3 for both actuator configurations.



Figure 10.5 Uncorrected surface deformations due to gravity: (a) first actuator configuration and (b) second actuator configuration.



Figure 10.6 Influence functions of the center actuator: (a) first actuator configuration and (b) second actuator configuration.



Figure 10.7 Corrected surface deformation for both actuator configurations.

Table 10.3 Surface RMS errors and correctabilities for adaptive hexagonal mirror.

RESULT TYPE	ACTUATOR CONFIGURATION 1	ACTUATOR CONFIGURATION 2
R_U	0.510 waves	0.021 waves
R_B	0.224 waves	0.008 waves
R_C	0.006 waves	0.006 waves
C_{Tot}	98.8%	71.4%
C_{RB}	56.1%	61.9%
C_{Els}	97.3%	25.0%

Notice that even though the corrected surface RMS error of each actuator configuration is the same, the correctabilities are very different. This difference is attributed to the difference in the uncorrected surface error between the two designs. The correctability of the second actuator configuration design is lower than that of the first actuator configuration design only because its uncorrected performance is superior. What is most informative in this case from a design standpoint is that each actuator design produces the same corrected surface.

The various correctability factors are useful in understanding how well an adaptive mirror corrects common load cases or a set of Zernike polynomial surface distortions. When comparing designs of similar architecture, the correctability factors are a good measure of performance. However, use of correctabilities can be misleading when comparing different design concepts.

10.3 Use of Augment Actuators

For optics that rely on specific disturbances being removed elsewhere in the optical system, augment actuators may be used to represent such removal. An example of this is the removal of rigid-body motions and power of an optical surface being interferometrically tested by a test set that cannot absolutely quantify the orientation and power of an optical surface.

Notice that the actuator-influence set may contain influence function displacement vectors of any scalable surface modification process. Such influence-function displacement vectors include those due to actuators, of course, but could also represent other processes that are not associated with the adaptive control system itself. For example, if an astronomical telescope includes capability to adjust the position of a downstream fold mirror to recalibrate the system for best focus, such adjustment may be equivalent to adding the $2\rho^2 - 1$ Zernike term to the primary mirror's optical prescription. Therefore, an actuator-influence function that represents such a profile may be included in the actuator-influence matrix so that the effects of adaptive and focus controls are considered simultaneously. Such displacement vectors in the actuator-influence matrix are referred to as augment actuators. As another example, when performing adaptive

control simulation of an architecture, such as that shown in Fig. 10.2(a), it is sometimes convenient to include the influence-function displacement vectors that represent control of the rigid-body motion of the optical surface as augment actuators rather than as influence function cases in the finite element analysis. Control of rigid-body motion may be analytically generated in the actuator-influence set by three individual displacement vectors that can linearly combine to represent bias and two orthogonal tilts of the optical surface. It should be noted that analytically generated augment actuators do not contain possible higher-order effects of the influence functions they are representing. An example of such effects could be elastic deformation caused by moments imparted to the system from flexures used in rigid-body motion control hardware. Inclusion of these higher-order effects can only be accomplished through the inclusion of the actuation in the set of influence functions predicted by the finite element analysis.

It is important to note that if linear combinations of one or more of the actuator influence functions are included as augment actuators, then a singularity will result in computing the actuator control inputs. One common way of doing this is to include augment actuators of rigid body motion in an influence function set that already has the capability to control rigid-body motion. A second common adaptive control capability that is redundantly specified through augment actuators is the control of power.

10.3.1 Example of augment actuators

One example of the use of augment actuators is in the simulation of an optical test in which the optical measurement system is unable to quantify surface orientation and power. Fig. 10.8 shows an optic being tested on two airbag rings, as might be done to accommodate mounting hardware. The rings will be inflated to minimize surface RMS after focus. The surface error introduced by the test setup may be predicted by performing adaptive control simulation with an influence function for each airbag and a set of polynomial augment actuators to remove the components of the disturbance that cannot be measured by the optical test. For instance, polynomial influence functions representing bias, two orthogonal tilts, and power may be introduced to the set of influence functions to represent the inability of the test to measure the orientation and power of the optical surface.

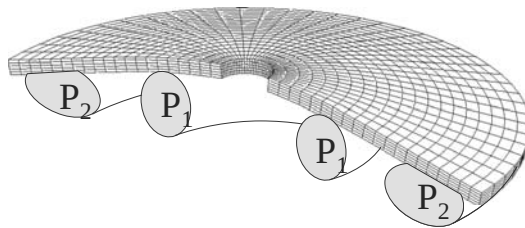


Figure 10.8 Finite element model of a mirror supported on a double annular air bag test set.

10.4 Slope Control of Adaptive Optics

For some applications, control of the slope of the optical surface is important. One example application is the interferometric testing of an optical surface. The error equation may be augmented with the surface slope terms:

$$E = (1-c) \sum_{i=1}^N w_i \left(ds'_i - \sum_{j=1}^M A_j f_{ji} \right)^2 + (c)(L) \sum_{i=1}^N w_i \left(d\Theta'_i - \sum_{j=1}^M A_j \Theta_{ji} \right)^2 + (c)(L) \sum_{i=1}^N w_i \left(d\Phi'_i - \sum_{j=1}^M A_j \Phi_{ji} \right)^2, \quad (10.7)$$

where c represents the weighting fraction of slope data versus displacements. Because slopes and displacements have different units, it is necessary to multiply slopes by a representative length L to combine terms. A possible choice for L is given by

$$L = \sqrt{A/N}, \quad (10.8)$$

where A is the surface area, and N is the number of nodes on the surface.

Note that solid elements in most FEA programs do not have rotational stiffness, so rotations of nodes are not calculated by these tools and are generally reported as zero or a numerically small number. The above equations for slope control would yield erroneous results if applied to a displacement set with zero rotations. If the optical surface is coated with a thin very compliant layer of shell elements with bending stiffness, then valid nodal slopes are calculated over most of the surface. However, nodes near the edge of the optic will exhibit fictitious edge behavior due to the incompatibility of the cubic shape functions of the bending elements compared to the linear shape functions of the solid elements. A suggested remedy to this fictitious behavior is to continue the mesh of the compliant shell layer down the outside edge of the optic.

10.5 Actuator Failure

In adaptive control applications actuator failure is a common concern for orbiting telescopes. If a force actuator fails in an architecture such as that described by Fig. 10.2(a), there is some loss of control, but the other actuators can generally provide some compensation. If a displacement actuator fails in an architecture such as that described by Fig. 10.2(b), then the failed actuator locks the optic to the reaction structure at that location. Therefore, correctability degrades faster with failed displacement actuators than with force actuators. The following example demonstrates the form of the correctability verses the number of actuators failed. The details of the curve will vary with the input disturbance

shape, the total number of actuators, and which actuators are considered to have failed.

Fig. 10.9 shows a finite element model of an adaptively controlled mirror with eighteen actuators distributed as shown. Adaptive control of an astigmatism surface disturbance was simulated. An actuator failure analysis was first run with displacement actuators only and then repeated with force actuators. The results of this actuator failure analysis are shown in Fig. 10.10. When no actuators fail, the correctability for both the force and displacement actuators is 97.2%. Failed actuators were then picked at random and the adaptive control simulation was repeated omitting the failed actuators. The correctability of both actuator architecture types was found for various numbers of failed actuators. Failed displacement actuators, with their associated grounding, degrade the correctability faster than the force actuators.

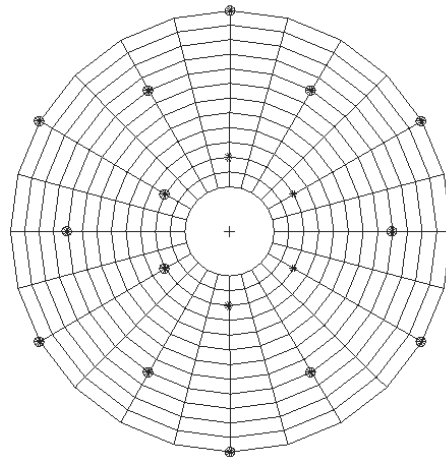


Figure 10.9 Adaptively controlled mirror with 18 actuators.

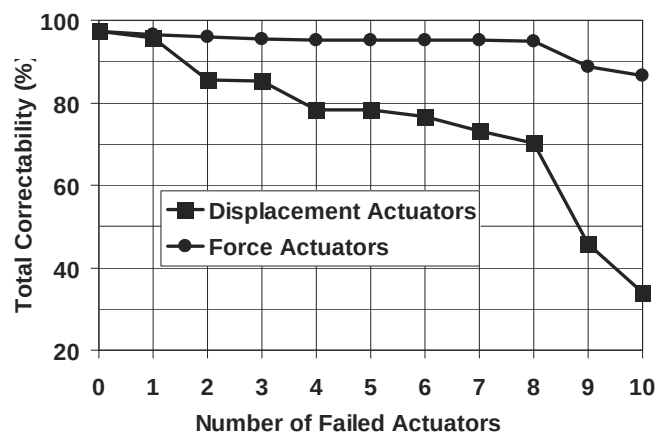


Figure 10.10 Total correctability vs. number of failed actuators.

10.6 Actuator Stroke Limits

Consideration of actuator stroke limits is important for adaptive systems that are designed to use the full useable stroke range of at least a subset of their actuators. It is important that the method of including stroke limits in the adaptive simulation analysis be consistent with the operation of the hardware. One method uses a technique similar to the analysis of failed actuators as shown in Fig. 10.11. This method begins with unbounded adaptive control simulation, followed by a comparison of the resulting actuator strokes to their stroke limits. The most violated actuator is then constrained to its maximum allowable stroke and the cycle of unbounded adaptive control simulation and constraint of the most violated actuator is repeated with the remaining unconstrained actuators. The process converges when an adaptive control simulation is performed and no additional actuators are found to be violated. The disadvantage of this method is that there is no allowance for an actuator constrained at full stroke to become reactivated as other actuators are constrained.

An improvement to the method (shown in Fig. 10.11) uses quadratic programming techniques to find the global optimum control within the stroke limits. The eighteen-actuator, adaptively controlled mirror in the example described in Section 10.5 shows the correctability curve when stroke limits are imposed. The method used to perform the stroke-limited adaptive control simulation is that used in Fig. 10.12.

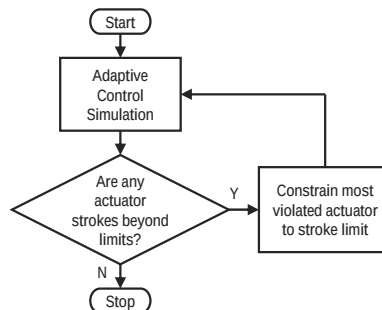


Figure 10.11 Flow chart of one method of adaptive control simulation with stroke limits.

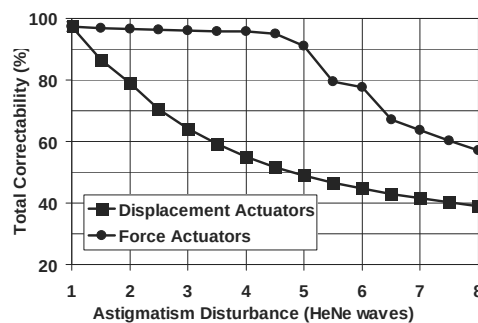


Figure 10.12 Correctability vs. disturbance amplitude.

In this analysis, the 1.0 HeNe waves of astigmatism is the amplitude of the disturbance at which the first actuator stroke limit is exceeded. As the amplitude is increased, more of the actuators reach their stroke limits. As can be seen from the curves, the correctability degrades faster with displacement actuators than with force actuators. The details of the curve will vary with the input disturbance shape, the total number of actuators, and actuator location.

10.7 Actuator Resolution and Tolerancing

In Section 10.2.1, it was assumed that actuator strokes were continuous variables. However, in many implementations actuators can only be controlled to discrete values separated by a step size. Monte Carlo techniques may be employed to quantify the error introduced by this effect.²

The nodal displacements U_{ij} are determined by the following equations:

$$x_{ik}^* = \sigma_k \times \gamma_{ik}, \quad (10.9)$$

$$U_{ij} = U_{Corr_j} + \sum_k \frac{df_{jk}}{dx_k} x_{ik}^*, \quad (10.10)$$

where i is an index on the Monte Carlo analyses, k is an index on the actuators, j is an index on the number of nodes, x_{ik}^* given by Eq. (10.9) is a random actuator input associated with the stepping induced error, σ_k is the actuator step size of the k^{th} actuator, γ_{ik} is a random number uniformly distributed between -0.5 and 0.5 , and U_{Corr_j} is the j^{th} nodal displacement of the best-corrected surface. The partial derivative ($\partial f_{jk} / \partial x_k$) of the j^{th} nodal displacement, with respect to the k^{th} actuator control input x_k , is

$$\frac{\partial f_{jk}}{\partial x_k} = \frac{f_{jk}}{x_k}, \quad (10.11)$$

where f_{jk} is the nodal displacement of the j^{th} node and the k^{th} actuator influence function, and x_k is the actuator input value associated with f_{jk} .

Using Eqs. (10.9) through (10.11), a set of Monte Carlo realizations can be generated to find the statistical behavior of the corrected surface due to the effect of actuator resolution. Statistical measures such as the mean, median, and cumulative probability can be computed from the results of the Monte Carlo analysis.

10.7.1 Example of actuator resolution analysis

The adaptively controlled mirror shown in Fig. 10.13 is mounted on three bipod flexure pairs and controlled with 15 force actuators. If the force actuators have a resolution of 0.20 lbf, the Monte Carlo analysis predicts that the mean residual surface-RMS-error component due to the actuator stepping is 0.060 HeNe waves, and the 95th percentile surface RMS error would be 0.079 HeNe waves. The analysis results can be used to determine the actuator resolution required to meet optical performance requirements on surface RMS.

The cumulative probability as a function of surface RMS error is plotted in Fig. 10.14. This plot illustrates the probability with which the corresponding surface RMS error is an upper bound on the error contribution due to actuator resolution. For example, Fig. 10.14 shows that 0.060 HeNe waves is a bound on the surface RMS error contribution 50% of the time.

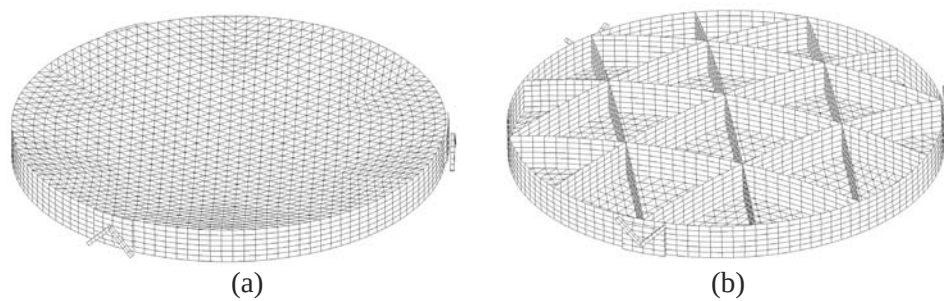


Figure 10.13 Finite element model of adaptively controlled mirror controlled by force actuators with 0.2 lbf resolution.

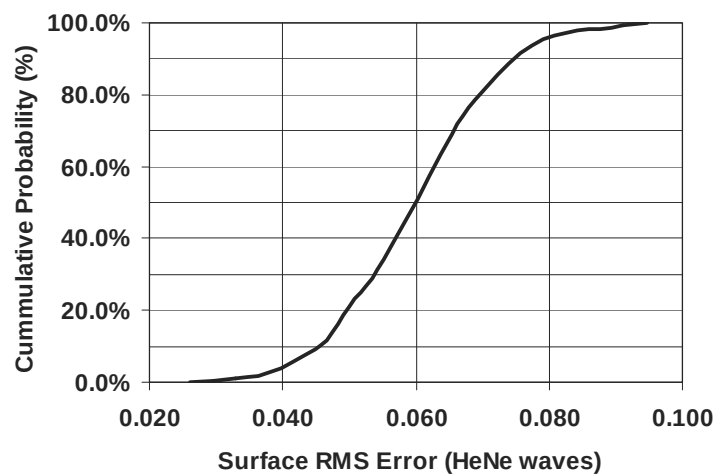


Figure 10.14 Cumulative probability function for surface RMS error contribution due to effect of 0.20 lbf actuator stepping.

10.8 Design Optimization of Adaptively Controlled Optics

Design optimization techniques can be powerful methods of improving existing designs or developing new design concepts that meet desired requirements. A strength of design optimization is that the algorithms used can handle multiple performance requirements and multiple design variables, each of various types. Adding the simulation of adaptive control to these techniques is an effective way to address the highly coupled complex nature of the many design variables and requirements associated with the development of adaptively controlled optical systems.

10.8.1 Adaptive control simulation in design optimization

The simulation of adaptive control as defined in Eqs. (10.1) through (10.6) must be implemented with the finite element design optimization process through the use of an external subroutine called by the finite element software as shown in Fig. 10.15. In the case of adaptive control simulation, Eq. (10.5) cannot be evaluated by the internal equation features of any finite element software. An external subroutine is required for any design response calculations that cannot be computed by the internal equation features within the capabilities internal to the finite element software.

As shown in Fig. 10.15, the design optimization process loops through subprocesses of response calculations, a convergence check, design sensitivity computation, and generation of a new design. The responses computed may include any quantity that is to be minimized, maximized, or constrained. These quantities are generally metrics associated with design requirements such as

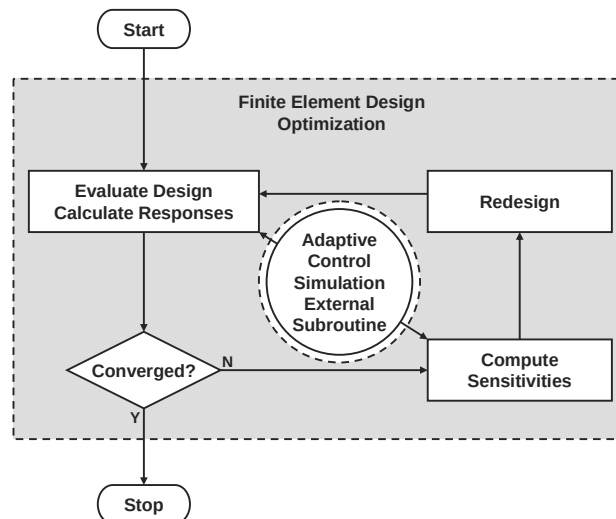


Figure 10.15 Flowchart of design optimization utilizing an external subroutine to perform adaptive control simulation.

weight, natural frequency, maximum stress levels, and other quantities predictable by finite element analysis. The computation of adaptively corrected surface error and its sensitivities with respect to the design variables is performed at the same time as the calculations for all other responses but uses a call to an external subroutine.

10.8.1.1 Example: Structural design optimization of an adaptively controlled optic

An example telescope model is used to demonstrate the optimization of an orbiting telescope's adaptive primary mirror. A plot of the telescope model is shown in Fig. 10.16(a). The primary mirror is of lightweighted construction composed of a lightweighted triangular cell core with front and back faceplates. A primary mirror reaction structure provides mounting locations for three displacement actuators and six force actuators. The layout of the mounts and actuators on the mirror is shown in Fig. 10.16(b). The primary mirror reaction structure also supports the focal plane unit behind the primary mirror. A cylindrical shell is used to meter a secondary mirror assembly comprising the secondary mirror and spider structure. Six main struts are used to mount the telescope to the spacecraft bus.

The goal of the optimization is to minimize the weight of the adaptively controlled primary mirror while constraining the optical performance of the telescope. The optical performance is measured by the wavefront error at the exit pupil of the telescope system. This system-level optical performance is computed by the external subroutine shown in Fig. 10.13 using the techniques shown in Section 4.5.^{3,4} Several design variables relating to the design of the primary mirror are defined. These variables include the depth of the mirror core, the thicknesses of the facesheets, and the wall thicknesses of various regions of the mirror core, as shown by the shading in Fig. 10.17.

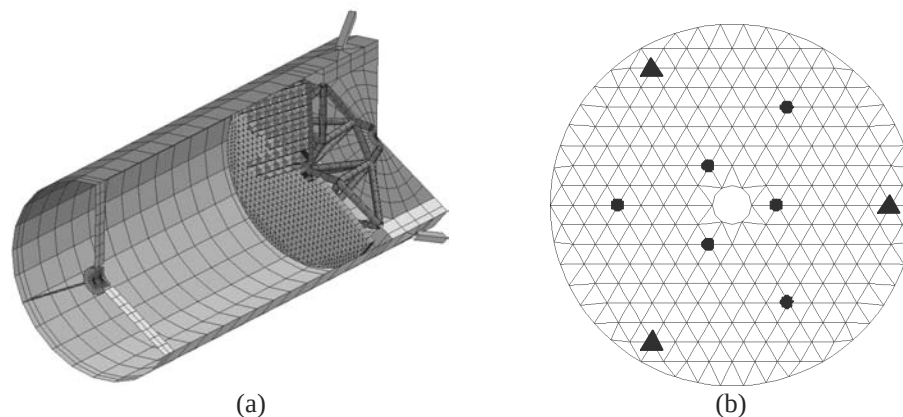


Figure 10.16 (a) Finite element model of a telescope with an adaptively controlled primary mirror and (b) a plan view of the primary mirror showing displacement actuator mounts with triangles and force actuators with circles.

The thickness variables were defined such that thicknesses could be designed near each of the mounts and actuators. Additional requirements were imposed consisting of a minimum natural frequency and a launch stress allowable. The optimization problem is formally defined as follows:

MINIMIZE:

Weight of primary mirror

DESIGN VARIABLES:

Optical facesheet thickness: $0.18 \text{ inch} < t_f < 0.25 \text{ inch}$

Back facesheet thickness: $0.10 \text{ inch} < t_b < 0.25 \text{ inch}$

Interior core wall thicknesses: $0.04 \text{ inch} < t_c < 0.25 \text{ inch}$

Inner and outer core wall thicknesses: $0.08 \text{ inch} < t_c < 0.25 \text{ inch}$

Core depth: $0.25 \text{ inch} < t_c < 5.0 \text{ inch}$

SUBJECT TO:

Thermally induced system wavefront error $< 20 \text{ nm RMS}$

Gravity-release-induced system wavefront error $< 60 \text{ nm RMS}$

Peak launch induced stress in PM $< 1000 \text{ psi}$

First mounted PM natural frequency $> 200 \text{ Hz}$

The analysis results for the initial design and the optimized design are shown in Table 10.4 alongside the requirements. Notice that the optimizer reduces the

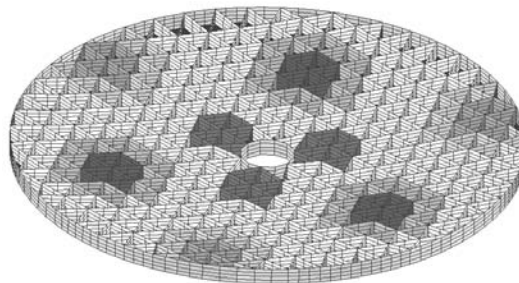


Figure 10.17 Finite element model of an adaptively controlled mirror to be optimized. Core thickness variables are shown by shading.

Table 10.4 Results of design optimization of adaptively controlled primary mirror.

Response	Initial Design	Optimized Design	Requirement
Thermally induced wavefront error	9 nm	20 nm	20 nm
Gravity-release-induced wavefront error	54 nm	60 nm	60 nm
Peak launch stresses	1000 psi	1000 psi	1000 psi
First natural frequency	231 Hz	221 Hz	200 Hz
Weight	20.8 kg	9.9 kg	Minimum
Areal density	53.0 kg/m ²	25.2 kg/m ²	Minimum

weight of the primary mirror by over 50% while the constraints on system wavefront error, launch stresses, and natural frequency are obeyed. It is important to notice that the stress constraint is already active in the initial design while the gravity-induced wavefront error constraint is nearly active.

10.8.2 Actuator placement optimization

The development of the optimum locations of actuators for an adaptively controlled optic is a manually iterative and time-consuming process without the implementation of an automated optimization technique. Such a manual process becomes prohibitive when multiple disturbance cases (e.g., gravity and thermo-elastic deformations) need to be considered in the development of a single-actuator layout.

Genetic algorithms offer a robust optimization method that is well suited to the combinatorial nature of actuator placement optimization.^{5,6} Additionally, their design enables them to find global optimums even in design spaces that contain many local optimums. This is achieved by simultaneously developing designs across the entire design space.

The basic method, shown in Fig. 10.18, is an iterative procedure that operates on a set of actuator layouts in order to develop a new set of layouts with improved adaptive-control performance in each iteration. Actuator layouts are constructed by selecting actuator locations from a finite set of candidate actuator locations. Each layout in the set of actuator layouts is represented by a series of binary digits, where each digit corresponds to a candidate actuator location. The iterative procedure includes operations such as mating selection, crossover, and mutation that mimic the processes of Darwin's theories of evolution and natural selection in order to find actuator layouts with successively improved, adaptively controlled performance. The iterative process ends when a convergence evaluation determines that development of additional sets of layouts is no longer beneficial. The optimum is the individual with the best performance of all individuals considered in the process.

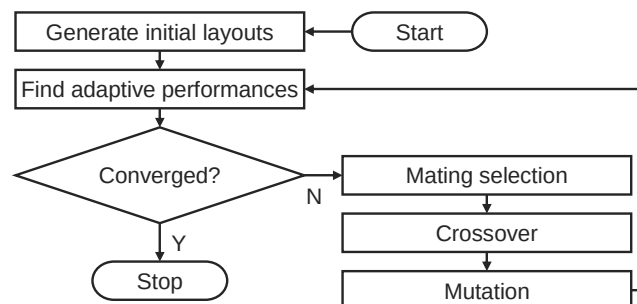


Figure 10.18 Flowchart of a genetic optimization algorithm.

10.8.2.1 Example: Actuator layout optimization of a grazing incidence optic

An actuator layout optimization is to be performed on a grazing-incidence optic to be used in an adaptively controlled x-ray telescope. Two thermo-elastic load cases are considered, with the first consisting of an axial thermal gradient and a second consisting of a circumferential thermal gradient. A set of 200 candidate actuators is distributed evenly, as shown in Fig. 10.19.

Using the actuator placement optimization in SigFit, the optimum sets of 20, 40, 60, and 80 actuators were found. A plot of the residual error versus the number of actuators is given in Fig. 10.20.

The locations of the actuators in the optimum actuator layouts for the 20-actuator case and the 40-actuator case are shown in Fig. 10.21. Notice that the set of actuators found by the optimizer results in force pairs creating effective edge moments to control the bending caused by the thermal gradients. This corrective loading is consistent with fundamental elasticity theory.

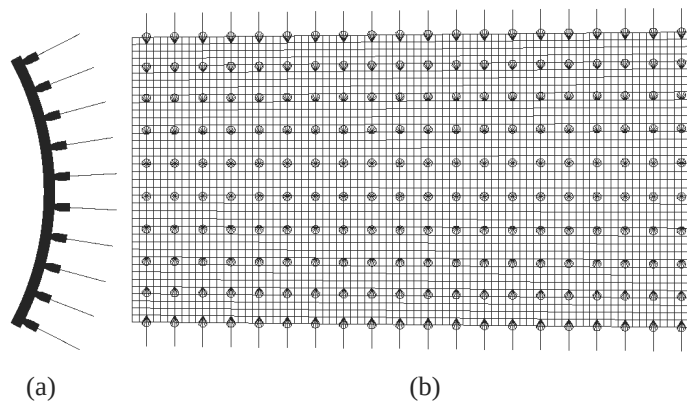


Figure 10.19 Plots of the finite element model of a grazing incidence optic with candidate actuator locations shown by the force vectors with which the actuators act.

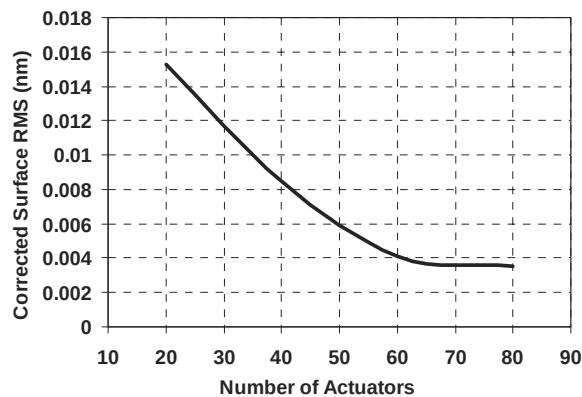


Figure 10.20 Plot of residual error vs. number of actuators.

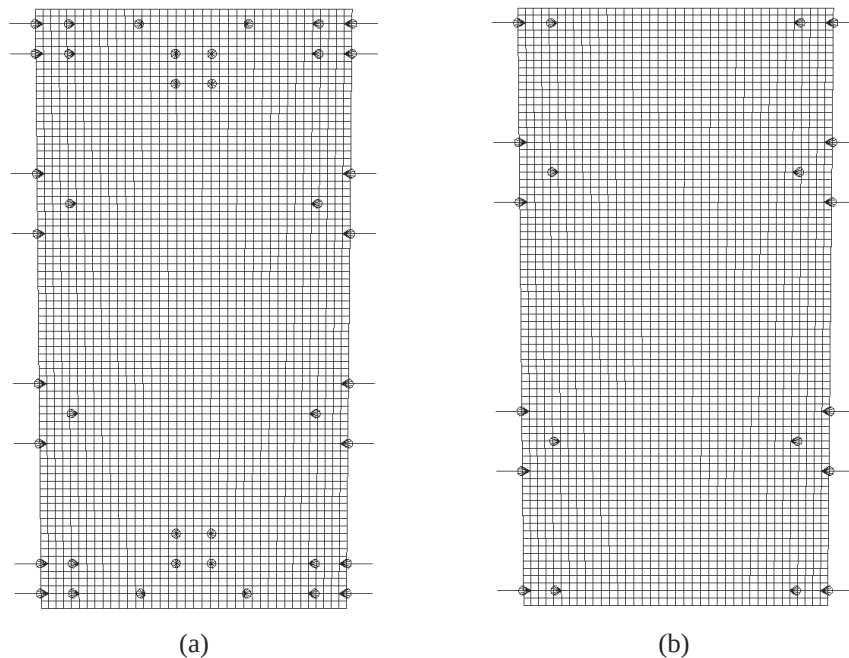


Figure 10.21 Optimum actuator sets for (a) 40 actuators and (b) 20 actuators.

10.9 Stressed-Optic Polishing

Stressed-optic polishing is a manufacturing technique allowing efficient fabrication of complex aspheric surface figures.^{7–10} Like most figuring processes, the technique is an iterative process involving cycles of figuring and measurement. However, within the process of stressed-optic polishing is the need to perform adaptive control simulation to maximize the convergence rate.

10.9.1 Adaptive control simulation in stressed-optic polishing

To show the role of adaptive control simulation within the process of stressed-optic polishing it is helpful to illustrate the full process. Polishing an initially curved surface to obtain a flat surface is less expensive and less time-consuming than polishing a flat or spherical surface into an aspheric surface. The process of stressed-optic polishing employs this advantage as shown in Fig. 10.22. The process begins with a blank that is figured either flat or to the best-fit sphere of the final desired surface geometry. An initial actuation of the blank to be figured is performed to impart the inverse of the desired change in figure. A polishing operation is then performed to bring the surface figure back to the starting blank geometry. During polishing, intermittent measurements of the surface figure are used in combination with the actuator loads, actuator influence functions, and analytical backouts to predict the surface figure of the unstressed blank in its operational configuration. The purpose of the analytically predicted backouts is

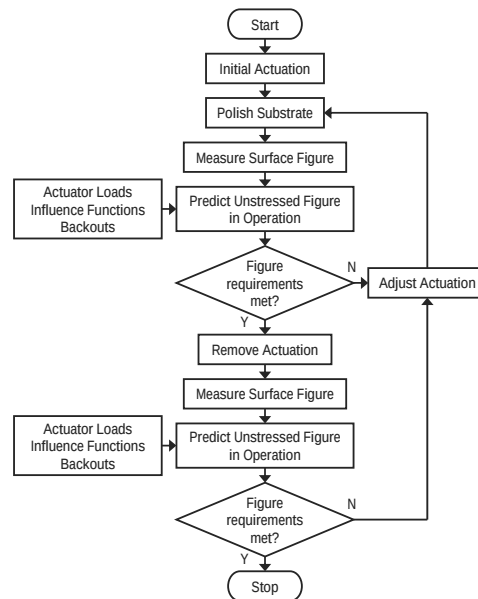


Figure 10.22 Finite element model of an array of mirror segments.

to remove the effect of test errors. This information can then be used to adjust the actuation of the blank before polishing is continued. The process continues until the figure requirements are met. Finally, the actuation loads may be removed so that direct measurements of the unstressed blank may be combined with the analytical backouts. The iterative process may continue, if required, by reapplying new actuation loads and continuing the polishing process.

An adaptive control simulation must be employed each time an adjustment in the actuator loads is made. This simulation is performed to find the actuation loads that best correct the results of a measurement of the surface figure with the desired backouts applied. The frequency with which measurements of the surface figure are made to support adjustment of the actuators is dependent on many factors associated with the details of figuring the surface, which are outside the scope of this text.

10.9.2 Example: Stressed-optic polishing of hexagonal array segments

Consider the segmented mirror array whose finite element model is shown in Fig. 10.23. The optical prescription of this array is an asphere centered at the center of the array.

One approach to fabrication of an individual segment starts with an oversized circular spherical blank, as shown in Fig. 10.24(a), and to deform the blank so that the surface possesses the inverse of the desired departure from the initial sphere. The desired departure from the initial spherical surface for an

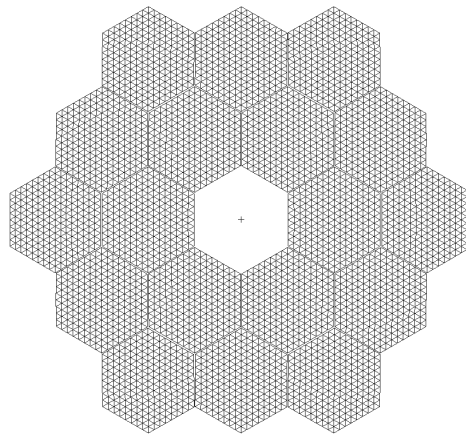


Figure 10.23 Finite element model of an array of mirror segments.

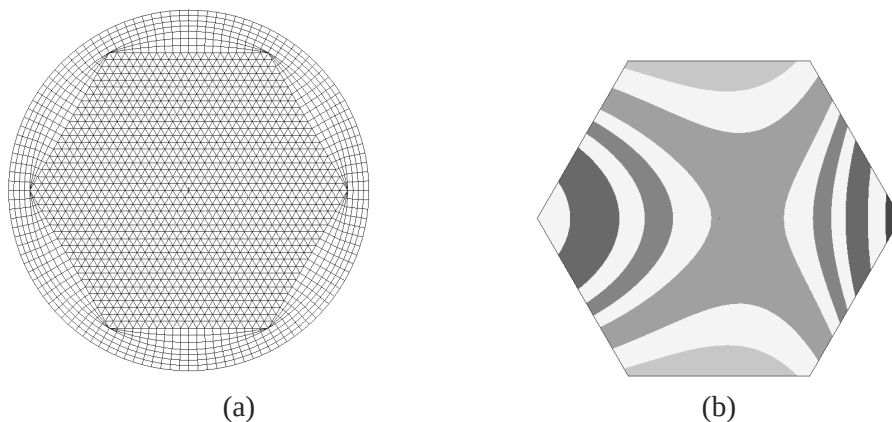


Figure 10.24 (a) Oversized spherical blank and (b) desired departure from the initial sphere after best-fit plane and power are removed in SigFit.

outside segment is shown in Fig. 10.24(b) with the best-fit plane and power removed. This shape can be determined using the off-axis slumping techniques described in Chapter 5. Once the desired figure is achieved on the circular blank, the blank is cut to the hexagonal shape. Undesired figure changes associated with this cutting process may be addressed by other polishing tools to obtain a final figure.

The optimum edge placement of actuators shown in Fig. 10.25(a) is found from the actuator-placement optimization techniques discussed above. The residual surface RMS error in generating the desired shape for stressed-optic polishing is $1.12\text{ }\mu\text{m}$. This residual surface error is shown in Fig. 10.25(b). If a set of 109 actuators are used, as shown in Fig. 10.26(a), the error is reduced to $0.007\text{ }\mu\text{m}$, as shown in Fig. 10.26(b).

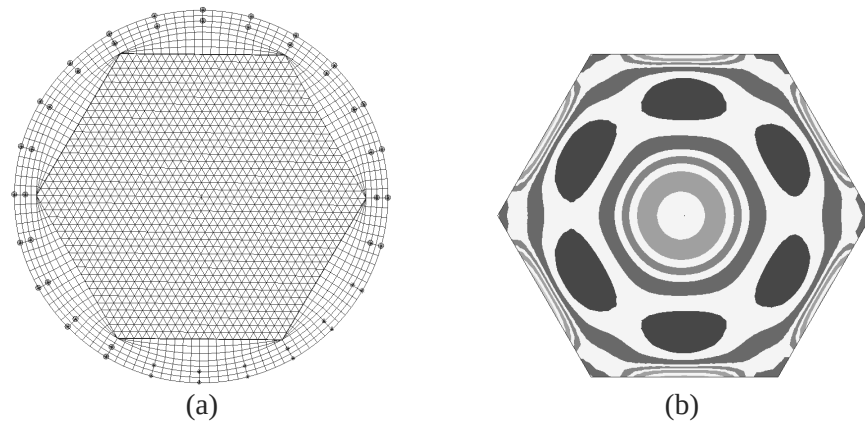


Figure 10.25 (a) Optimum 48-edge actuator arrangement and (b) residual surface error ($1.12 \mu\text{m}$).

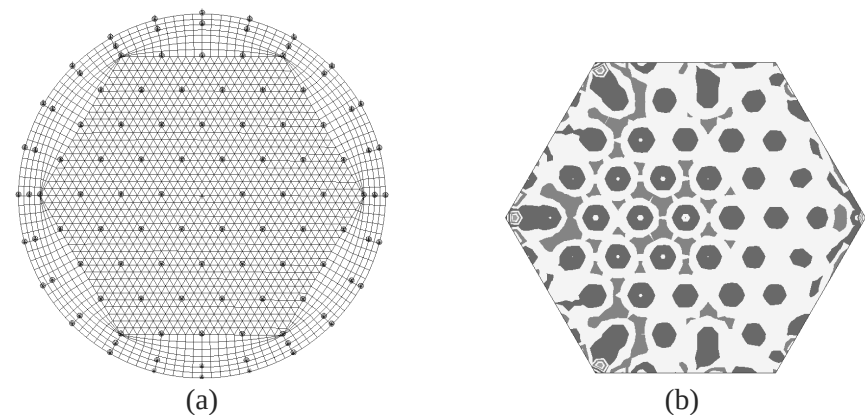


Figure 10.26 (a) 109-actuator arrangement and (b) residual surface error ($0.007 \mu\text{m}$).

It is important that stresses induced by the actuation of the blank be considered. The need to apply such limits on the actuators often is the driving factor in requiring the use of a blank initially figured to the best-fit sphere of the prescription. If the predicted stress levels are still found to be high when applied to an initially spherical blank, then the adaptive control simulation analysis may be used to find the appropriate stroke or force limits on the actuators. Such limits on actuation may require more frequent measurements in the stressed-optic polishing cycle in order to achieve convergence to the desired figure.

10.10 Analogies Solved via Adaptive Tools

Adaptive analysis as described in this chapter is the linear solution of scalar factors on a series of influence functions to obtain a minimum surface RMS error. The disturbances to be corrected and the influence functions used to correct them may come from FEA predictions, test data (such as interferometric arrays), prescribed polynomials, any other source, or any combination of sources. With

these varied definitions of disturbances and influence functions, many nonadaptive problems may be solved using adaptive analysis. Many of these applications are instances of the need to correlate unknown behaviors to measured test data. A requirement, however, when using the adaptive control simulation techniques presented in Eqs. (10.1)–(10.6) is that the relationship between the factors being correlated and the surface deformation of the optical surface must be linear.

10.10.1 Correlation of CTE variation

Adaptive control simulation may be used to find an unknown CTE variation attributed to the fabrication process. If the optic is tested with an isothermal temperature change on a kinematic mount, the results of such optical test data may be used to correlate the variation in CTE through the use of adaptive control simulation. The measurement of the change in surface deformation due to the change in temperature may be represented as a grid array or as Zernike polynomials for use as a disturbance in adaptive control simulation. Influence functions may be synthetically generated as effective temperatures representing basis shapes of spatial CTE variation. The basis shapes of spatial CTE variation may be functions of the lateral and axial directions. The resulting CTE profile to be found may then be related to the combination of such basis shapes. That is,

$$CTE(\rho, \theta, \zeta) = \sum_{i=1}^N A_i p_i(\rho, \theta, \zeta), \quad (10.12)$$

where ρ , θ , and ζ are parametric axes convenient to the shape of the optic, and A_i is the coefficient of the i^{th} polynomial basis shape $p_i(\rho, \theta, \zeta)$. The basis shapes may be any convenient spatial variation that is hypothesized to contribute to the spatial profile of the CTE variation. Influence functions are generated as effective temperatures at each nodal location:

$$T_{eff_i}(\rho, \theta, \zeta) = \frac{p_i(\rho, \theta, \zeta)}{CTE_0} + T_{ref}, \quad (10.13)$$

where $T_{eff_i}(\rho, \theta, \zeta)$ is the effective temperature at (ρ, θ, ζ) for the i^{th} basis function p_i , CTE_0 is the value of CTE used in the finite element model, and T_{ref} is the reference temperature used in the finite element model. The actuator values found by the adaptive control simulation will be the values of the coefficients A_i in Eq. (10.12).

In the process of correlation to measured test data, the analyst must use careful interpretation of the correlated results. If high-order basis shapes are used to correlate to behavior that is best represented by low-order descriptions, then an ill-posed correlation may result. It is advisable to attempt correlation with a small number of low-order basis shapes before adding higher-order basis shapes.

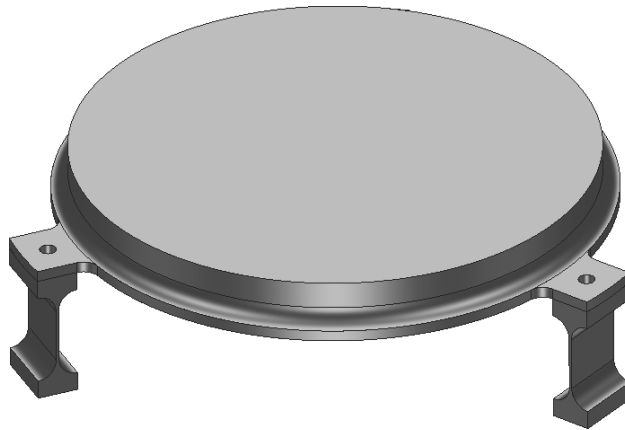


Figure 10.27 Mirror mounted on flexures to be bolted to an interface.

10.10.2 Mount distortion

Adaptive control simulation may be used to find flatness errors over redundant or flexured mounting interfaces. If nonplanarity is present between the three precision-machined interfaces at the base of the flexures of the mirror assembly shown in Fig. 10.27, then the process of bolting the assembly to the interface will induce deformation of the mirror's optical surface. If the optical figure is measured, then it may be used as a disturbance in adaptive control simulation to quantify the flatness errors in the interface. The influence functions are developed through nine load cases, each exercising a single flatness error of the interface at the base of a single flexure. The flatness errors at the base of each flexure can be described by a displacement along the optical axis of the mirror, and one tilt about each of the two lateral axes. Note that the other three degrees of freedom at each flexure base are not associated with flatness errors but may be included as well.

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◀Chapter 11▶

Optimization of Optomechanical Systems

In a conventional design-development process, engineers perform parametric trade studies, iterating through trial designs until a satisfactory or feasible design is found. This is a trial-and-error effort that requires intuition and insight. If there are a significant number of design variables, the process can be prohibitively complex and time consuming, which can exhaust available funding and time. For this reason, this manual trade-study process is incomplete in realizing the full benefit from the design variables available and results in nonoptimal and underperforming designs.

Optimization theory offers a methodology to improve the design process, including design sensitivity and nonlinear programming (NLP) techniques. When incorporated into a general purpose FEA program, optimization methods offer new opportunities for design improvement. Automated design-optimization features in the major FEA tools will sequentially improve a starting design to obtain an optimum design. The optimum design is generally limited by the starting design and the choice of design variables. Because of the sequential nature of NLP, this optimum design may be a local optimum rather than a global optimum. Even with these shortfalls, design optimization is a powerful tool when employed by a knowledgeable user. Table 11.1 lists some of the common advantages and disadvantages of using design optimization methods.

Table 11.1 Advantages and disadvantages of employing design optimization in the design process.

ADVANTAGES:

- ◀1▶ Provides logical, systematic, and complete design approach
- ◀2▶ Facilitates development of complete problem statement with all design requirements
- ◀3▶ Reduces design time; allows higher-level design trades
- ◀4▶ It generally works, since even a local optimum is an improvement

DISADVANTAGES:

- ◀1▶ Requires computer tools, optimizer, and compatible FE program
- ◀2▶ Requires knowledge of the tools and the theory
- ◀3▶ May get trapped in local optima
- ◀4▶ May have difficulty with ill-posed problems

11.1 Optimization Approaches

There are four techniques to employ optimization of optical structures with optical performance constraints:

- Level 1 is characterized by manual iteration to improve the predicted performance of a design. In this approach, a finite element analysis is performed to find the structural deflections. The FE results are processed in a postprocessor to write surface deformations in a format readable by optical analysis software, as described earlier in this text. The optical analysis software is then used to compute optical performance. Intuition and experience are important in this process to recognize how the design should be modified to improve performance.
- Level 2 is characterized by the use of equations of optical performance within the FE model. These equations can be written for optical performance quantities at the single-surface level, such as surface RMS error after bias, tilt, and power have been removed, or at the system level, such as RMS wavefront error or line-of-sight jitter. The internal optimizer in FE software can then optimize the optical design directly without the need for manual iterations.
- Level 3 is characterized by calculation of optical performance through an external subroutine linked to the FE software for use by the FE program's optimizer. This approach may be used to perform optimization using design performance metrics that cannot be computed by the equations used in Level 2. One such example is the design optimization of an adaptively controlled mirror in order to minimize the corrected surface figure.
- Level 4 is characterized by combining the capabilities of CAD, FE, and optical analysis within a single optimization program. This level of implementation allows coupled design variation of the optical prescription and the mechanical design. There has been some notable progress in this approach, but it is not yet commonplace.

This chapter includes a brief overview of optimization theory and its terminology. However, the main emphasis is on the application of optimization tools to optomechanical systems. In the design optimization of a typical optical structure the predicted quantities relating to performance of the system are referred to as design responses. Example design response types are shown in Table 11.2. Generally, only one of these design responses may be specified to be minimized or maximized by the optimizer and is referred to as the objective. All other design responses may have performance limits applied to them consistent with the requirements of the design. The applications of such limits in the optimizer are referred to as design constraints. In order to define how the optimizer is allowed to modify the design, several types of parameters and the

Table 11.2 Typical design response quantities used in the optomechanical design optimization.**TYPICAL DESIGN RESPONSE QUANTITIES:**

- ◁1▷ Structural: System weight, center-of-gravity, mass-moment-of-inertia
- ◁2▷ Structural: Stress, buckling, natural frequency, dynamic response
- ◁3▷ Optical: Image motion, jitter, MTF
- ◁4▷ Optical: Surface RMS error
- ◁5▷ Optical: System wavefront error

Table 11.3 Typical design-optimization problem statement.**DEFINITIONS:** **X** = vector of design variables, such as sizing, shape, material **R** = vector of design responses, typically nonlinear functions of X **F** = objective = a design response to minimize or maximize **g** = design constraint on a response as either an upper or lower bound

$$R \leq R^U \Rightarrow g = (R - R^U) / R^U \leq 0 \quad (11.1)$$

MATHEMATICAL DESIGN PROBLEM STATEMENT:Minimize $F(X)$ subject to $g \leq 0$ and $X^L \leq X \leq X^U$

behavior constraints

side constraints (11.2)

manner with which they relate to the structural design may be specified. These parameters are referred to as design variables. The design variables often have specific allowable limits and are referred to as side constraints. Side constraints differ from design constraints in that side constraints are applied to design variables, whereas design constraints are applied to design responses. Table 11.3 further illustrates the definitions of a design optimization problem and shows a complete design-optimization problem statement.

Current technology allows for structural optimization using optical performance constraints (Section 11.3) or multidisciplinary thermal-structural-optical optimization (Section 11.4). This chapter does not address some other problems that could broadly fall under optomechanical design, such as optical beam path length optimization¹ in which optimization is used to solve a difficult geometry problem.

11.2 Optimization Theory

In this text, optimum design refers to the application of nonlinear programming techniques to find the best solution of the mathematical statement of the design problem.

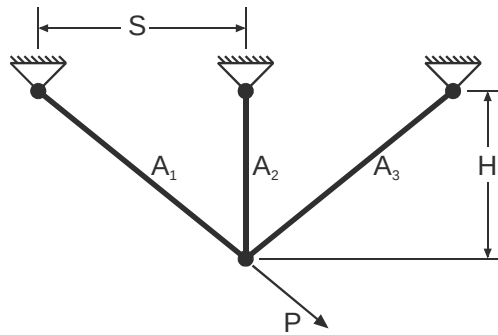


Figure 11.1 Three-bar truss with truss member of areas A_1 , A_2 , and A_3 as labeled.

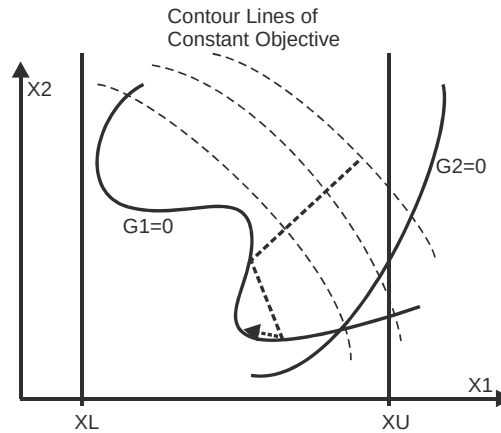


Figure 11.2 Two-variable design space.

If the design goal is to maximize the objective F , the problem can be stated in standard form by minimizing $-F$. If a response is limited by an equality constraint, it may be treated as two inequality constraints:

$$h = 0 \Rightarrow h \leq 0 \text{ and } h \geq 0. \quad (11.3)$$

Fig. 11.1 shows a simple three-bar truss to be optimized with cross-sectional area sizing variables A_1 , A_2 , and A_3 , and shape variables S and H . The objective is to minimize the weight of the structure while satisfying performance constraints on displacement and stress, and obeying side constraints on size and shape.

The design space is an N -dimensional space with an axis for each of the N design variables, which is impossible to visualize if $N > 3$. A two-variable design space is depicted in Fig. 11.2. In most problems, the constraints are generally nonlinear functions of X and are often found numerically, which makes them expensive and difficult to plot, even in a 2D space. In the five-variable truss example, the stress and displacement are found via FEA, and all responses are nonlinear in S and H .

There is a variety of NLP techniques available² that move through the design space in a sequential manner. The most efficient techniques are gradient-based, requiring first derivatives (sensitivities) of the response quantities with respect to the design variables (dR/dX).

A common approach is to use finite differences to calculate sensitivities. Let X_0 represent a starting design point:

$$X_0 = (A_1, \dots, A_j, \dots, A_n), \quad (11.4)$$

which is evaluated via FEA:

$$K_0 U_0 = P_0 \Rightarrow U_0. \quad (11.5)$$

The derivative of displacement with respect to design variable A_j is found by perturbing the design:

$$X_j = (A_1, \dots, A_j + \Delta A_j, \dots, A_n). \quad (11.6)$$

Then, re-evaluating with FEA,

$$K_j U_j = P_j \Rightarrow U_j, \quad (11.7)$$

and computing a finite difference derivative:

$$U_j' = dU / dX_j = (U_j - U_0) / \Delta A_j. \quad (11.8)$$

This is a very general technique, but quite expensive computationally.

A more efficient technique uses implicit derivatives of the initial equation [Eq. (11.5)]:

$$K_0 U' + K' U_0 = P'. \quad (11.9)$$

The derivative U' can be solved from

$$K_0 U' = P' - K' U_0 = P^*, \quad (11.10)$$

which is the equivalent computational cost of an additional load case P^* in the original solution. Note that K' and P' are relatively computationally inexpensive to calculate in most cases. For the example truss problem,

$$k = AE / L \Rightarrow k' = dk / dA = E / L. \quad (11.11)$$

$$P = AL\rho g / 2 \Rightarrow P' = dP / dA = L\rho g / 2. \quad (11.12)$$
$$d\sigma/dX = (d\sigma/dU)U' \Rightarrow d\sigma/dU = E/L. \quad (11.13)$$

- <1> give a design X_q at design cycle q ,
- <2> run a full FE analysis along with design sensitivity,
- <3> create approximate problem (response surface) via Taylor series:

$$g^* = g(X_q) + g'(X_q)/(X - X_q), \quad (11.14)$$

- In this approach (shown in Fig. 11.3), Step 4 requires hundreds of computationally inexpensive optimizations, while the computationally expensive FEA in Step 2 is typically 10–20 analyses.

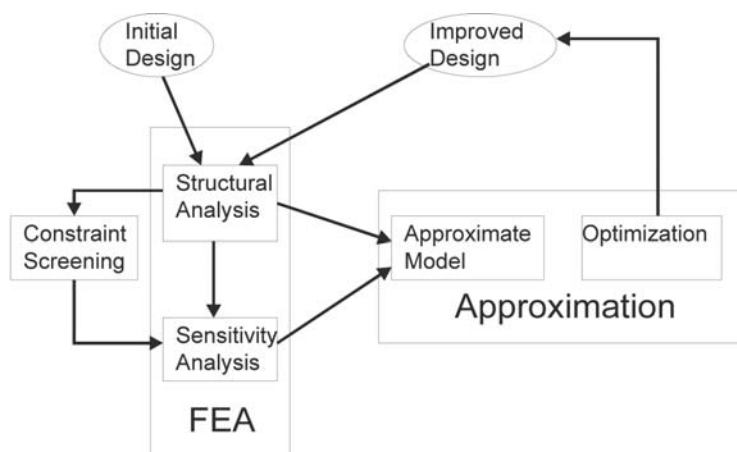


Figure 11.3 Optimization flow using approximation theory.

11.3 Structural Optimization of Optical Performance

11.3.1 Use of design response equations in the FE model

In this section, it is assumed that the optical design is fixed, leaving only the structural variables to design. Commercially available software allows some of the optical response quantities to be incorporated into the structural FEA optimization model.³⁻⁶ The specific capabilities listed in this section are found in MSC/Nastran and Sigmadyne/SigFit⁷ software packages.

The optical performance metrics most easily incorporated into equations are image motion and defocus. In Chapter 7, sample equations of image motion for a simple telescope are presented as multipoint constraint (MPC) equations. For small motions, these are linear equations that can be incorporated into any FEA code that allows linear equation input. In the development of the equations for image motion and defocus, the average surface motion must be calculated for each of the optical surfaces. An interpolation element can approximate the average motion without affecting the stiffness. However, an interpolation element cannot include the effects due to radial deformation of the optical surfaces that can affect the rigid body motions significantly, especially in thermo-elastic cases. A better representation of surface tilts, bias, and even radius of curvature should be calculated and written to the MPC format with the effects of radial correction included. The details of radial correction may be found in Chapter 4.

Wavefront error budgets typically specify a surface RMS or peak-to-valley (P-V) requirement for surface deformation under a variety of test and operational load conditions. These budgets commonly require that the pointing and focus terms are subject to one wavefront error budget and that the residual surface deformation is subject to a separate wavefront error budget. Separation of these quantities in writing the design response equations may be accomplished by writing the Zernike polynomials (or any other polynomial) as MPC equations,^{5,8} subtracting the tilt, bias, and focus terms, and then calculating the residual RMS or P-V using a nonlinear equation feature. The procedure is outlined as follows:

$$\begin{aligned} U_k &= \text{displacement of node } k \text{ from finite element solution,} \\ C_j &= j^{\text{th}} \text{ Zernike coefficient,} \\ F_{jk} &= \text{node } k \text{ displacement due to unit value of } j^{\text{th}} \text{ Zernike;} \\ Z_k &= \sum_j C_j F_{jk} = \text{Zernike representation of node } k \text{ displacement.} \end{aligned} \quad (11.15)$$

When fitting polynomial coefficients C to a deformed shape U , the error E ,

$$E = \sum_k W_k (U_k - Z_k)^2, \quad (11.16)$$

is minimized with respect to the coefficients. That is,

$$dE/dC_j = 0, \quad (11.17)$$

resulting in the linear system of equations

$$[H]\{C\} = \{R\}. \quad (11.18)$$

Solving for C ,

$$\{C\} = [H^{-1}]\{R\} = [A]\{U\}, \quad (11.19)$$

which can be represented as MPC equations with the C as scalar degrees of freedom. In SigFit, the coefficient calculation shown in Eq. (11.19) is modified to include radial correction. Another set of MPC equations subtract the user-selected Zernike terms and place the residual error into a dummy surface mesh:

$$E_k = U_k - \sum_j C_j F_{jk}. \quad (11.20)$$

The residual surface RMS error or P–V is calculated using design-response equation features that allow nonlinear relationships to the nodal displacements. That is,

$$\text{RMS} = \sqrt{\sum_k W_k E_k^2}, \quad (11.21)$$

$$\text{P} - \text{V} = \max(E_k) - \min(E_k). \quad (11.22)$$

Any of the above responses may be treated as design constraints or as the objective in the optimization process. The telescope example in Chapter 13 shows the application of these techniques.

When optimizing lightweight mirrors, use of the 2D and 3D equivalent models discussed in Chapter 5 is especially useful in development of the preliminary design. Important design quantities such as cell size B are easily incorporated as design variables in an equivalent stiffness design through the use of design-variable to property relations. However, such variables are impossible to incorporate as design variables in a full 3D model because they cause topological changes in the geometry. For a mirror with symmetric front- and back-plate thicknesses and a hexagonal core, a typical design flow involves finding B , T_p , T_c , and H_c from design optimization with a 2D or 3D equivalent stiffness model, as shown in Table 11.4. The 1-g or polishing-induced quilting effects can be included in the effective models as design constraints by using the quilting equations presented in Chapter 5. For complex cell geometry involving

Table 11.4 Common design variables for lightweighted mirrors.

<u>DESIGN VARIABLES FOR LIGHTWEIGHT MIRROR</u>
B = cell-size (inscribed circle diameter)
T_p = front and back faceplate thickness
T_c = core wall thickness
H_c = core height
<u>2D EQUIVALENT STIFFNESS (ALL PLATE SIZE VARIABLES)</u>
$H = H_c + 2T_p$ = core height
$\alpha = T_c/B$ = core solidity ratio
$T_m = 2T_p + \alpha H_c$ = membrane thickness (size variable)
$I_b = [H^3 - (1-\alpha)H_c^3] / 12$ = bending inertia
$R_b = 12I_b / T_m^3$ = bending ratio (size variable)
$S = [H^2 - (1-\alpha)H_c^2] / \alpha$ = transverse shear
$R_s = 8I_b / ST_m$ = bending ratio (size variable)
$NSM = \alpha H_c \rho$ = additional nonstructural mass
<u>3D EQUIVALENT STIFFNESS MODEL (SIZE, SHAPE, AND MATERIAL VARIABLES)</u>
$\alpha = T_c/B$ = core solidity ratio
$E^* = \alpha E$ = effective modulus of core (material variable)
$\rho^* = 2\alpha \rho$ = effective density of core (material variable)
profile changes = moves grid position for H (shape variable)
T_p = plate property faceplate thickness (size variable)

cathedral ribs, a separate breakout model of a single cell can be included in the overall optimization model for the purposes of predicting quilting deformation for a cell geometry with no available analytical equation. The results of the 2D or 3D equivalent stiffness optimization can subsequently be used to create a full 3D model, which can then be optimized again to refine values for T_p , T_c , and H_c for additional 3D effects.

11.3.2 Use of external design responses in FEA

For optical performance quantities that are not easily represented as bulk data equations, some FEA tools allow the linking of an external subroutine to compute design responses. This allows an external subroutine to compute a design response quantity in the optimization loop for use in constraint or objective calculations. For example, the external subroutine could be called to calculate the surface RMS error due to random loading or MTF due to vibration, which are quantities impossible to calculate from within any finite element tool. Additionally, optimization can be combined with the simulation of adaptive control to improve the design of adaptive optics.⁹ In this case the FEA software calculates the surface deformations and actuator influence functions for each design cycle. The FEA software calls the external subroutine, as shown in Fig.

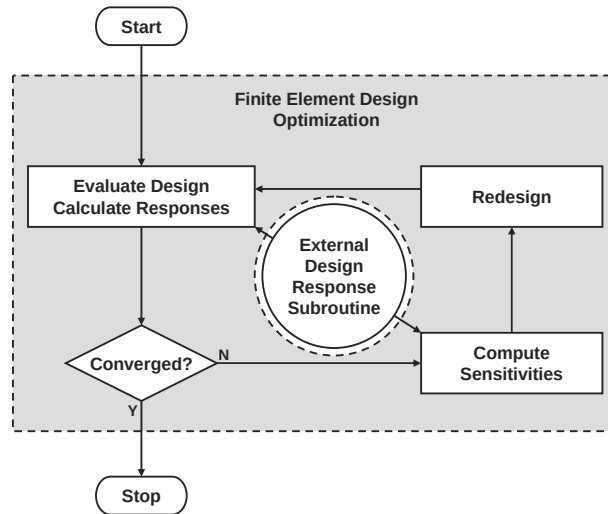


Figure 11.4 Flowchart of design optimization utilizing an external subroutine.

11.4, to calculate the actuator strokes and resulting best-corrected surface RMS error to be used as a response to constrain or minimize. An example of design optimization of an adaptively controlled optic is given in Section 10.3.1.2 of Chapter 10.

11.4 Integrated Thermal-Structural-Optical Optimization

To achieve higher performance than realizable in the optimization techniques described above, an integrated design optimization approach based on multidisciplinary design optimization (MDO) is required.^{10,11} In such an approach design, optimization is performed simultaneously on the structural, optical, and thermal control design. Without the ability to work concurrently, the disciplines of thermal control, structural design, and lens design impose worst-case performance requirements on each other so that each specialty can contribute to a design independently. In a typical design approach, the optical systems engineer creates a performance error budget that dictates deformation limits to the structural engineer, who then dictates limits on temperatures and gradients to the thermal engineer. Requirements are derived and then flowed down. The thermal and structural engineers attempt to achieve these limits under all operational conditions. Such an approach satisfies the optical performance requirements, but at a cost of overdesign due to stacked-up margins. For example, temperature gradients in a mirror-support structure are inconsequential so long as the required optical performance is achieved, yet derived limits on such gradients often become a design driver for thermal-control specialists. To achieve higher performance, an integrated design approach based on MDO is required.

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◀Chapter 12▶

Superelements in Optics

12.1 Overview

Large optical systems such as an orbiting telescope involve several organizations to supply the spacecraft, the metering structure, the primary mirror, the remaining optics, and the science instruments. Each component must be analyzed individually, in various subassemblies, and in a full-assembly analysis of launch and orbiting configurations. In FEA, superelements (SEs) can be used to represent each component and subassembly in an efficient analysis approach. Superelements provide an easy method to swap component models into and out of system-level models to account for local design changes and modeling updates. Superelements also allow organizations to protect proprietary information within a component model.

12.2 Superelement Theory

Superelement is another name applied to substructure analysis. A component FE model with many degrees of freedom (DOF) is partially solved and reduced to a much smaller matrix representation involving boundary (or connection) DOF and some number of internal DOF. Superelements can be treated just like any other finite element with a mass and stiffness matrix. They can be assembled with other SE or standard finite elements to build a system level matrix (called the residual structure), which is then solved. In Fig. 12.1 the SE are arranged in a tree structure. The process order starts at the tips of the tree, working downward to the base of the tree called the residual structure. When the residual structure is solved, the analysis works back up the tree to recover results internal to the SEs.

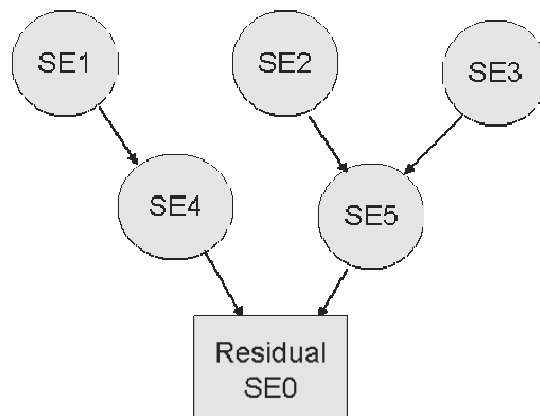


Figure 12.1 Superelement tree.

In a telescope model, SE1 can represent a primary mirror that is then joined to its mounts and support ring to become a primary mirror assembly in SE4. The primary mirror assembly is then merged with the secondary mirror assembly, science instruments, and metering structure to become a full telescope model (residual structure or SE0). Each tip SE can be created, verified, and run as a separate component model. For example, the primary mirror model (SE1) can be used to analyze the mirror during polishing and testing. The primary mirror assembly (SE4) can be used for analysis support of assembly testing. In this manner, the SE approach mimics the actual buildup of the hardware allowing analysis of each assembly level.

12.2.1 Static analysis

In static analysis, an SE is just a partial solution of the equilibrium equation. The SE operation is exact, with no approximations. The following set notation is used in this chapter:

G = all DOF in model
 M = dependent DOF from rigid bodies and equation input (MPC)
 N = independent DOF = $G - M$
 S = specified DOF from boundary conditions (SPC)
 F = free DOF = $N - S$
 O = omitted DOF or slave DOF reduced out by substructuring
 A = analysis DOF = $F - O$

The full static equilibrium equation after dependent DOF (M) and specified DOF (S) have been reduced out is

$$[K_{FF}]\{U_F\} = \{P_F\}. \quad (12.1)$$

If the free DOF (F) are partitioned into the omitted DOF (O) and the analysis DOF (A), then Eq. (12.1) becomes

$$\begin{bmatrix} K_{OO} & K_{OA} \\ K_{AO} & K_{AA} \end{bmatrix} \begin{Bmatrix} U_O \\ U_A \end{Bmatrix} = \begin{Bmatrix} P_O \\ P_A \end{Bmatrix}. \quad (12.2)$$

If the upper equation is used to solve for U_O , then

$$K_{OO}U_O + K_{OA}U_A = P_O \Rightarrow U_O = K_{OO}^{-1}P_O - K_{OO}^{-1}K_{OA}U_A. \quad (12.3)$$

Substituting into the lower equation and regrouping terms,

$$\begin{aligned} (K_{AA} - K_{AO}K_{OO}^{-1}K_{OA})U_A &= P_A - K_{AO}K_{OO}^{-1}P_O, \\ \bar{K}_{AA}U_A &= \bar{P}_A. \end{aligned} \quad (12.4)$$

The overbar represents the SE reduced to the analysis DOF (A). The analysis DOF include the boundary nodes that connect to other structures and any other internal DOF of special interest.

12.2.2 Dynamic analysis

In dynamic analysis, the mass matrix must be reduced along with the stiffness matrix. There are two common approaches to reduce the mass matrix given in the next two sections.

12.2.2.1 Guyan reduction

If the same static reduction that was applied to the stiffness matrix in the above section is applied to the mass matrix, the result is called Guyan reduction or static condensation. If P_O is ignored, then

$$\begin{aligned} \bar{U}_O &= -K_{OO}^{-1}K_{OA}U_A = G_{OA}U_A, \\ G_{OA} &= -K_{OO}^{-1}K_{OA}, \\ \bar{K}_{AA} &= K_{AA} + K_{OA}^T G_{OA}, \\ \bar{M}_{AA} &= M_{AA} + M_{OA}^T G_{OA} + G_{OA}^T M_{OA} + G_{OA}^T M_{OO} G_{OA}. \end{aligned} \quad (12.5)$$

This is usually a poor approximation because inertial loads on the omitted DOF are ignored. To reduce the error in this approximation, the analysis DOF (A) must include all large masses, rotational inertias, and a sprinkling of DOF throughout the “interior” of the structure. This early-reduction technique has been replaced by component mode synthesis.

12.2.2.2 Component mode synthesis

Component mode synthesis (CMS) is sometimes referred to as Craig–Bampton modes. In this approach, a selected set of internal modes of the SE (component) are calculated with the boundary nodes fixed using traditional eigenvalue techniques:

$$\begin{aligned} \Phi_K &= \text{Eigenvector}(\text{ModeShape}), \\ z_K &= \text{Modalmultiplier}. \end{aligned} \quad (12.6)$$

The constraint modes are calculated for each boundary DOF. A constraint mode is the static solution of imposing a unit displacement on single-constraint DOF while all others are held fixed:

$$\begin{bmatrix} K_{OO} & K_{OA} \\ K_{AO} & K_{AA} \end{bmatrix} \begin{bmatrix} \Psi_{OA} \\ I_{AA} \end{bmatrix} = \begin{bmatrix} 0 \\ R_{AA} \end{bmatrix}. \quad (12.7)$$

In the above equation, I_{AA} is an identity matrix of imposed unit displacements, and R_{AA} is the resulting reactions. From the top row of the partition,

$$\Psi_{OA} = -K_{OO}^{-1} K_{OA}. \quad (12.8)$$

The constraint matrix is

$$\Psi_c = \begin{bmatrix} \Psi_{OA} \\ I_{AA} \end{bmatrix} = \begin{bmatrix} -K_{OO}^{-1} K_{OA} \\ I_{AA} \end{bmatrix}. \quad (12.9)$$

The full response of the SE is the sum of the internal modes and the constraint modes:

$$U = \Phi_k z_k + \Psi_c z_c. \quad (12.10)$$

When combined with other components, equilibrium and compatibility are enforced at the boundary DOF:

$$\begin{aligned} F_A^{SE1} + F_A^{SE2} &= 0, \\ U_A^{SE1} &= U_A^{SE2}. \end{aligned} \quad (12.11)$$

In the above discussion, boundary nodes were assumed as “fixed boundary,” as in classical Craig–Bampton modes. However, the boundary modes could be “free boundary” as well. For a more complete discussion, see Reference 1.

CMS can be made as accurate as desired by including more internal modes in the analysis.

12.2.3 Types of superelements

There are two general types of superelements based on the amount of data stored within the SE: conventional and external.

12.2.3.1 Conventional superelement

In a conventional superelement, all finite element data is stored along with the reduced matrices. To combine with other SEs, the analyst must resolve issues of duplicate node and element numbers, just as if the model were run without SEs. After a system model is run at the residual level, the results can be passed back up the SE tree to determine any results internal to the SE. This is the most complete approach, requiring the most knowledge of each SE. Because the full

model of all components is present, it is difficult to protect proprietary data in any component of the SE model. Conventional SEs require significant database management.

12.2.3.2 External superelement

In an external element, only the reduced matrices are passed down the SE tree. Lower-level SEs do not see the internal nodes and elements of higher-level SEs. This allows each SE to have a numbering scheme independent of all others. The analyst is not required to resolve any duplicate numbering issues. This approach also protects any proprietary information contained in the SE model. If results internal to an SE are required on the data recovery cycle, there is an option to pass an output transformation matrix (OTM) down the tree along with the reduced matrices. The OTM can provide stress or force in key members or displacements of key nodes internal to the SE. External SE requires minimal database management.

12.3 Application to Optical Structures

Superelements are most efficient when the number of boundary nodes is small compared to the number of internal nodes. In optical structures, components are commonly joined together with kinematic mounts to minimize interaction.

12.3.1 Kinematic mounts

Kinematic (or statically determinate) mounts are described in Chapter 6. The big advantage of kinematic mounts is that internal distortions in one component cause no internal distortion of the neighboring component. For example, a primary mirror on a kinematic mount made of a different material will move stress- and distortion-free under an isothermal temperature change. A kinematic mount can have only six attachment DOFs. The low number of attachment DOFs makes each kinematically mounted structure an excellent candidate for being treated as SEs in a system-level model.

12.3.2 Segmented mirrors

Many modern telescopes have large segmented primary mirrors. Each segment is usually an identical copy of all other segments. The only variation is in the final figuring of an aspheric surface, which varies with radius from the assembly optical axis. In Fig. 12.2, a parabolic primary mirror has 6 segments of shape A, 6 of shape B, and 6 of shape C.

Mechanically each segment is nearly identical. Because the aspheric departure is usually quite small, a single-segment model would be sufficient for dynamic analysis. If thermo-elastic effects including the aspheric geometry are desired, then only three unique segments are required.

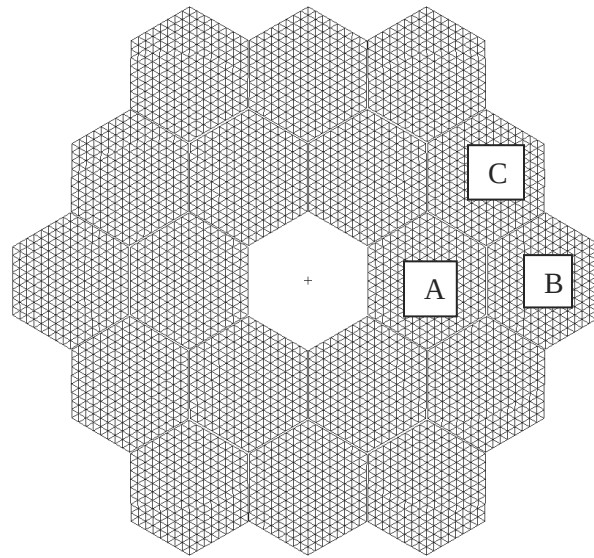


Figure 12.2 Segmented mirror.

Without SE, the analyst can create a single model of segment A and then create 17 copies to produce a model of the full mirror. Each copy must have nonconflicting node and element numbers. In the external SE approach, the analyst can reduce the single-segment A model to its reduced matrices. To get a full model, the reduced matrices of segment A are placed in the 18 segment locations, greatly reducing the computer resources required.^{2,3}

12.4 Advantages of Superelements

There are several advantages to using superelements:

1. More efficient re-analysis. A one-pass analysis with SE may take longer than without SE. However, each subsequent re-design and re-analysis can be much more efficient because only the changed SE and those below it in the tree need re-analysis.
2. Individual-component SE models can be replaced in a systems model as component designs are updated.
3. External SE allows components to be represented only by their boundary matrices. This can protect proprietary information in a component model from being passed among associate contractors.

12.5 Telescope Example

The telescope example described in Chapter 13 is an ideal candidate for superelements. Each component has a small number of interface nodes that connect to neighboring components.

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◀Chapter 13▶

Integrated Optomechanical Analysis of a Telescope

13.1 Overview

A simple two-mirror telescope will be used to demonstrate integrated analysis techniques common in optomechanical systems. Although this design was created for analysis demonstration only, the performance requirements placed on this system are representative of real applications. The level of detail in these models is quite coarse yet consistent with a conceptual-design study model; it also keeps the model files small and readable. The models are available for download from www.sigmadyne.com. The models are in MSC/Nastran and Zemax format because both programs are commonly used to model telescopes. Most FE preprocessors can read Nastran data files and convert to other FE codes. The Readme.txt file explains the filenames used for each analysis described below.

IN THIS CHAPTER, THE FOLLOWING NOTATION WILL BE USED:

PM = primary mirror (just the optic)
PMA = primary mirror assembly (optic plus mount)
SM = secondary mirror (just the optic)
SMA = secondary mirror assembly (optic plus mount)
FP = focal plane = detector
LOS = line-of-sight error = image motion

The flow of this chapter represents the flow of some of analyses required to support the design of a telescope.

- Section 13.2: The optical model is usually developed first to determine optical performance of the nominal design.
- Section 13.3: The structural model is developed to determine on-orbit performance. The model is broken into numbering ranges and files so that multiple engineers can design individual components.
- Section 13.4: Because the PM is the long-lead item, it must be designed first. A 3D equivalent model is used during design optimization.
- Section 13.5: Once a concept model of the complete telescope is developed, line-of-sight equations are determined.
- Section 13.6: Using the LOS equations, an on-orbit jitter analysis is conducted to see if the concept will meet performance requirements.
- Section 13.7: The surface RMS under random loads is considered.

- Section 13.8: Once the design meets jitter requirements, a detailed design of the PM is conducted with a full shell model. This model can be dropped right into the modular model, replacing the equivalent stiffness model.
- Section 13.9: During detailed studies of the PM, the question of bond design and material must be studied. The tradeoff of soft RTV verses stiffer epoxy is analyzed in this section.
- Section 13.10: The telescope assembly must be analyzed in various 1-g test configurations. Results are presented as Zernike polynomials. When polynomials do not represent the surface due to high-order quilting, then grid arrays can represent the data for further optical analysis. There is often a requirement to determine the optical performance over a subaperture (or cookie) for off-axis field points.
- Section 13.11: Isothermal temperature conditions are always required to be analyzed. After radial correction, these distortions are well represented by Zernike polynomials.
- Section 13.12: Polynomial coefficients can be determined by writing MPC equations in the model file, as shown in this section. However, the residual RMS cannot be represented as linear MPC equations.
- Section 13.13: All telescopes require assembly conducted in a 1-g environment. Depending on the assembly process, it is possible to create locked-in strain, resulting in distortions at zero gravity.

13.2 Optical Model Description

The primary mirror (PM) and secondary mirror (SM) are both centered on the z axis, as is the detector, with the global origin at the PM vertex. The local coordinate systems for the PM, SM, and FP are right-handed and identical except for their z positions (see Fig. 13.1). The optical prescription is given in Table 13.1.

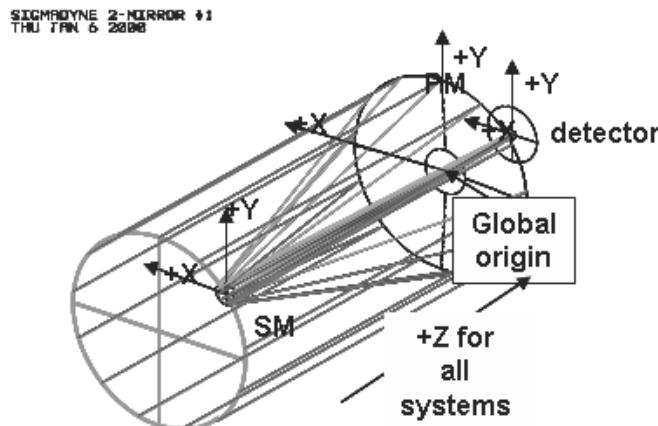
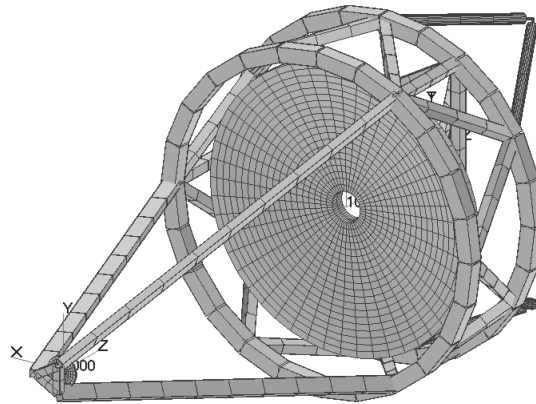


Figure 13.1 Telescope optical layout.

Table 13.1 Optical prescription.

	<u>RADIUS</u>	<u>CONIC CONSTANT</u>	<u>AXIAL (z) POSITION</u>
PM	$R_1 = -101.431649''$	-1.002273	0
SM	$R_2 = -12.382587''$	-1.499313	-45.1200''
FP	flat	NA	+12.95392''

**Figure 13.2** Structural model.

Note that the z axis of the PM points into the mirror, and thus the radius of curvature is negative, and the center of curvature is on the negative z axis.

13.3 Structural Model Description

A cutaway plot of the finite element structural model is shown in Fig. 13.2. In this model, the metering structure is graphite/epoxy (GREP) composite. The lightweight PM is fused silica and represented as a 3D equivalent stiffness model on three bipod flexures of titanium. A full 3D shell model is used for some of the later analyses. The PM mount pads (Fig. 13.3) are represented as a titanium plate with the RTV adhesive bond as a 3D solid with effective properties. The SM (Fig. 13.4) is solid fused silica and held by three titanium blade flexures, represented as beam elements, and the RTV bond is represented as solid elements with effective properties. The focal plane is a single node connected to the GREP aft metering structure (Fig. 13.5) by beam elements. The structure has three planes of symmetry in stiffness, but there are two star trackers mounted to the metering structure at $+60$ and -60 deg from the x axis, leaving only the xz plane as a symmetry plane. The mass asymmetry will cause the modes for x bending to differ slightly from y bending.

Coordinate systems in the structural model match the coordinate systems in the optical file. The MSC/Nastran model of the telescope is subdivided into components for easy manipulation. The data files are heavily commented so that a reader can follow the input and modify as desired.

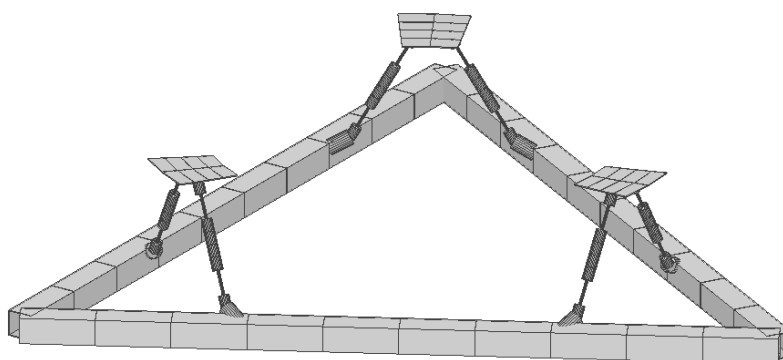


Figure 13.3 PM mount structure.

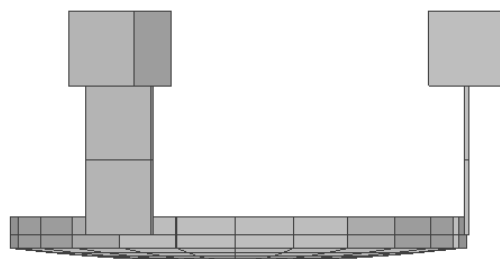


Figure 13.4 SM mirror and flexures.

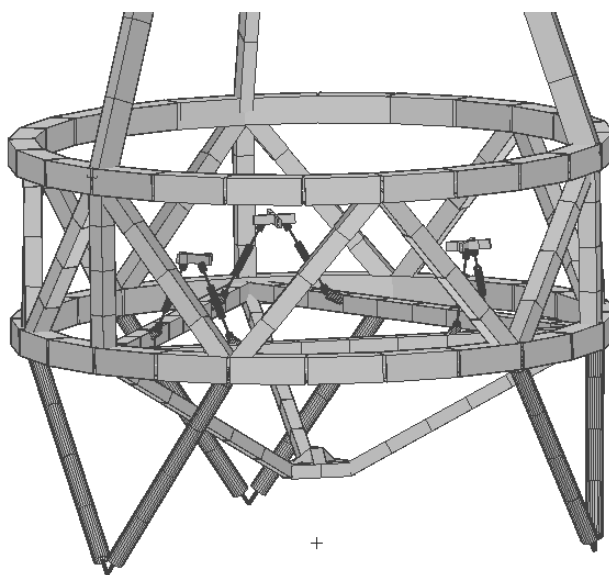


Figure 13.5 Metering structure.

13.4 Optimizing the PM with Optical Metrics

To reduce the weight of the primary mirror, an optimization using optical measures¹ was performed using the techniques in Chapter 11. This optimization was conducted early in the design phase using a 3D equivalent stiffness model. By using an equivalent model, cell size B can be treated as a design variable (Table 13.2) without creating a new model for each cell size. The mirror and flexures are closely coupled in behavior, so the flexure neckdown diameter is included in the optimization statement. The lighter and more flexible the PM becomes, the less bending moment the flexures are allowed to transmit.

The Nastran control file contains load cases for launch loads with stress constraints, 1-g test requirements, and a subcase with natural frequency constraints. Polishing quilting and 1-g quilting requirements are included as equations. The 1-g test requirement is that the surface RMS be less than a specified value after pointing and focus correction (i.e., after bias, tilt, and power are removed). The equations for calculating the surface RMS after correction were written in Nastran format (MPC, DRESP2, DEQATN) by SigFit.² Table 13.3 lists the initial and final design values for variables and responses.

Once a suitable cell size B is chosen, a full 3D model can be created for further optimization with more detailed design criterion. With actual cell

Table 13.2 Design variables.

<u>DESIGN VARIABLES FOR LIGHTWEIGHT MIRROR = B, T_p, T_c, H_c</u>	
B	= cell size (inscribed circle diameter)
T_p	= front- and back-faceplate thickness
T_c	= core-wall thickness
H_c	= core height
D	= diameter of flexure neckdown

Table 13.3 PMA optimization results.

<u>DESIGN VARIABLES</u>	<u>START</u>	<u>END</u>	
Faceplate thickness	0.07	0.05	
Core thickness	0.06	0.04	
Core height	1.45	1.45	
Cell size	3.00	2.60	
Strut diameter	0.12	0.10	
<u>RESPONSES</u>	<u>START</u>	<u>END</u>	
Weight	15.1	12.5	Minimize
Surface RMS	0.2234	0.225	Limit < 0.225
Quilting	0.0202	0.0224	Limit < 0.0224
Natural frequency	79.5	88.2	Limit > 88
Strut stress	48200.00	48700.00	Limit < 50,000

geometry modeled, quilting is predicted directly by the model, eliminating the need for quilting equations. This full shell model could be further optimized to fine-tune faceplate thickness, core thickness, and core height.

13.5 Line-of-Sight Calculations

After the primary mirror assembly has been optimized, meaningful dynamic analyses can be run since the PM weight, stiffness, and flexure stiffness are known. The most important response for on-orbit performance is jitter. To calculate jitter, the line-of-sight (LOS) equations must be obtained.

The LOS sensitivity matrix in image space can be obtained from the optics model by perturbing each optic a small amount in each coordinate direction and about each coordinate axis. The amount that the image moves is then divided by the perturbed input to get a sensitivity term. This operation must be done for each surface to obtain a full system-level LOS matrix. Care must be taken to account for surface numbering, model units, angle units, coordinate system orientations, and left-handed rotations. Because this is an error-prone operation, the resulting LOS equations should be verified with a rigid-body error check as in Chapter 7.

SigFit has an automated LOS calculation that makes LOS calculations more efficient and less error prone. As in an optics program, the raytracing is used to find the LOS coefficients. SigFit has the option to create and write LOS equations in the FE model format as linear equations. This approach eliminates the need to convert units, surface numbering, and left-handed rotations, with the added benefit that the equations use the existing FE-model node numbers. If the equations are added to the FE model, LOS becomes a standard FEA output. A second option in SigFit is to calculate the LOS during a SigFit run of fitting static displacements, running adaptive analysis, or calculating dynamic response.

The LOS sensitivity matrix for the example telescope is given in Table 13.4. As a data check, the rigid-body error check is given in Table 13.5.

In the rigid-body error check, the optical system is moved in six DOF. For unit translations in x , y , and z , the net LOS motion is essentially zero. For unit rotations about x and y , the net LOS is the focal length in image space. Thus the LOS equations pass this check.

13.6 On-Orbit Image Motion Random Response

With LOS equations, the on-orbit jitter can be calculated. For dynamic excitation, a “base shake” input PSD (Fig. 13.6) was applied through the main telescope mounts to represent on-board disturbances. A random response analysis was run in Nastran to calculate the 1σ response of the LOS as 1.629×10^{-4} in. The Nastran model could not predict the MTF loss due to jitter, and could not break the LOS effects into drift and jitter components.

Table 13.4 LOS sensitivity matrix.

Surface	Surface Motion	LOS - X	LOS - Y
Primary Mirror	Translation X	10.3797	0.0000
Primary Mirror	Translation Y	0.0000	10.3797
Primary Mirror	Translation Z	0.0000	0.0000
Primary Mirror	Rotation X	0.0000	1052.8330
Primary Mirror	Rotation Y	-1052.8330	0.0000
Primary Mirror	Rotation Z	0.0000	0.0000
Secondary Mirror	Translation X	-9.3799	0.0000
Secondary Mirror	Translation Y	0.0000	-9.3799
Secondary Mirror	Translation Z	0.0000	0.0000
Secondary Mirror	Rotation X	0.0000	-116.1478
Secondary Mirror	Rotation Y	116.1478	0.0000
Secondary Mirror	Rotation Z	0.0000	0.0000
Focal Plane	Translation X	-1.0000	0.0000
Focal Plane	Translation Y	0.0000	-1.0000
Focal Plane	Translation Z	0.0000	0.0000
Focal Plane	Rotation X	0.0000	0.0000
Focal Plane	Rotation Y	0.0000	0.0000
Focal Plane	Rotation Z	0.0000	0.0000

Table 13.5 Rigid-body error check.

Rigid Body Motion	LOS - X	LOS - Y
Translation X	0.000	0.000
Translation Y	0.000	0.000
Translation Z	0.000	0.000
Rotation X	0.000	526.417
Rotation Y	-526.417	0.000
Rotation Z	0.000	0.000

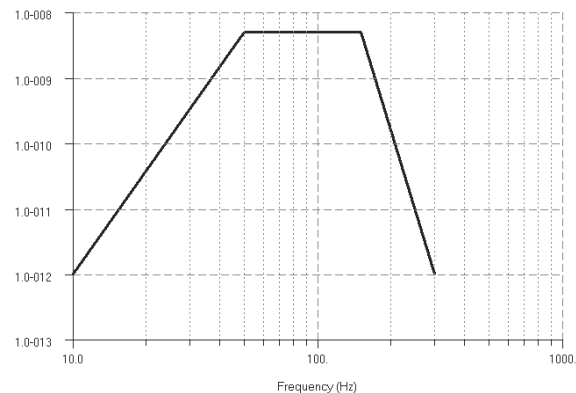
**Figure 13.6** Base-shake input PSD curve.

Table 13.6 LOS response.

```

For integration time = 0.0010
LoS drift  = 1.6149E-04
LoS jitter = 2.1504E-05
Strehl ratio factor = 9.9416E-01
-----
For integration time = 0.0100
LoS drift  = 6.2407E-05
LoS jitter = 1.5049E-04
Strehl ratio factor = 7.9102E-01
-----
For integration time = 0.0000
LoS drift  = 0.000
LoS jitter = 1.6292E-04
Strehl ratio factor = 7.6627E-01

```

Table 13.7 Modal contributors to jitter.

Each mode's % contribution to LoS jitter		
Mode	Freq	LI-TV
4	66.15	0.000
5	66.71	61.798
6	76.11	0.000
7	76.12	24.966
8	120.92	0.000
9	121.39	13.021
10	122.76	0.000
11	156.22	0.000
12	164.24	0.000
13	164.80	0.004
14	168.03	0.000

To obtain more information about the dynamic response, a SigFit random response analysis was conducted using Nastran calculated natural frequencies and mode shapes. The random response analysis directly output the LOS response drift and jitter response in Table 13.6 and identifies the key modal contributors³ to that response in Table 13.7.

In this example, sensor integration times of 0.01 sec and 0.001 sec were used to break the LOS error into slowly varying “drift” and rapidly moving “jitter” terms.⁴ If the integration time is set to zero, then any response is considered jitter.

This zero-integration-time result agrees exactly with the Nastran-calculated LOS motion. In many optical systems, the jitter terms are more important than the drift term. A useful way to study the results is to look at the MTF effect of the jitter. In Fig. 13.7, the nominal MTF of the telescope is plotted along with the

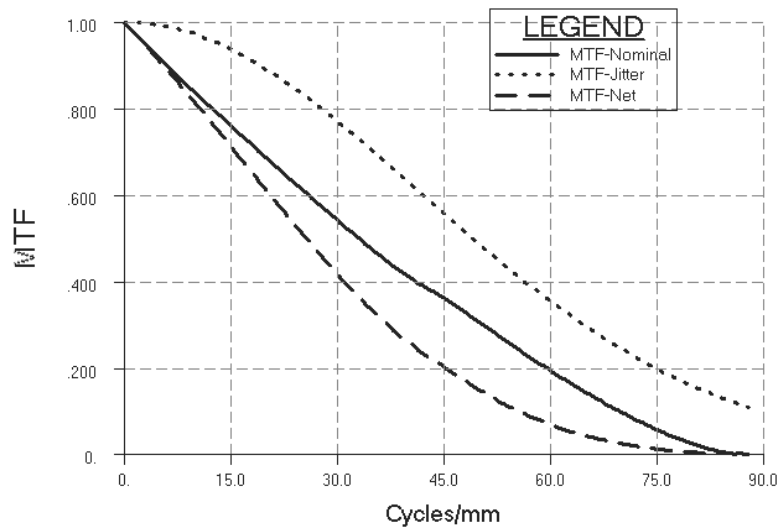


Figure 13.7 MTF due to jitter calculated in SigFit.

jitter MTF for a sensor integration time of 0.01 sec. The product of those two curves is the net MTF. To obtain a single number as a design measure, the Strehl ratio factor (SRF) is obtained by dividing the area under the net MTF curve by the area under the nominal MTF curve. This factor can be used to multiply the Strehl ratio of the nominal (unperturbed) telescope.

At this point in the design cycle, the engineer must decide if the predicted on-orbit jitter response is acceptable. To decrease the effect of jitter, look at the modal contribution to jitter response in Table 13.7. Modes 5, 7, and 9 are the major contributors to jitter PSD. If the strain energy density is plotted in those two modes, the biggest strain energy is in the PM flexures and the main spacecraft flexures. If the model is rerun with both sets of flexures doubled in diameter, the SRF for the doubled design was 0.83 (verses 0.79 for the original design at 0.01 sec integration time). The penalty for increasing the PM flexures is a 10% increase in mirror surface RMS for thermal loads. These results must be compared to the performance requirements to determine the proper design improvements.

13.7 On-Orbit Surface Distortion in Random Response

The line-of-sight calculations in the previous section account for the rigid-body motions of the optical surfaces. Elastic surface distortion caused by random loads also degrades the image quality. When the random response is conducted in a finite element program, the resulting surface motion is represented as an envelope of response. In Fig. 13.8, the response envelope from an FEA is shown for three nodes. That response could be all rigid body as in Fig. 13.9(a) or all elastic as in Fig. 13.9(b). Since all phasing is lost, there is no way to tell the

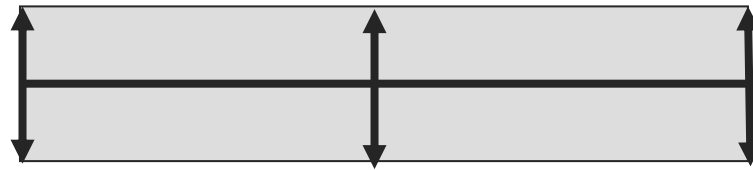


Figure 13.8 Envelope of random response of three surface nodes.



Figure 13.9 Two possible forms of response within the envelope: (a) all-rigid-body motion and (b) all-elastic motion.

Table 13.8 Random response PSD of PM.

Surface	Surface Motion	Units	1-sigma
Primary Mirror	Translation X	inch	1.021E-05
Primary Mirror	Translation Y	inch	1.969E-13
Primary Mirror	Translation Z	inch	3.818E-08
Primary Mirror	Rotation X	radians	3.485E-15
Primary Mirror	Rotation Y	radians	8.563E-08
Primary Mirror	Rotation Z	radians	3.122E-15
Primary Mirror	Surface RMS	waves	6.048E-04

difference. If the mode shapes are decomposed into rigid-body and elastic behavior before the random analysis, then the results can be presented separately.

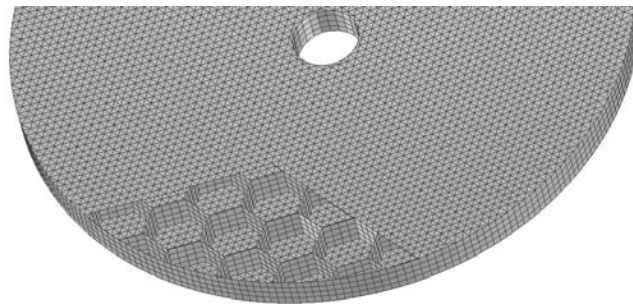
The PM average rigid body motions and surface RMS with rigid body motion subtracted are presented in Table 13.8 with their key modal contributors in Table 13.9. The response tables for the PM show that the 1σ surface RMS error for random base shake in x direction is 0.0006 waves. The 3σ response of 0.0018 waves is 3 times the 1σ response. Since the surface RMS is after rigid-body motion has been removed, it represents the elastic distortion of the mirror and directly affects the image quality. The rigid-body motion of the optic contributes to the LOS error. These key modes could now be investigated by plotting strain energy density to see if design improvements could reduce the surface distortions.

13.8 Detailed Primary Mirror Model

After the early design trades found an acceptable PM design, the effort is made to construct an accurate 3D shell model of the mirror. The design variables determined in the optimization were used. The model is shown in Fig. 13.10

Table 13.9 Modal contributors to PM PSD.

Mode	Freq	RB-Tx	RB-Ty	RB-Tz	RB-Rx	RB-Ry	S-RMS
4	66.15	0.00	0.13	0.00	0.00	0.00	0.00
5	66.71	95.78	0.15	17.23	0.00	0.02	70.94
6	76.11	0.00	50.41	0.00	49.98	0.00	0.00
7	76.12	3.32	49.31	0.62	50.02	88.82	13.49
8	120.92	0.00	0.01	0.00	0.00	0.00	0.00
9	121.39	0.90	0.00	3.05	0.00	10.81	7.64
10	122.76	0.00	0.00	0.00	0.00	0.00	0.00
11	156.22	0.00	0.00	0.00	0.00	0.00	0.00
12	164.24	0.00	0.00	0.00	0.00	0.00	0.00
13	164.8	0.00	0.00	46.31	0.00	0.18	1.71
14	168.03	0.00	0.00	22.19	0.00	0.00	0.09

**Figure 13.10** PM shell model.

with the front faceplate partially erased. Again, this model is still quite coarse to keep file size to a minimum. A full verification model would contain much more detail. The shell model and its mount pads can drop right into the existing metering structure for continued analyses. Because the full model was organized with separate component files and numbering ranges, each component may be replaced with a revised model as the design progresses.

Developing a lightweight shell model with curved geometry can be quite time consuming. An efficient modeling technique is to create a 1/6 model then reflect and rotate to create a full model. The extra modeling time to fit partial cells at the boundary is reduced significantly. The model was created flat for ease of modeling. The curvature was added in two slumping steps as described in Chapter 5. In the first step, the full mirror substrate was slumped to a sphere, similar to a physical slumping or molding process. In the second step, the optical face was slumped to an asphere, similar to a polishing operation. This approach produces a highly accurate aspheric model, which is necessary for proper thermoelastic response. For this example, slumping was performed in SigFit which provides contours of the sag added to the surface in Fig. 13.12.

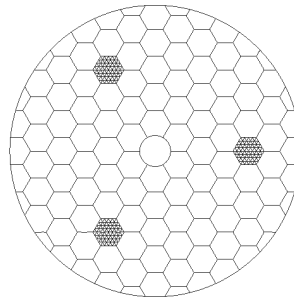


Figure 13.11 PM core and mount pads.

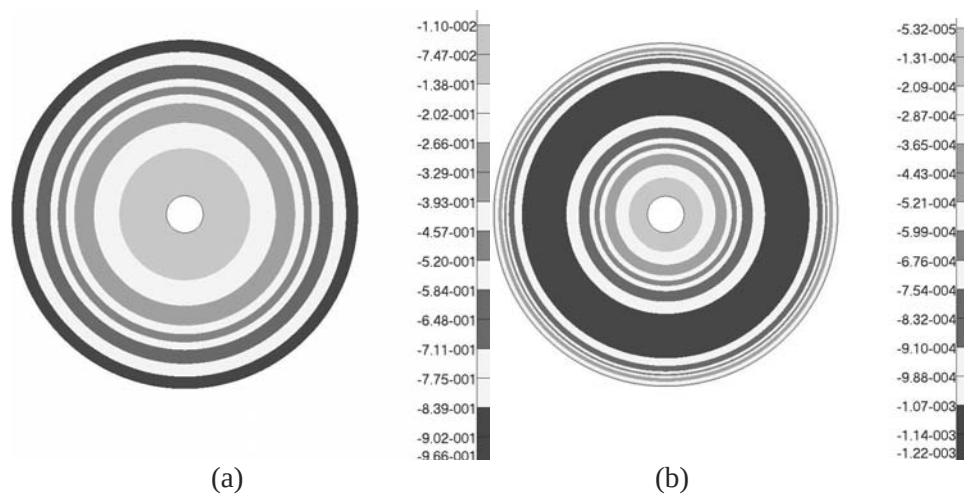


Figure 13.12 (a) Sag added to create spherical geometry; (b) sag added to sphere to get aspheric geometry.

Table 13.10 Comparison of primary mirror models.

	Equiv	Shell	
	Model	Model	Difference
Wt (Lb)	7.43	7.55	2%
Mode 1 (Hz)	597	585	-2%
Mode 2 (Hz)	597	585	-2%
Mode 3 (Hz)	947	916	-3%

As a sanity check, the new shell model was compared to the previous 3D equivalent stiffness model for mass and natural frequencies. The comparison in Table 13.10 shows good agreement, so any design decisions based on the early model are still valid. The shell model is slightly heavier because the core cell over the mount pad was made thicker. The extra mass and extra flexibility of the more-detailed model causes the modes to be slightly lower, as expected.

Table 13.11 Material properties.

	RTV	Epoxy
Modulus (psi)	500	300,000
Poisson ratio	0.499	0.400
Bond thickness (in)	0.040	0.010
CTE (PPM/C)	240	100
Cure Shrinkage (%)	0.33%	0.12%

Table 13.12 Comparison of RTV and epoxy bonds on surface RMS after rigid-body removed.

	RTV	Epoxy	Change
Case	RMS (λ)	RMS (λ)	(%)
1g X	0.1314	0.1293	-1.6%
1g Z	0.2575	0.2469	-4.1%
$\Delta T +10C$	0.0339	2.5142	7316.5%
Cure	0.0479	0.0531	10.9%

Table 13.13 Comparison of RTV and epoxy bonds on frequencies.

	RTV	Epoxy	Change	Mode shape
	Freq (Hz)	Freq (Hz)	(%)	
Mode 1	87.4	95.4	9.2%	PM Lateral
Mode 2	87.4	95.4	9.2%	PM Lateral
Mode 3	161.8	162.3	0.3%	PM bounce Z
Mode 4	164.7	202.3	22.8%	PM torsion
Mode 5	253.9	262.8	3.5%	PM Rocking
Mode 6	253.9	262.9	3.5%	PM Rocking
Mode 7	448.3	449.0	0.2%	Frame bending
Mode 8	448.3	449.0	0.2%	Frame bending
Mode 9	554.9	603.5	8.7%	Mirror bending
Mode 10	555.0	603.5	8.7%	Mirror bending
Mode 11	651.9	656.6	0.7%	Frame bending
Mode 12	672.1	716.5	6.6%	Frame & mirror bending

13.9 RTV vs Epoxy Bond

The choice of bond material for the PM pads must be decided based on the performance requirements. Two common bond materials were compared, RTV and epoxy, with properties given in Table 13.11. Due to the high Poisson ratio of the RTV, equivalent properties were used, as discussed in an earlier chapter on modeling bonds. To study various effects, the PMA (primary mirror, bond, mount pad, flexures, and delta frame) with kinematic supports was used.

From the results in Table 13.12, the big change is in thermal load case. The epoxy bond has a higher CTE and is much stiffer in shear. The shear stiffness allows the Invar mount pad's expansion to add to the mirror distortion. The RTV has very low in-plane shear stiffness so little of the pad expansion affects the

mirror. The cure shrinkage of the stiffer epoxy causes more mirror distortion than the softer RTV, which has more shrinkage.

The natural frequency table shows that the biggest difference appears in modes that involve shearing of the bond (modes 1, 2, and 4). Modes 1 and 2 are important because of LOS jitter effects. The mirror-torsion mode (4) is not significant because it has little impact on optical performance, and it will not be easily excited by standard loads. Because optical performance is the primary concern, RTV is chosen for the design.

13.10 Gravity Static Performance

Because all systems must be tested in a 1-g environment, the predicted 1-g static performance in typical test orientations must be analyzed. The Nastran FEA results for various orientations were analyzed in SigFit to create Zernike coefficient tables and Zemax files for rigid-body motion and Zernike coefficients. The results for PM due to 1-g +z are given in Table 13.2.

The raw FE displacements are dominated by rigid-body effects. If rigid-body motion is subtracted, the resulting surface is shown in Fig. 13.13(a) on the FE model. There is significant quilting, which cannot be represented by polynomials. For further optical processing, the FE data is interpolated in SigFit to a rectangular array (101×101) in interferogram format, as shown in Fig. 13.13(b). This data is in a form that may be directly input into optical-design programs, and it may be directly compared to experimental interferograms for correlation, or used as “backouts” to subtract 1-g effects for on-orbit predictions⁵ from optical tests.

A complete fringe Zernike fit to the axial gravity case is given in Table 13.14, where the wavelength is 23.6 micro-inches (0.60 microns).

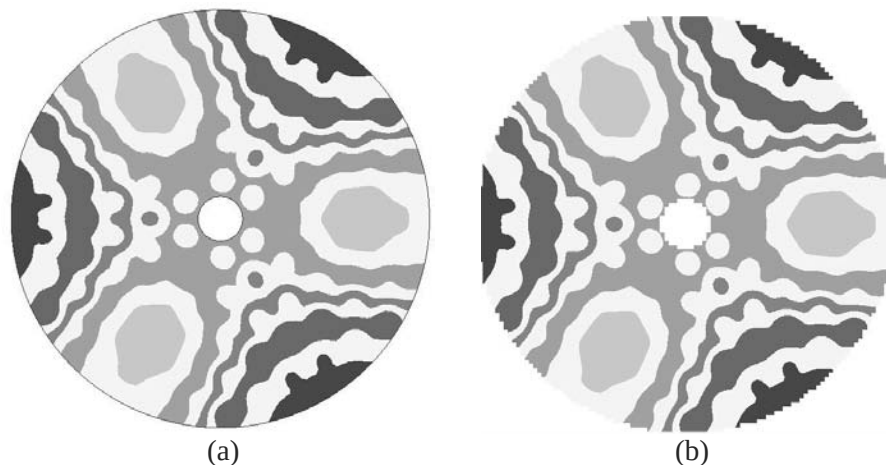


Figure 13.13 Contours of z displacement for 1-g +z: (a) contours on FE model and (b) interpolated grid array.

The quilting seen in the detailed PM model could not be predicted in the equivalent stiffness model that did not have individual cells modeled. However, the quilting RMS can be predicted from the equations given in the mirror modeling chapter. The high-order quilting may be assumed to be independent of the fringe Zernikes, which allows the quilting RMS to be combined with equivalent stiffness RMS by an RSS technique. The residual from the equivalent stiffness model after all Zernikes are subtracted is 0.028λ , which represents higher-order mount errors. If this is combined via RSS with the quilting RMS of 0.022λ , the combined result is 0.35λ . This compares closely to

Table 13.14 Fringe Zernike fit to axial gravity case using detailed PM model.

Order			Aberration	Magnitude (Waves)	Phi (deg)	Residual RMS	Residual P-V
K	N	M					
			Input(wrt zero)			0.2575	1.0720
1	0	0	Bias	0.00066	0.0	0.2575	1.0720
2	1	1	Tilt	0.00000	7.0	0.2575	1.0720
3	2	0	Power (Defocus)	-0.14494	0.0	0.2431	0.9582
4	2	2	Pri Astigmatism	0.00034	1.0	0.2431	0.9580
5	3	1	Pri Coma	0.00010	176.0	0.2431	0.9581
6	4	0	Pri Spherical	-0.06900	0.0	0.2414	0.9481
7	3	3	Pri Trefoil	0.62913	0.0	0.0904	0.4241
8	4	2	Sec Astigmatism	0.00011	-88.6	0.0904	0.4241
9	5	1	Sec Coma	0.00005	5.0	0.0904	0.4241
10	6	0	Sec Spherical	0.05030	0.0	0.0884	0.4116
11	4	4	Pri Tetrafoil	0.00008	-42.5	0.0884	0.4116
12	5	3	Sec Trefoil	0.26873	-60.0	0.0416	0.1985
13	6	2	Ter Astigmatism	0.00006	-1.4	0.0416	0.1985
14	7	1	Ter Coma	0.00001	-50.6	0.0416	0.1985
15	8	0	Ter Spherical	0.01379	0.0	0.0415	0.2049
16	5	5	Pri Pentafoil	0.00002	7.3	0.0415	0.2049
17	6	4	Sec Tetrafoil	0.00004	3.4	0.0415	0.2049
18	7	3	Ter Trefoil	0.09661	0.0	0.0332	0.1691
19	8	2	Qua Astigmatism	0.00002	72.7	0.0332	0.1691
20	9	1	Qua Coma	0.00002	-179.3	0.0332	0.1691
21	10	0	Qua Spherical	-0.01465	0.0	0.0329	0.1687
22	12	0	Qin Spherical	-0.00108	0.0	0.0329	0.1687
RNORM: normalizing radius (FE units) = 1.4000E+01 Polynomials normalized to have unit magnitude at RNORM Fit: axial displacement (dz) vs. radial position (r) Displ: ALL R-B subtracted prior to polynomial fit							

the residual RMS at the bottom of the Zernike table (0.033λ) for the detailed mirror model.

There is often a requirement to determine performance over a subaperture, commonly called a cookie [as in cutting cookies from a large (full-aperture) piece of cookie dough]. Fig. 13.14 shows the full front surface of the detailed primary mirror with three cookies shown. Fig. 13.15(a) is the full surface normal deformation in 1 g on the three-point mount. Figs. 13.15(b)–(d) show the same deformation within the cookie apertures. The cookie apertures can be fit with polynomials or interpolated to grid arrays for representation in an optics program. In SigFit, each cookie aperture is treated as a surface with its own local fitting coordinate system and aperture. Since nodes can belong to multiple surfaces in SigFit, the cookie analysis requires no special operation. There must be enough nodes in any cookie to fit the desired polynomial order.

13.11 Thermo-Elastic Performance

Thermo-elastic distortions are important both during on-ground test and during on-orbit conditions. The FE model includes coefficient of thermal expansions (CTEs) for all materials and rigid elements. A very important issue in processing thermo-elastic performance is the radial correction of axial displacements as discussed in Chapter 3.

In this example, an isothermal change of $+10^\circ\text{C}$ is run in Nastran. This is a linear analysis, so it is scalable to any other isothermal change. Because the radius of curvature (RoC) is negative, the initial power is negative. With radial correction, the power change in the PM due to $+10^\circ\text{C}$ is positive, becoming less negative in total as the mirror flattens, as in Fig. 13.16(a). Without radial correction as discussed in Chapter 4, the z displacements in Fig. 13.16(b) predict the power change is negative, meaning that the mirror becomes more curved. It is well known that an increase in temperature should flatten the mirror, which agrees with the radially corrected displacements. Zernike coefficients are given in Table 13.15.

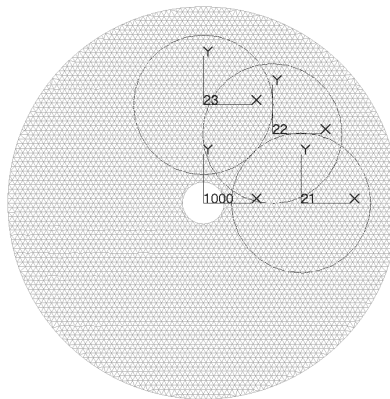


Figure 13.14 Full aperture and cookies.

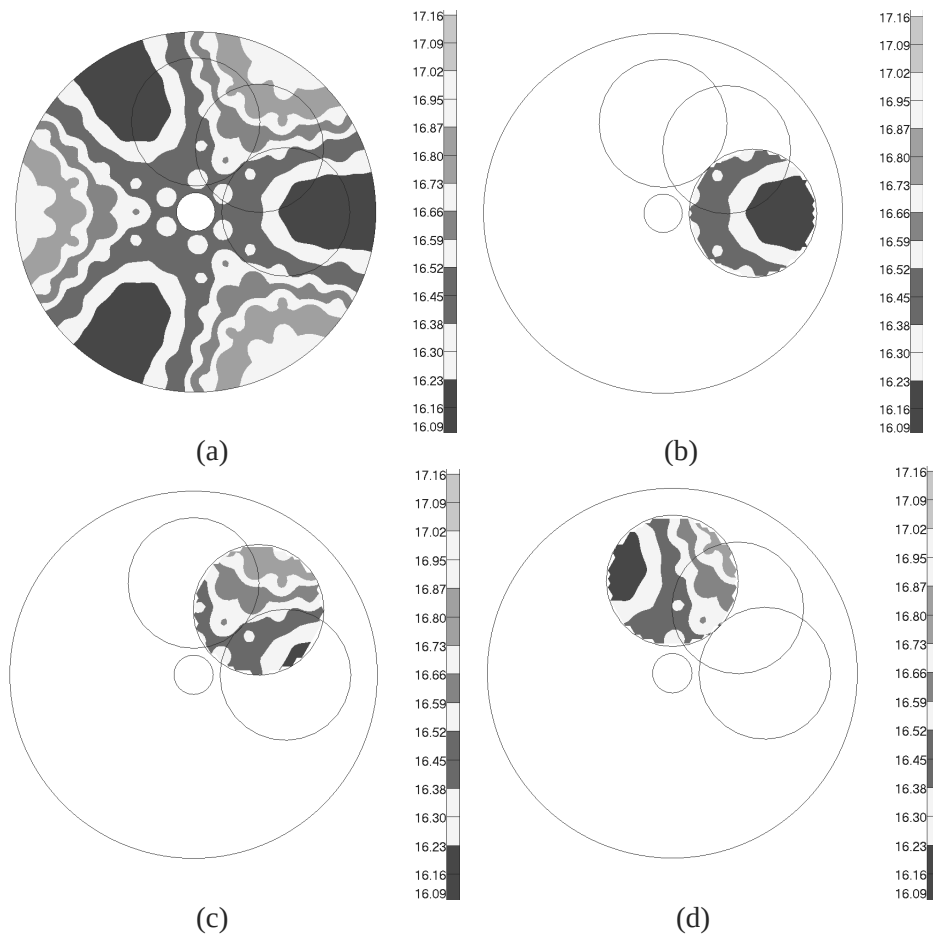


Figure 13.15 Surface distortion over the full aperture and 3 cookies.

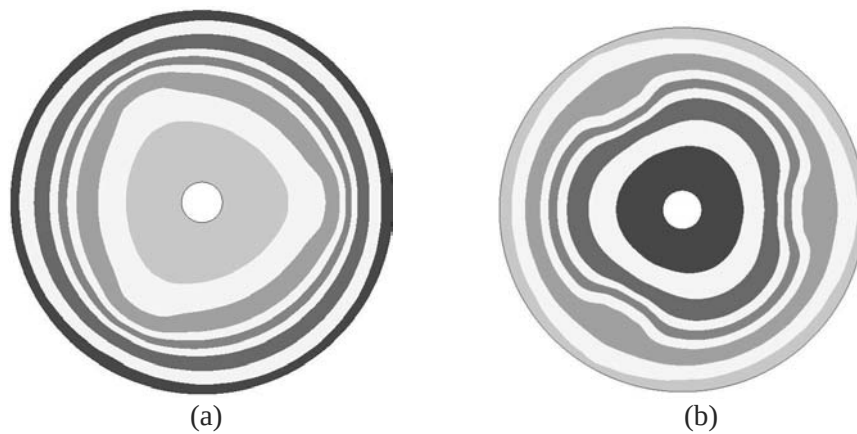


Figure 13.16 z displacement contours for $+10\text{ }^{\circ}\text{C}$ (a) without radial correction high in the center and (b) with radial correction low in the center.

Table 13.15 Fringe Zernike fit to +10 °C isothermal case using detailed PM model.

----- EXAMPLE TELESCOPE MODEL ISOTHERMAL +10°C -----						
Optic-Id = 2			Optic Label = PM		Wavelength = 2.3622E-05 in	
Order	Aberration		Magnitude (Waves)	Phi (deg)	Residual RMS	Residual P-V
K	N	M				
			Input(wrt zero)			0.0464 0.1694
1	0	0	Bias	-0.00103	0.0	0.0464 0.1694
2	1	1	Tilt	0.00000	68.4	0.0464 0.1694
3	2	0	Power (Defocus)	0.08058	0.0	0.0070 0.0373
4	2	2	Pri Astigmatism	0.00001	64.3	0.0070 0.0373
5	3	1	Pri Coma	0.00001	-119.7	0.0070 0.0373
6	4	0	Pri Spherical	-0.00845	0.0	0.0060 0.0306
7	3	3	Pri Trefoil	0.00309	0.0	0.0059 0.0307
8	4	2	Sec Astigmatism	0.00000	72.2	0.0059 0.0307
9	5	1	Sec Coma	0.00001	59.5	0.0059 0.0307
10	6	0	Sec Spherical	0.00437	0.0	0.0056 0.0306
11	4	4	Pri Tetrafoil	0.00000	-29.1	0.0056 0.0306
12	5	3	Sec Trefoil	0.01630	60.0	0.0031 0.0211
13	6	2	Ter Astigmatism	0.00001	-29.1	0.0031 0.0211
14	7	1	Ter Coma	0.00000	56.7	0.0031 0.0211
15	8	0	Ter Spherical	0.00197	0.0	0.0030 0.0227
16	5	5	Pri Pentafoil	0.00000	24.3	0.0030 0.0227
17	6	4	Sec Tetrafoil	0.00000	14.8	0.0030 0.0227
18	7	3	Ter Trefoil	0.00799	0.0	0.0022 0.0139
19	8	2	Qua Astigmatism	0.00000	50.4	0.0022 0.0139
20	9	1	Qua Coma	0.00001	-120.2	0.0022 0.0139
21	10	0	Qua Spherical	-0.00247	0.0	0.0021 0.0133
22	12	0	Qin Spherical	0.00024	0.0	0.0021 0.0132
RNORM: normalizing radius (FE units) = 1.4000E+01 Polynomials normalized to have unit magnitude at RNORM Fit: axial displacement (dz) vs. radial position (r) Displ: ALL R-B subtracted prior to polynomial fit						

13.12 Polynomial Fitting

Polynomial fitting is usually done as a postprocessing operation on FE-generated nodal displacements. In Section 13.4, the PM is optimized with a criterion being

Table 13.16 Comparison of Zernike coefficients calculated from MPC equations with those from a postprocessing fit of displacements.

Loadcase = 1g X				Fit to	Poly	
#	N	M	TERM	Displ	MPC	Diff
2	1	1	cos	-4.797E-05	-4.795E-05	-0.03%
5	2	2	cos	2.720E-01	2.720E-01	0.00%
7	3	1	cos	2.333E-02	2.333E-02	0.00%
12	4	2	cos	-7.340E-02	-7.340E-02	0.00%
14	5	1	cos	-4.437E-04	-4.433E-04	-0.08%
17	4	4	cos	-4.387E-02	-4.387E-02	0.00%
21	6	2	cos	1.912E-02	1.911E-02	0.00%
23	7	1	cos	-4.959E-03	-4.961E-03	0.04%
26	5	5	cos	2.855E-02	2.854E-02	0.00%
28	6	4	cos	4.430E-02	4.430E-02	0.00%
32	8	2	cos	4.322E-03	4.322E-03	0.00%
34	9	1	cos	1.336E-03	1.335E-03	-0.10%

Loadcase = +10C				Fit to	Poly	
#	N	M	TERM	Displ	MPC	Diff
1	0	0	cos	-1.056E-03	-1.056E-03	0.00%
4	2	0	cos	8.018E-02	8.018E-02	0.00%
9	4	0	cos	-8.725E-03	-8.726E-03	0.02%
10	3	3	cos	1.132E-03	1.131E-03	-0.15%
16	6	0	cos	4.673E-03	4.673E-03	0.00%
19	5	3	cos	-1.699E-02	-1.699E-02	0.00%
25	8	0	cos	2.169E-03	2.170E-03	0.08%
30	7	3	cos	8.342E-03	8.342E-03	0.00%
36	10	0	cos	-2.392E-03	-2.392E-03	-0.03%
37	12	0	cos	-1.533E-04	-1.532E-04	-0.06%

the residual RMS after best-fit-plane and power are removed. As a check on the accuracy of the polynomial equations (which were written in Nastran MPC format by SigFit), the MPC coefficients are compared to the conventional postprocessing fitting. The comparison of coefficients is shown at the top of Table 13.16 for the lateral gravity case, and at the bottom of Table 13.16 for the isothermal case. Numbers that are less than 1.0×10^{-5} waves are eliminated as noise. The comparisons show excellent correlation, even for large radial growth in the isothermal condition that requires radial correction. The downside of the MPC coefficients is that there is no indication of how good the polynomial fit is because the residual RMS is not calculated as with the postprocessing fitting routine.

13.13 Assembly Analysis

When the PM is attached to the PM mount structure, some residual strain is locked in. The PM is supported at three points on the outer diameter in a 1-g

field, which causes some distortion of the optic. The PM mount is supported at the three corners of the delta frame in the same 1-g field. When the two structures are brought together, epoxy is applied to the flexure–mount pad joint, bonding the structures in their deformed geometry. In the Nastran model, the joining process is achieved by turning on a set of connecting MPC equations. To determine the locked-in strain, the edge support condition on the PM is removed, and gravity is turned off. The resulting PM surface distortions represent the locked-in mount strain that would be seen on-orbit. For assembly in a vertical configuration, the locked-in RMS is only 0.0002λ . If the optical axis is horizontal during assembly, the locked-in RMS is 0.0194λ . Contours of both locked-in distortions is given in Fig. 13.17.

13.14 Other Analyses

The above analyses are peculiar to an optical system. There are several other analyses that are required for any spacecraft,⁶ which are not documented here. The most obvious analysis required is stress under launch loads. This may be treated as an envelope of static load cases, which encompass the dynamic loads. The static loads may involve hundreds of load cases for different g-levels in different directions with temperature extremes for lift-off, maximum acoustic loads, and stage-1 separation, to name a few. The advantage of the static-load approach is that the mathematics are real (not complex), and equivalent stresses (Von Mises and Max Principle) are easily calculated. Composite materials have several failure modes measured by a failure index. The processing of the many static loads can be automated by special postprocessing software.

The alternative to a static-load envelope is a series of dynamic analyses, including transient, harmonic, and random. It can be difficult to calculate all of the various failure criteria in these dynamic analyses. Acoustic loads and shock loads require special techniques, as discussed in Chapter 7.

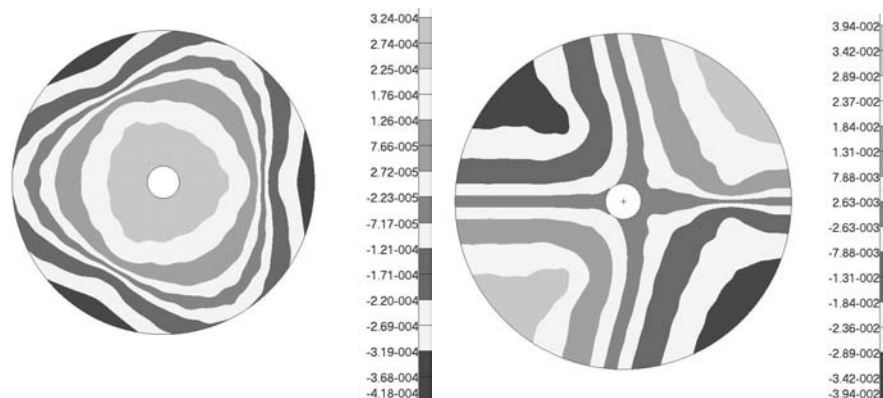


Figure 13.17 Locked-in surface deformations in waves for assembly with optical axis vertical (left) and horizontal (right).

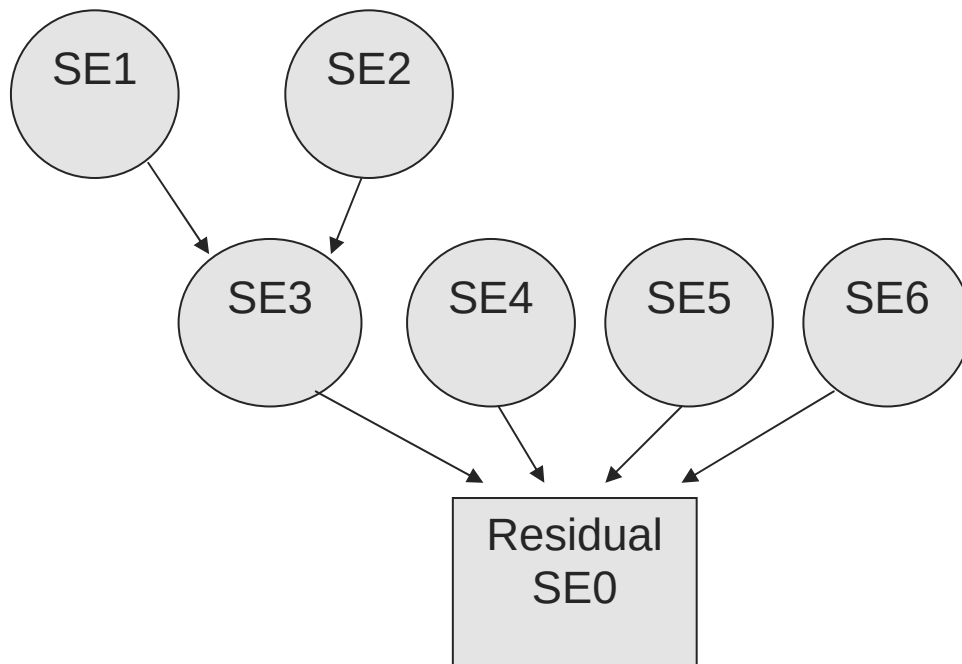


Figure 13.18 Superelement tree.

Thin flexures require a buckling analysis to verify survivability. Because eigenvalue buckling calculates an upper bound on buckling load (non-conservative), a large safety factor (2.0 to 4.0) must be used on flexure buckling.

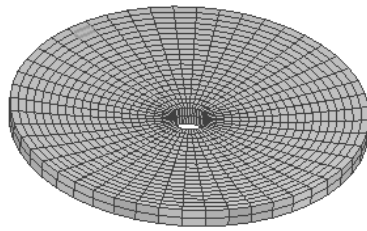
Additional analyses must be performed for handling, transportation, and storage loads and may involve extreme temperatures.

13.15 Superelements

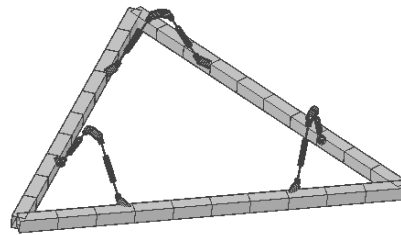
This example telescope shows how conveniently an optical system can be broken into superelements. The SE tree is shown in Fig. 13.18, and the corresponding plots are shown in Fig. 13.19.

- SE1: Primary mirror and mount pads that connect to the bipod mount at a single node for each bipod. There can be a SE for the coarse equivalent stiffness model, which can be replaced by the detailed mirror model at any time.
- SE2: Primary mirror flexures and delta frame that connects to the metering structure at three nodes.
- SE3: The primary mirror assembly (PMA) is the combination of SE1 and SE2.
- SE4: The secondary mirror and mount connects to the metering structure at three nodes.

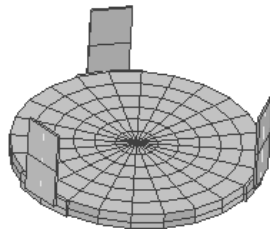
- SE5: The aft metering structure that supports the detector and other instruments is connected to the metering structure at three nodes.
- SE6: The metering structure connects to other SE at a minimal number of nodes.
- SE0: The residual structure is just the interface face nodes at which the SE join, including the attachment to the spacecraft.



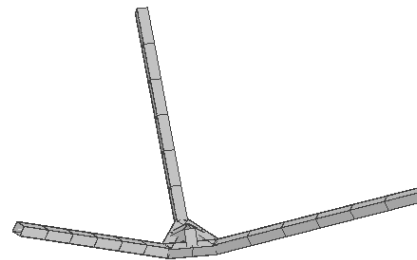
SE1 = PM with pads
3 attach nodes to mount



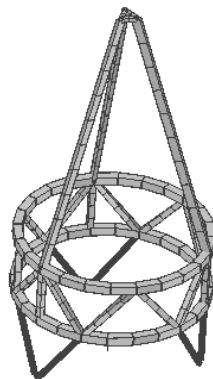
SE2 = PM Mount
3 attach nodes to PM
3 attach nodes to frame



SE4 = SM
3 attach nodes to frame



SE5 = Aft metering
3 attach nodes to frame



SE6 = Metering 12 attach nodes
3 each to PM mount: SM, aft metering, base

Figure 13.19 Plots of superelements.

As each component gets an updated design, it can replace the corresponding SE matrix in the analysis. Only the SEs that are below the replacement SE in the SE tree need to be re-run. There are many other ways that the telescope can be broken into superelements. In static analysis, SEs are exact. In dynamic analysis, however, SEs are approximate, but the approximation can be excellent if component mode synthesis is used.

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◀Chapter 14▶

Integrated Optomechanical Analyses of a Lens Assembly

Thermal, structural, and optical analyses are performed to predict the optical performance of a double Gauss and a seven-element lens assembly due to on-axis heat loads. In the double Gauss lens assembly, optical performance is computed as a function of power, and in the seven-element lens assembly, optical performance is computed as a function of time for a fixed power.

14.1 Double Gauss Lens Assembly

Thermal, structural, and optical modeling tools are used to predict the optical performance of a double Gauss lens assembly subject to an on-axis heat load as shown in Fig. 14.1. System specifications are listed in Table 14.1. A fraction of the heat load is absorbed by the optical elements via bulk volumetric absorption, resulting in thermal gradients in the lens assembly. The on-axis wavefront error, point spread function, modulation transfer function, and the change in focus is computed for incident powers of 40, 80, 120, 160, and 200 W. The surface numbering for the double Gauss lens is shown in Fig. 14.2.

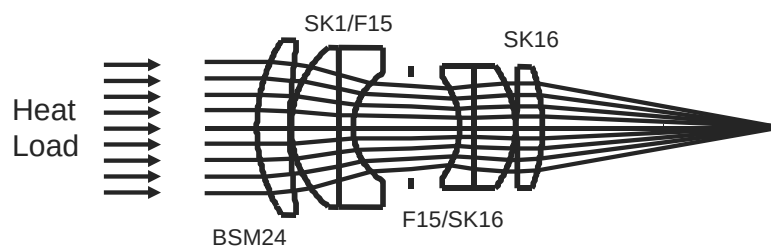


Figure 14.1 Double Gauss lens assembly.*

Table 14.1 Double Gauss lens assembly specifications.

OPTICAL SPECIFICATIONS	
Wavelength	587 nm
EPD	25 mm
f_{eff}	100 mm
$F/\#$	4.0
Housing	SS416

* U. S. Patent 2,532,751.

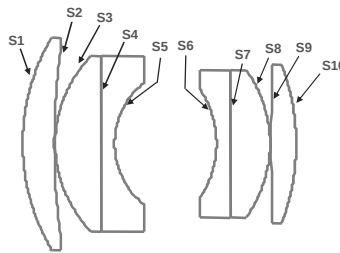


Figure 14.2 Double Gauss surface numbering.

14.1.1 Thermal analysis

A thermal analysis is performed to compute the temperature distribution in the lens assembly due to the bulk volumetric absorption of the heat load. A two-step modeling effort was conducted using the thermal analysis software Thermal Desktop. First, a heat-rate model was developed to compute the energy absorbed. A heat source was defined in the heat-rate model to emit a parallel radiation flux, and a mask was used to set the clear aperture for the lens assembly. Multiple surfaces of zero thickness with the appropriate indices of refraction were then defined for each lens element. The surface absorption coefficients were radially varied to account for the absorption characteristics of each glass and the energy distribution of the heat source. The radiation flux was varied to yield the desired incident power. The resulting heat rates were then used in a steady-state thermal analysis to compute the temperature distribution for powers of 40, 80, 120, 160, and 200 W. The resulting temperature distribution is a radial gradient with a slight axial variation due to surface effects as shown in Fig. 14.3. The approximate radial gradient as a function of power is listed in Table 14.2. The temperatures were subsequently mapped to a MSC/Nastran finite element model using the shape function interpolation algorithm in Thermal Desktop, as demonstrated in Fig. 14.4.

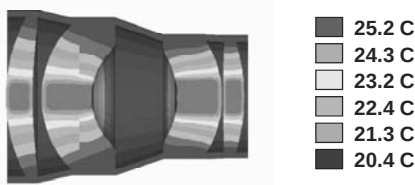


Figure 14.3 Temperature distribution due to heat load of 200 watts.

Table 14.2 Radial gradient as a function of heat load.

POWER (W)	RADIAL GRADIENT (°C)
40	1
80	2
120	3
160	4
200	5

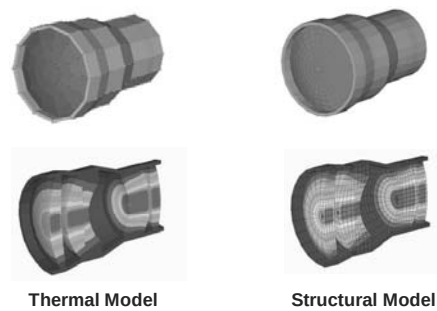


Figure 14.4 Temperature mapping using shape function interpolation.

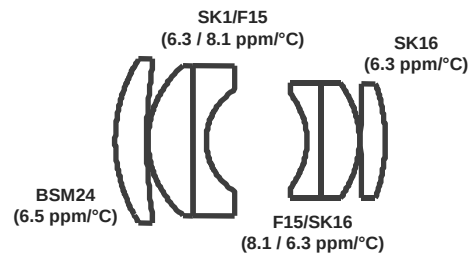


Figure 14.5 CTEs of optical glasses and housing.

14.1.2 Thermo-elastic analysis

A thermo-elastic analysis was performed using the finite element model to compute the rigid-body motions of the optical surfaces, the higher-order surface deformations, and the mechanical stress in the optical elements. The coefficient of thermal expansions for the optical glasses and housing materials are shown in Fig. 14.5. Due to the rotational symmetry of the system including the geometry and loading, aspheric polynomials were used to represent the higher-order surface deformations. An exaggerated view of the resulting deformed shape is shown in Fig. 14.6. The perturbed surface shapes are listed in Table 14.3, where ρ is the vertex curvature, and A, B, C, and D are the aspheric coefficients. The change in shape of the center surface for each of the two cemented doublets is ignored in this analysis (surfaces 4 and 7). The higher-order surface departure is plotted as a function of radius in Fig. 14.7.

Table 14.3 Perturbed optical surface shape for 200-W load.

PERTURBED OPTICAL SURFACE SHAPE					
<u>SURF</u>	<u>ρ</u>	<u>A</u>	<u>B</u>	<u>C</u>	<u>D</u>
S1	0.017410	-4.2E-09	-3.8E-11	1.7E-14	-6.1E-17
S2	0.005302	1.5E-08	-1.8E-11	-2.2E-14	5.5E-17
S3	0.028673	-6.3E-09	-5.0E-11	-6.6E-16	-7.0E-17
S5	0.046570	3.6E-08	-1.1E-10	-4.1E-13	9.7E-16
S6	-0.036980	-3.2E-08	7.3E-11	1.0E-12	-6.0E-15
S8	-0.028593	9.7E-09	9.7E-11	-2.0E-13	1.2E-15
S9	0.001709	-1.5E-08	-8.2E-12	2.6E-13	-1.2E-15
S10	-0.015848	6.3E-09	6.5E-11	-2.8E-13	1.4E-15

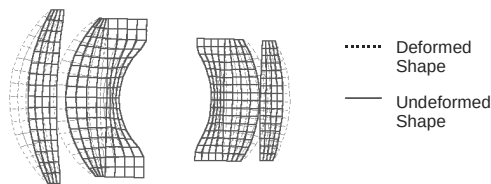


Figure 14.6 Double Gauss deformed and undeformed FEA results.

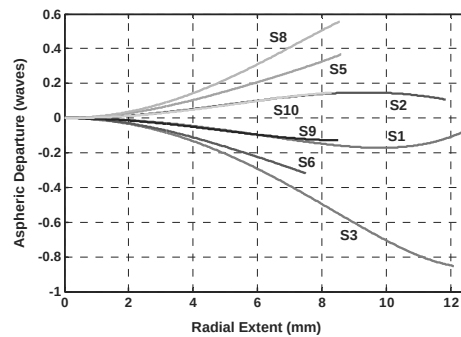


Figure 14.7 Optical surface departure.

Table 14.4 Stress-optical coefficients.

STRESS-OPTICAL COEFFICIENTS ($\times 10^{-8} \text{ IN}^2/\text{LB}_F$)		
GLASS TYPE	$-K_{11}$	$-K_{12}$
BSM24	0.55	2.00
SK1	0.48	2.07
F15	1.65	3.65
SK16	0.69	1.92

14.1.3 Stress birefringence analysis

The heat load produces stress in each of the optical elements, which creates wavefront error in the lens assembly. SigFit software is used to create OPD maps representative of the stress-induced wavefront error for each of the optical elements. This data is subsequently fit to Zernike polynomials and formatted into Code V wavefront interferogram files. The stress-optical coefficients for each of the glass types are listed in Table 14.4.

14.1.4 Thermo-optic analysis

A thermo-optic analysis is performed to compute the wavefront error due to index changes. The relative thermo-optic coefficients were used to simplify the analysis and are displayed with the temperature distribution for each of the optical elements in Fig. 14.8. In this case, the air is assumed to be the same temperature as the lens, which is an approximation. (Use of the absolute thermo-optic coefficient coupled with specifying the index of refraction of the

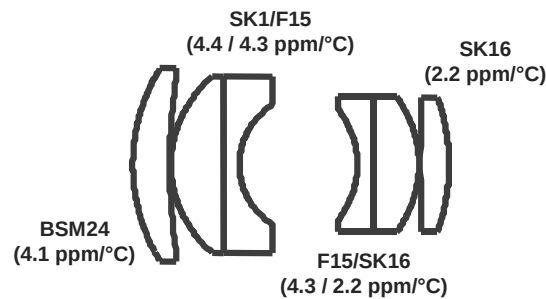


Figure 14.8 Thermo-optic coefficients for each glass.

surrounding air provides for greater accuracy, as discussed in Section 9.3.) SigFit software was used to compute OPD maps for each element by incrementally summing the OPD through the lens elements. The OPD maps are fit to Zernike polynomials and used to create Code V wavefront interferogram files.

14.1.5 Optical analysis

An optical analysis was performed using Code V to compute optical performance as a function of incident power. For each heat load, the mechanical perturbations were applied to the optical model. Each optical surface was repositioned along the optical axis, and the higher-order surface deformations were represented as aspheric surfaces. Mechanical stress and thermo-optic effects were accounted for in the optical model using wavefront interferogram files. On-axis performance of the lens assembly was then computed for heat loads of 40, 80, 120, 160, and 200 watts.

The change in focus of the lens assembly δ , as illustrated in Fig. 14.9, is listed as a function of heat load in Table 14.5. Peak-to-valley and RMS wavefront error as a function of power is listed in tabular form using the dominant Zernike terms in Table 14.6 and shown graphically using interferogram plots in Fig. 14.10. In an interferogram plot, the wavefront error or OPD is calculated by the number of fringes. A fringe is comprised of both a bright and dark band and represents a wave of error. The primary effect of the heat load is to create a focus error in the system.

A comparison of the wavefront error produced by the individual physical effects including thermo-elastic deformations, mechanical stress, and thermo-optic effects for the 200-W heat load is shown using interferogram plots in Fig. 14.11.

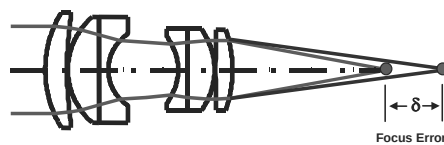


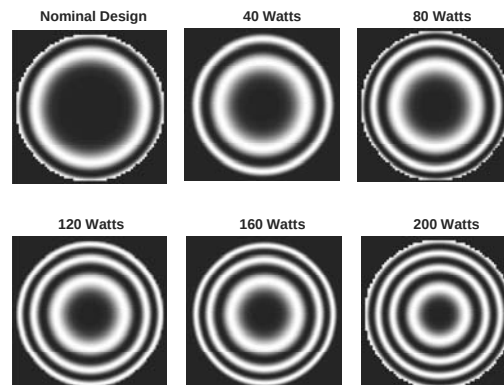
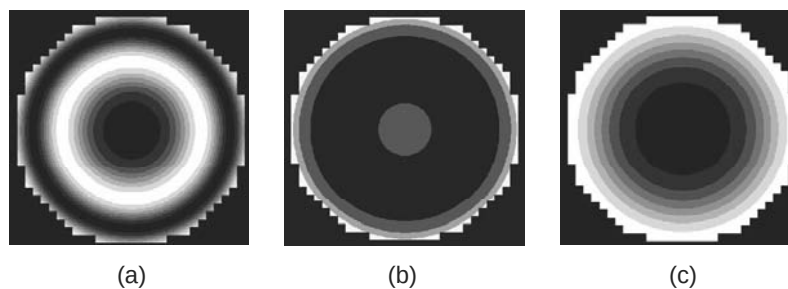
Figure 14.9 Heat-load-induced focus error.

Table 14.5 Focus error.

FOCUS ERROR δ	
LOAD CASE	ΔF (μm)
40 W	41
80 W	75
120 W	101
160 W	116
200 W	151

Table 14.6 Wavefront error fit to Zernike coefficients.

WAVEFRONT ERROR FRINGE ZERNIKE COEFFICIENTS					
LOAD CASE	PISTON	FOCUS	SPHERICAL	RMS	P-V
Nominal	0.56	0.82	0.25	0.48	1.6
40 W	0.92	1.09	0.16	0.63	2.2
80 W	1.10	1.32	0.21	0.76	2.6
120 W	1.38	1.50	0.11	0.84	3.9
160 W	1.46	1.59	0.12	0.92	3.2
200 W	1.91	1.83	-0.10	1.10	3.6

**Figure 14.10** Interferogram plots of wavefront error as a function of heat load.**Figure 14.11** Individual contributions to system wavefront error using interferogram plots for 200-W load: (a) thermo-elastic effects, (b) mechanical-stress effects, and (c) thermo-optic effects.

The results indicate that, for this example, the thermo-elastic effects contribute approximately three times the wavefront error as the thermo-optic effects. The stress-induced wavefront error represents a small fraction of the total wavefront error.

The effect of the heat loads on the PSF and the MTF is shown in Figs. 14.12 and 14.13, respectively. As the heat load is increased, the blur diameter of the PSF increases and the MTF cutoff frequency decreases.

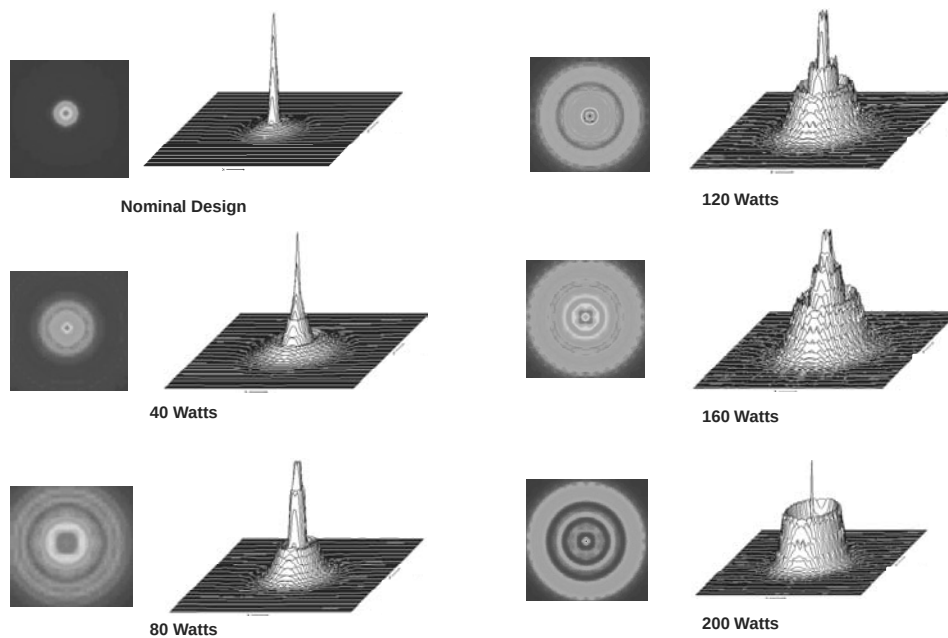


Figure 14.12 PSF as a function of heat load.

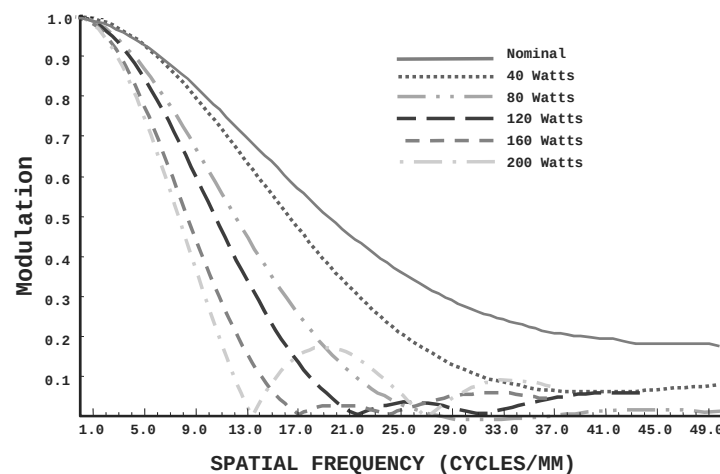


Figure 14.13 MTF as a function of heat load.

14.2 Seven-Element Lens Assembly

Thermal, structural, and optical analyses are performed to predict the transient on-axis optical performance of a seven-element lens assembly (Japan patent 61_6363 860226) subject to an on-axis heat load as shown in Fig. 14.14. The lenses are mounted in an aluminum housing with 60 W incident on the front lens. Optical performance is computed using a wavelength of 546 nm. It is assumed that the optical coatings absorb 0.1% of the incident energy on each surface and that each lens absorbs a fraction of the transmitted energy. The total bulk volumetric absorption of the optical glasses is 3.25%. A finite element model is developed for both the thermal and structural analyses. A thermal analysis is performed that computes the lens-assembly temperature distribution at four time steps (T1, T2, T3, T4), as shown in Fig. 14.15. A 9.0 °C radial gradient is experienced by the lens assembly at time step T4.

The four temperature distributions are used in a thermal elastic analysis to compute the resulting displacements and mechanical stress. The optical element displacements are separated into rigid-body errors and higher-order elastic-surface errors that are fit to Zernike polynomials and used to create new optical surfaces using the Zernike polynomial surface definition. OPD maps are created for each optical element at each time step due to the thermo-optic and stress-optic effects. These perturbations were added to the optical model using wavefront interferogram files.

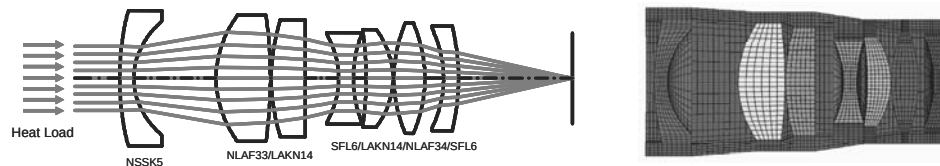


Figure 14.14 Optical and finite element models of a seven-element lens assembly.

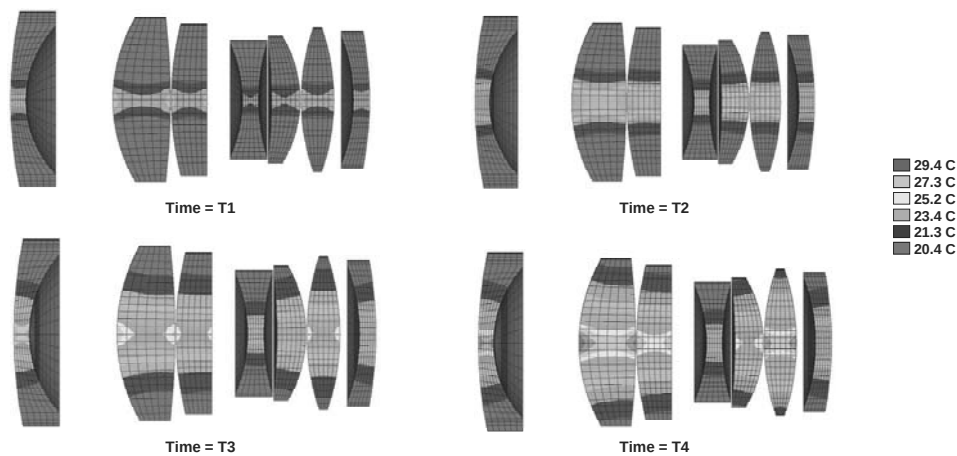


Figure 14.15 Temperature plots at four time steps in the seven-element lens assembly.

The effects of temperature on wavefront error for each time step are shown as interferogram files in Fig. 14.16 produced using Code V. A comparison of the wavefront error produced by the individual physical effects including thermo-elastic deformations, mechanical stress, and thermo-optic effects at time step T4 is shown using interferogram plots in Fig. 14.17. In this example, the thermo-elastic effects and the thermo-optic effects are approximately equal. The stress-induced wavefront error again represents a small fraction of the total wavefront error. Once a ‘perturbed’ optical model is created, any optical performance metric that is supported by the optical design code may be evaluated. For this example, in addition to wavefront error, the impact of the heat loads on optical resolution is shown in Fig. 14.18.

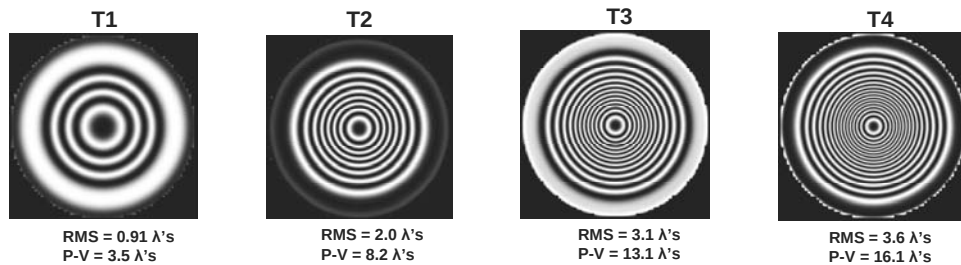


Figure 14.16 Wavefront error represented using interferogram files as a function of time.

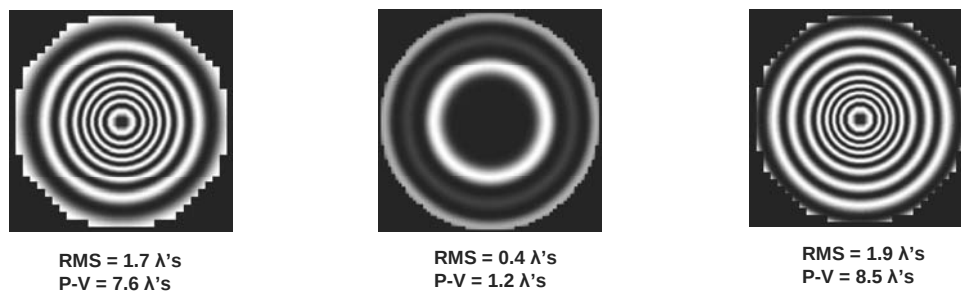


Figure 14.17 A comparison of the wavefront error contributions for thermo-elastic effects only (left), the effects of mechanical stress only (center), and thermo-optic effects only (right).

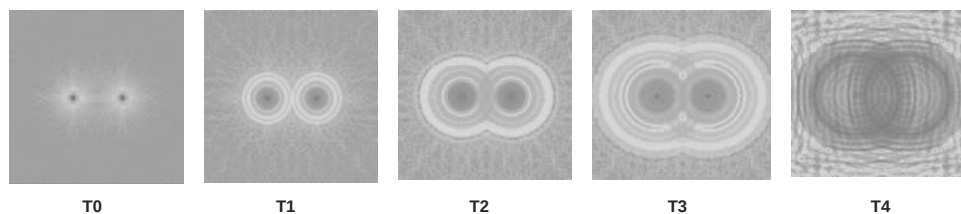


Figure 14.18 Optical resolution as a function of temperature.

Index

- actuator influence functions, 303
- adaptive optical system, 301
- adhesive bonds, 151, 153, 159
- air bags, 182
- Airy disk, 45
- aspheric polynomials, 77
- assembly, 174, 195–198
- augment actuators, 307
- automeshing, 107
- Beer's law, 293
- bending moment of inertia, 111, 118
- biaxial, 267
- blur diameter, 44
- bulk volumetric absorption, 293, 294, 371, 372
- cell shapes, 108, 110
- cell size, 109
- coating-cure shrinkage, 138, 141, 143
- coating-moisture absorption, 138
- coatings, 138
- coefficient of moisture expansion, 298
- coefficient of thermal expansion, 281, 290
- correctability, 303–307
- cut-off frequency, 49
- damping, 201, 245
- Delaunay triangulation, 94
- design optimization, 113, 116, 314, 317, 319, 322
- design sensitivity, 327, 332
- diffraction, 44, 58, 59, 61
- diffraction-limited depth-of-focus, 45, 287
- effective properties, 109, 119, 153, 154, 155, 161
- elasticity, 4
- electric field vector, 38, 39, 269
- encircled energy, 47, 57, 58, 59
- equivalent stiffness, 108, 109, 112–122
- f*-number, 45
- failure theories, 251
- Fick's law, 297, 298
- flexures, 118, 162–164, 167, 169, 172–176, 180, 195–197
- focus error, 286, 287, 375, 376
- Fourier's law, 297
- frequency domain, 51
- Grid Sag surface, 94
- hockey-puck bond, 154, 157, 158
- Hooke's law, 105, 115, 119, 149, 154, 156
- image contrast, 49
- image jitter, 221
- image motion, 333
- impulse response, 50
- incompressible bonds, 147, 198
- index ellipsoid, 266
- index of refraction, 37, 87, 269, 283, 293
- interferogram, 360
- interferogram files, 92, 276, 289, 374, 375
- isotropic materials, 5, 6
- kinematic mounting, 164
- Legendre–Fourier polynomials, 76
- lightweight mirror, 108
- mass density, 109, 112, 115, 117
- material coordinate system, 116, 157, 159
- maximum modulus, 149–151, 155, 159
- membrane thickness, 110, 112, 118
- modal analysis, 203
- mode shapes, 25
- model checkout, 28
- modulation transfer function, 48, 371

- Mohr's circle, 9
mounts, 114, 147, 162–167, 172, 181, 188, 195, 196
multidisciplinary design
 optimization (MDO), 336
natural frequencies, 199, 200, 354
neutral plane, 109, 110, 112, 118, 166, 172
nonlinear programming, 327, 329
nonstructural mass, 112, 119
obscuration, 57
optical frequency, 38
optical path difference (OPD), 40, 290
optical path length (OPL), 40
optical transfer function, 51
optimization, 327, 329, 332, 333, 335–339, 351
opto-thermal expansion coefficient, 285
orthotropic materials, 7
phase transfer function, 53
phase, 38
plane strain, 8
plane stress, 6, 7
point spread function (PSF), 46, 371
polarization, 38, 270, 273, 275, 276
principal stress, 9
pseudo-kinematic mounting, 164
quilting, 102, 112, 334
radius of curvature (ROC), 85, 97
random response, 209, 210, 352
rays, 40
redundant mounting, 164
resolution, 47
rigid-body error, 29, 81
rigid-body motion, 101–104, 113, 116, 163, 164
ring bonds, 161, 162
roller-chain test supports, 189, 190
shape function, 17
shape function interpolation, 100, 296
shape optimization, 116
single-point model, 103
sling test supports, 189
solid optics, 104
solidity ratio, 110, 118
spatial domain, 51
spot diagrams, 46, 99
stress analysis, 249
stress birefringence, 265, 374
stress-optical coefficient, 267–271, 276
structural analysis, 21
surface deformation, 101, 102, 138, 139, 174, 175
surface effects, 137–141
symmetry, 24, 28
tangency, 182–187
test supports, 181, 182, 189
thermal analysis, 22, 23
thermal-glass constant, 285, 286
thermal soak, 30
thermal strain, 280
thermo-optic coefficient, 283, 284, 290
thermo-optic constant, 288
thermoelastic expansion, 103, 174
three-dimensional element models, 105
transverse shear factor, 104
two-dimensional models, 104
Twyman effect, 138, 141, 145
uniaxial, 267
unstable mounting, 163
V-block, 189
vibrations, 199
wavefront, 40
wavefront error, 40, 88, 270, 274, 276, 371, 374–376, 379
wavelength, 38
X-Y polynomials, 74, 77
Zernike polynomials, 1, 63, 64, 89–91, 97, 290, 333



Keith Doyle has over 25 years of experience in the field of optomechanical engineering, working on a diverse range of high-performance optical instruments specializing in the multidisciplinary analysis and integrated modeling of optical systems. He is currently a Group Leader in the Engineering Division at MIT Lincoln Laboratory. He previously served in a variety of roles including Vice President of Sigmadyne, Inc., Senior Systems Engineer at Optical Research Associates, and a Structures Engineer at Itek Optical Systems. He received his Ph.D. from the University of Arizona in Engineering Mechanics with a minor in the Optical Sciences in 1993, and he holds a BS degree from Swarthmore College received in 1988. Dr. Doyle is an active participant in SPIE symposia, teaches short courses on optomechanics and integrated modeling, and has authored and co-authored over 30 technical papers in optomechanical engineering.



Dr. Victor Genberg PE has over 45 years of experience in the application of finite element methods to high-performance optical structures, and is a recognized expert in optomechanics. He is currently President of Sigmadyne, Inc. Prior to starting Sigmadyne, Dr. Genberg worked at Eastman Kodak for 28 years, serving as a technical specialist for commercial and military optical instruments. He is an author of SigFit, a commercially available

software product for optomechanical analysis. Dr. Genberg is also a full professor (adjunct) of Mechanical Engineering at the University of Rochester, where he teaches a variety of courses in finite elements, design, optimization, and optomechanics. He has over 50 publications. He received his Ph.D. from Case Western Reserve University in 1973.



Gregory Michels PE has worked for twenty years in optomechanical design and analysis, and is currently Vice President of Sigmadyne, Inc. He received his MS degree in Mechanical Engineering from the University of Rochester in 1994. He specializes in finite element analysis and design optimization of high-performance optical systems. Mr. Michels is also a software developer and technical support engineer for Sigmadyne's optomechanical analysis

software product, SigFit. Prior to co-founding Sigmadyne, he worked at Eastman Kodak for five years as a structural analyst on the Chandra X-Ray Observatory. Mr. Michels has authored or co-authored over 25 papers in the field of integrated optomechanical analysis and teaches short courses on finite element analysis and integrated modeling.